Particle resonances' domination over parametric instabilities and their mitigation

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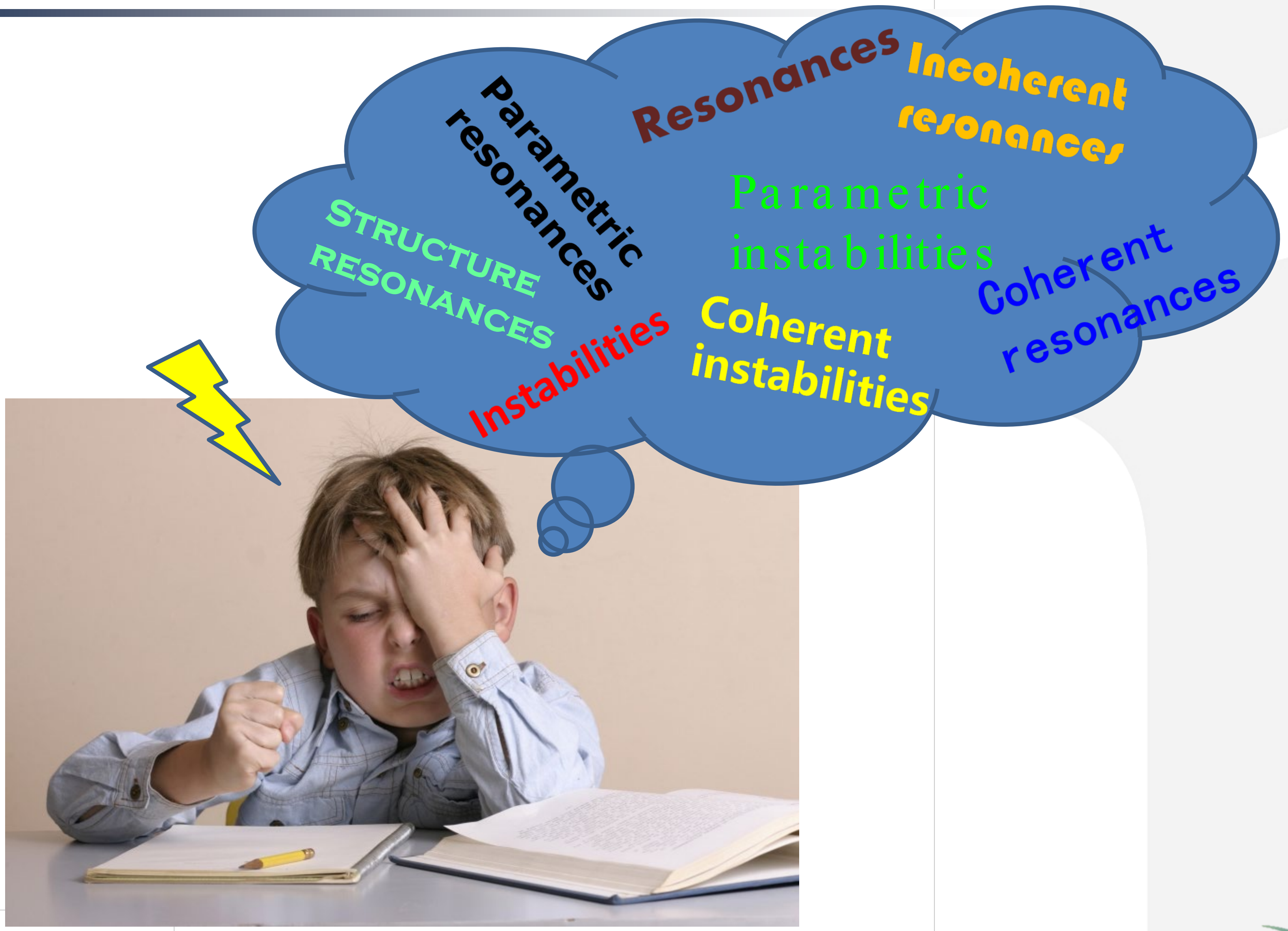
Drowned in a swamp of terms…?



Space-charge halo mechanisms

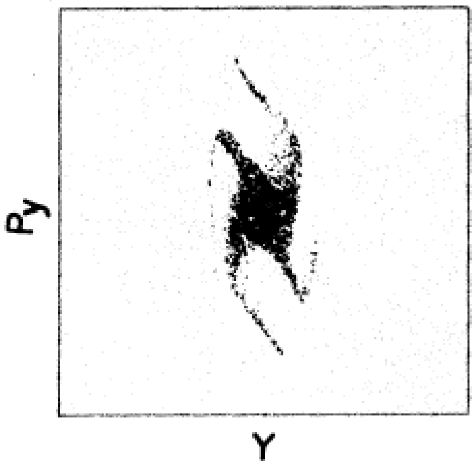
- There are two families of space-charge mechanisms and yet they need to be differentiated:
  (particle) resonances and parametric instabilities.

  - Particle resonances: a.k.a. single particle resonances, incoherent resonances …
  - Parametric instabilities: a.k.a. parametric resonances, mode instabilities, structure resonances, coherent resonances, coherent instabilities …

- They are totally different mechanisms.

Instabilities or Parametric Instabi

- 1959, Kapchinskij and Vladimirskij found a self-consist intensity beam distribution (called "KV distribution") and envelope equation: $x'' + k(s)x - \frac{\varepsilon^2}{x^3} - \frac{K(s)}{x} = 0.$



- 1979, Haber et al. found an instability of KV distributior
- 1983, Hofmann derived various instabilities of KV distributions from Vlasov-Poisson equations.
  - called "instabilities" in the paper (instabilities of beam eigenmodes)
  - showed that phase advance above 90° should be avoided
- The 2nd order instability is widely known as "the envelope instability".
- In the early days, they were called "instabilities". But recently they are called "parametric resonances" → causing confusion.

- Are they resonances of the beam particle?
  No. They are "resonances" of beam modes.

Parametric instabilities in actual linacs?

- Parametric instabilities have been observed for KV and waterbag distribution in multiparticle PIC simulations.
- But, not observed for Gaussian distribution.

multiparticle PIC simulation

Parametric instability	**KV distribution**	**waterbag distribution**	**Gaussian distribution**
2^{nd} order	O	O	O
3^{rd} order, 4^{th} order	O	O	X
higher order	O	X	X

- The 3^{rd} or 4^{th} order instabilities can be ignored in actual linacs, unless waterbag or KV distribution is generated.

Jeon, J. Korean Phys. Soc. 72, 1523 (2018);
Space Charge 2019

Particle Resonances
in high-intensity linacs

- The $4\sigma = 360°$ resonance in high-intensity linacs was discovered.
  [Jeon et al, PRST-AB 12, 054204 (2009)]

- The $6\sigma = 720°$ resonance was discovered.
  [Jeon et al, PRL 114, 184802 (2015)]

- The $6\sigma = 360°$ resonance is too weak to observe for Gaussian beam.
  [Jeon et al, PRL 114, 184802 (2015)]
- Weak sign is observed for waterbag beam.
  [Hofmann et al, PRL 115, 204802 (2015)]

- Higher order resonances were discovered:
  $8\sigma = 1080°$ resonance, $10\sigma = 1440°$ resonance [Hofmann, HB2016]

What is the difference?

Particle Resonances
or incoherent resonances

⇨ O resonance frequency component

Particle resonances → resonance islands in phase space

Parametric Instabilities
or parametric resonances
or coherent resonances

⇨ X resonance frequency component

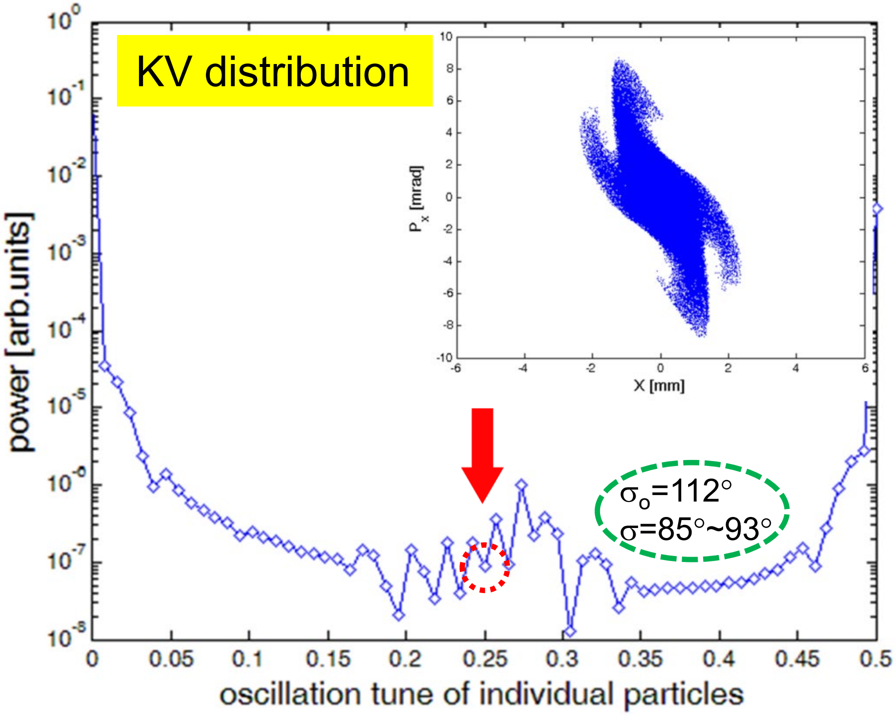
Parametric instabilities → no resonance islands in phase space

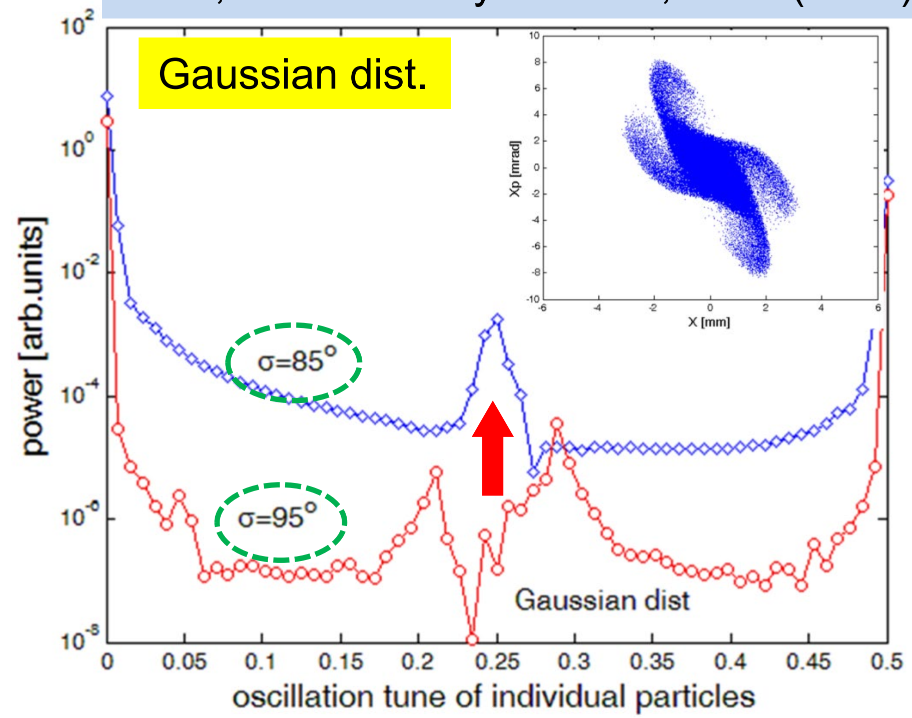
There are look-alike people

- There are look-alike people but not related at all.
- They have different parents.

Appearance can be deceiving

Jeon, J Korean Phys Soc 72, 1523 (2018)





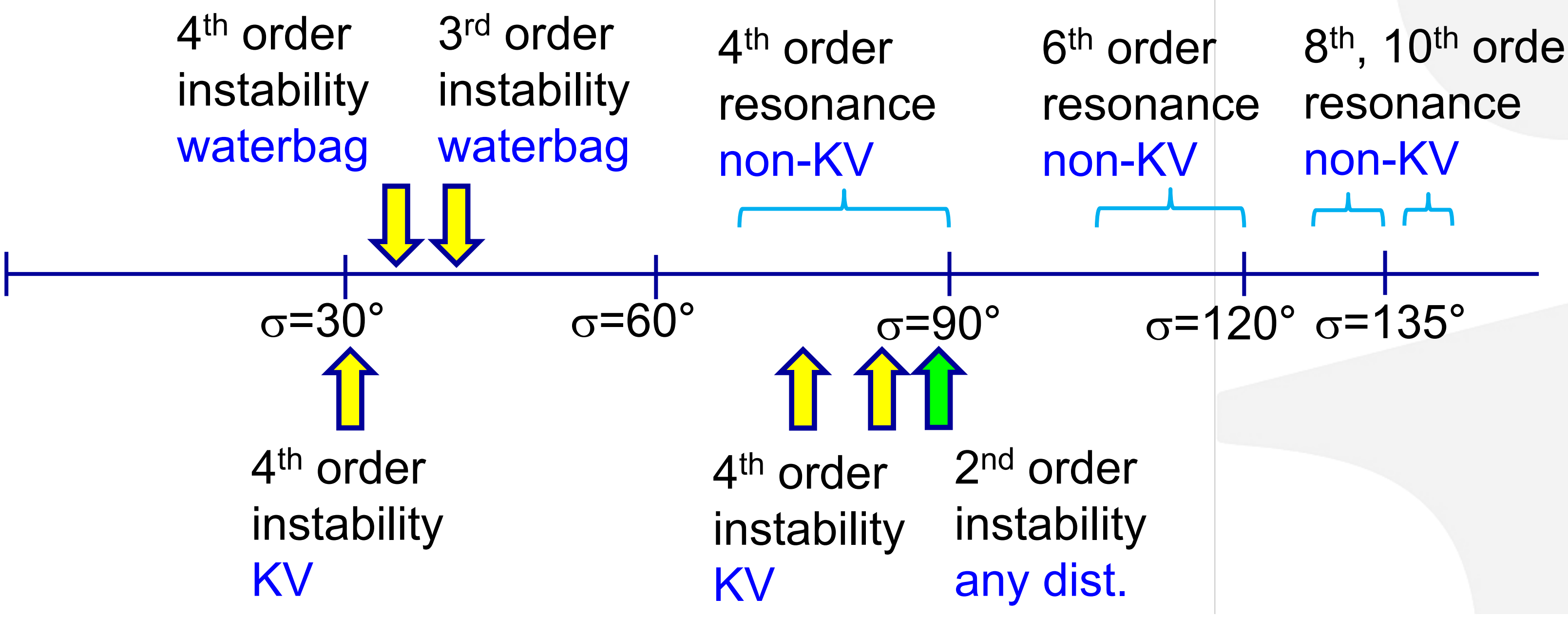
parametric instability (4th order)

particle resonance (4th order)

- Resonance frequency component is observed for the 4th order resonance.
- No resonance frequency component is observed for the 4th order parametric instability.
- For non-KV distribution, the 4th order particle resonance dominates.
- The 4th order particle resonance has been **verified in two experiments**.

Particle resonances dominate

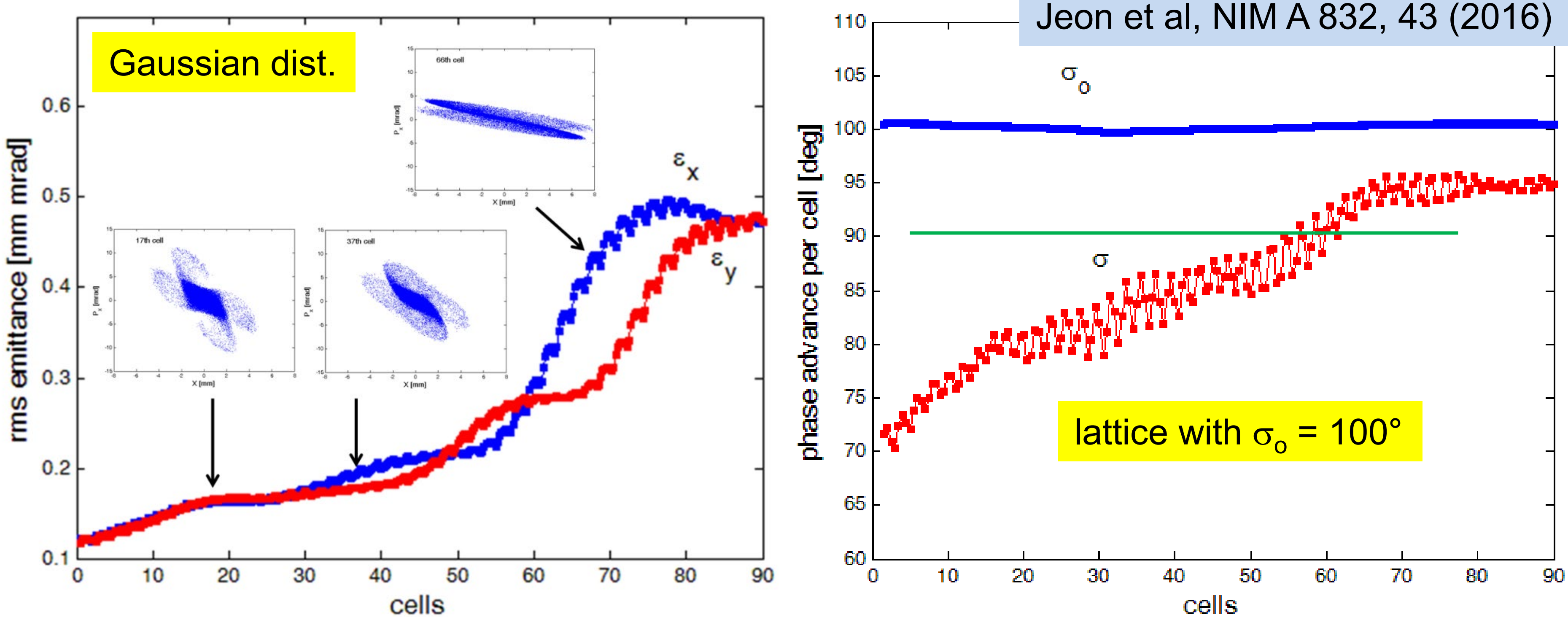
for non-KV distribution



- Particle resonances dominate over parametric instabilities in resonance stopbands for non-KV distribution.
- Envelope instability develops only when 4th order resonance fades away (as σ increases and reaches 90° in a constant-σ_o lattice) for non-KV distribution.
- Envelope instability is suppressed in constant-σ lattices.
- Only for waterbag distribution, the 3rd and 4th order parametric instabilities are observed away from particle resonances.

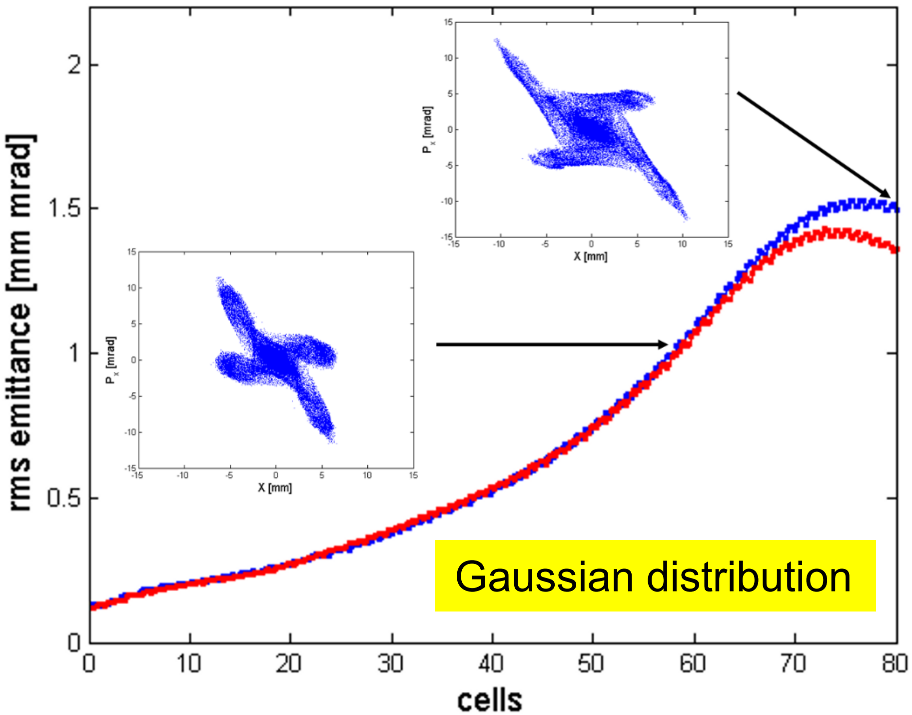
Envelope instability develops
when particle resonance fades away

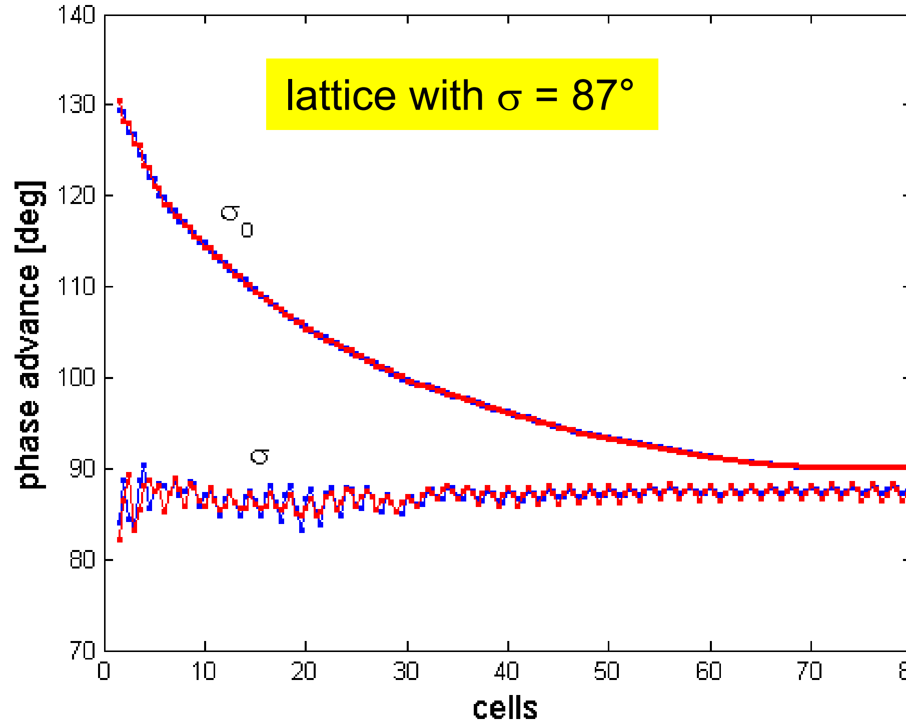
Jeon et al, NIM A 832, 43 (2016)



- 4th order resonance develops first for well-matched non-KV distribution.
- Envelope instability emerges because 4th order resonance fades away (as σ increases and reaches 90°).
- Halo particles in the resonance islands act as a mismatch, triggering the envelope instability.
- For fair comparison, the 4th order resonance should be maintained.

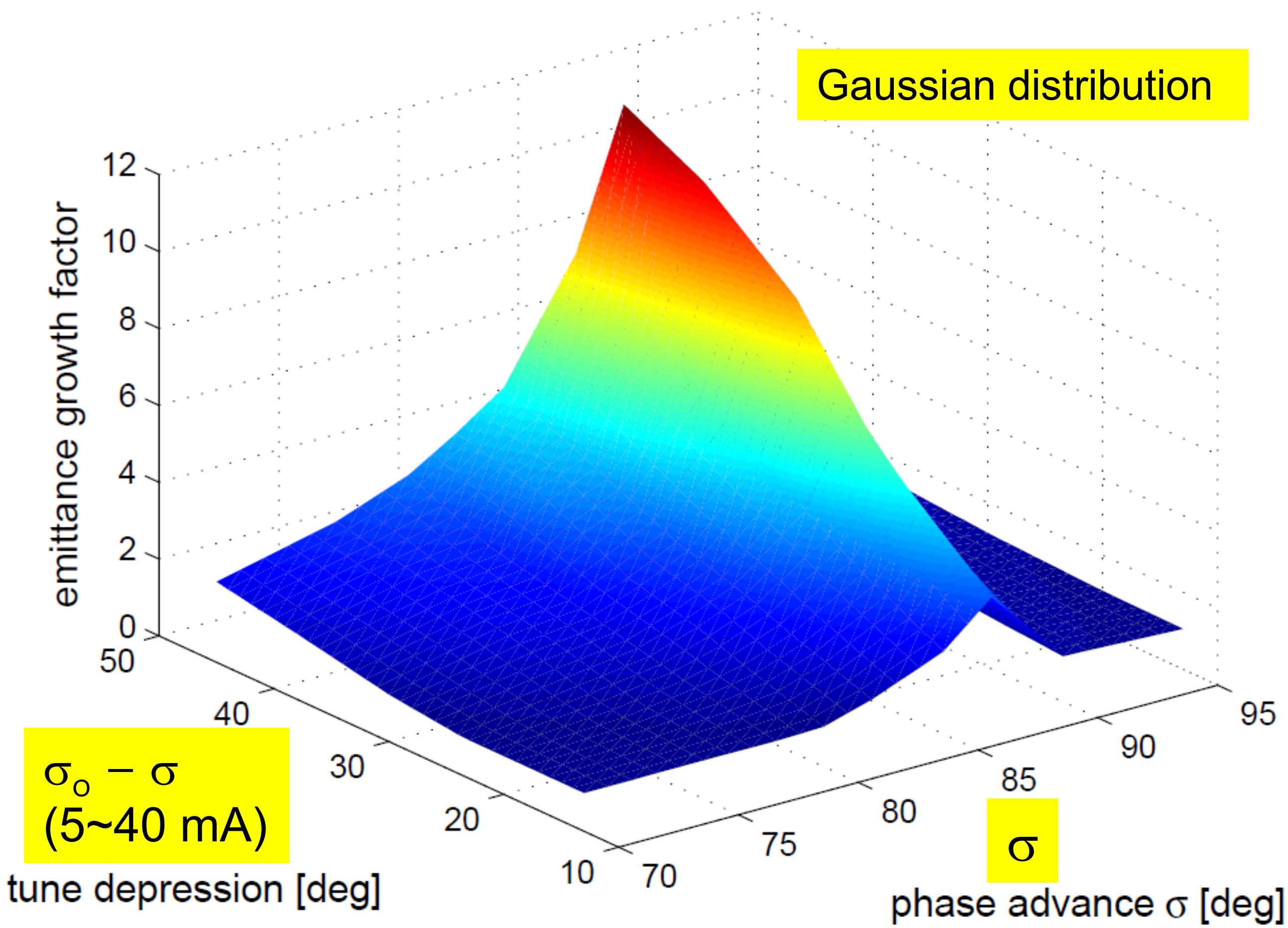
4th order particle resonance dominates over envelope instability





- In a constant-σ lattice, 4th order particle resonance is maintained and dominates over the envelope instability all the way.
- Envelope instability is suppressed.
- Resonance islands represent potential wells.
- 4th order particle resonance is the limiting mechanism in $\sigma_o > 90°$.

4th order particle resonance dominates over envelope instability



In constant-σ lattices, **envelope instability** is not observed.

well-matched 3D Gaussian input beam

- $\varepsilon_{final} / \varepsilon_{initial} (\sigma, \sigma_o - \sigma)$.
- Only the 4th order resonance is manifested in constant-σ lattices.
- 4th order resonance dominates over envelope instability.

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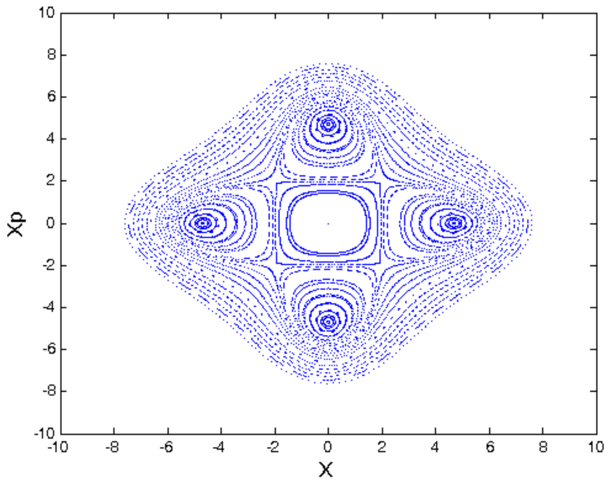
4^{th} order particle resonance dominates
for non-KV distribution

- The 4^{th} order particle resonance forms resonance islands (potential wells) in the phase space.
- Potential wells hold particles and suppress parametric instabilities.

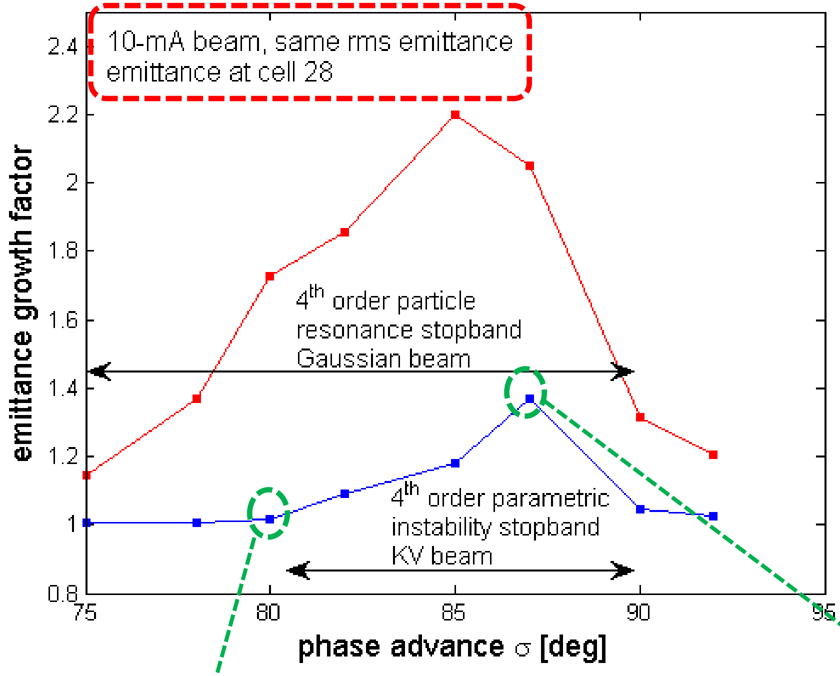
- In constant-σ lattices, 4^{th} order resonance dominates over the envelope instability.

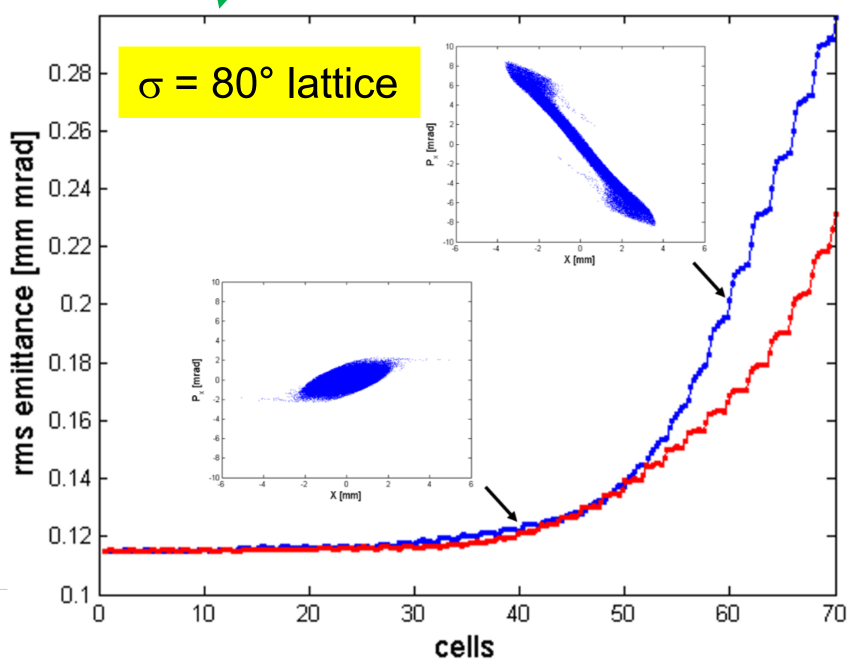
- For non-KV distribution, 4^{th} order resonance dominates over the 4^{th} order parametric instability.
- The 4^{th} order parametric instability does not have resonance islands, and constant-σ lattice does not suppress the envelope instability.

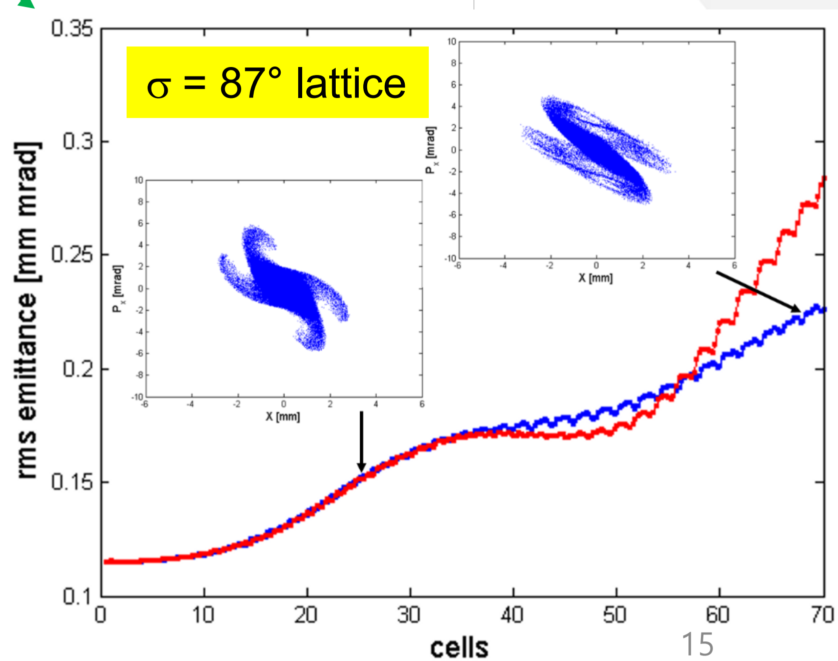


4^{th} order instability and envelope instability

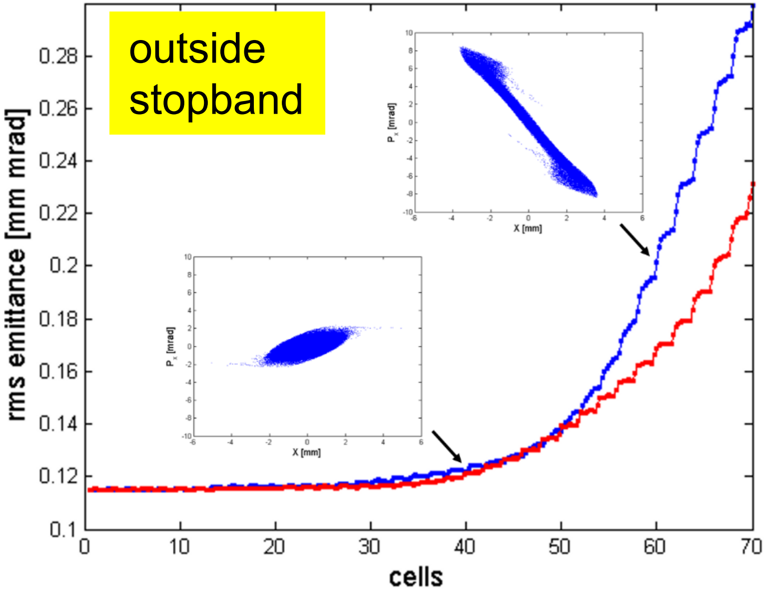
- Inside stopband, 4^{th} order instability appears first and envelope instability follows.
- Outside stopband, envelope instability appears alone in σ=80° lattice.
- Envelope instability is still induced in constant-σ lattices.

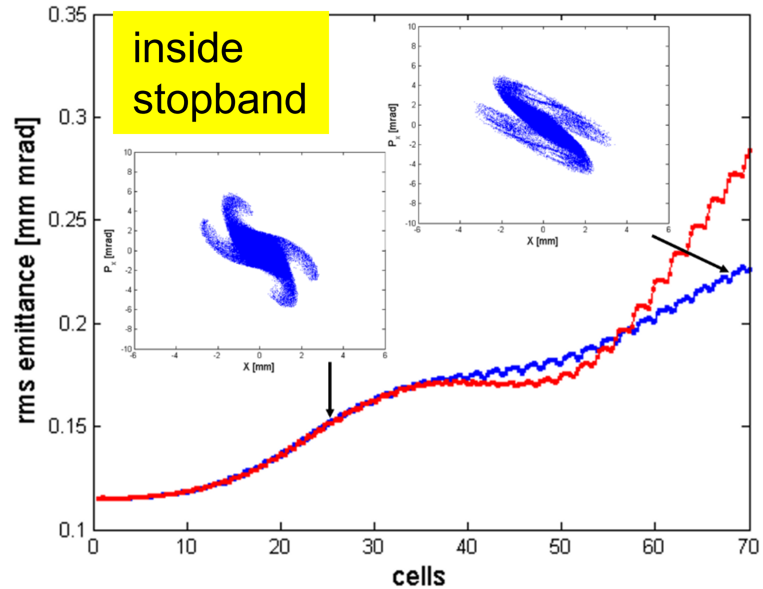


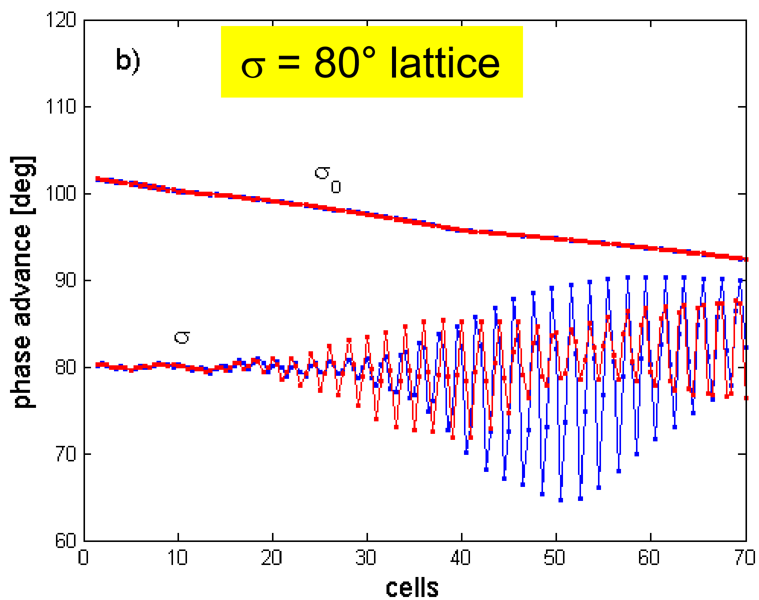


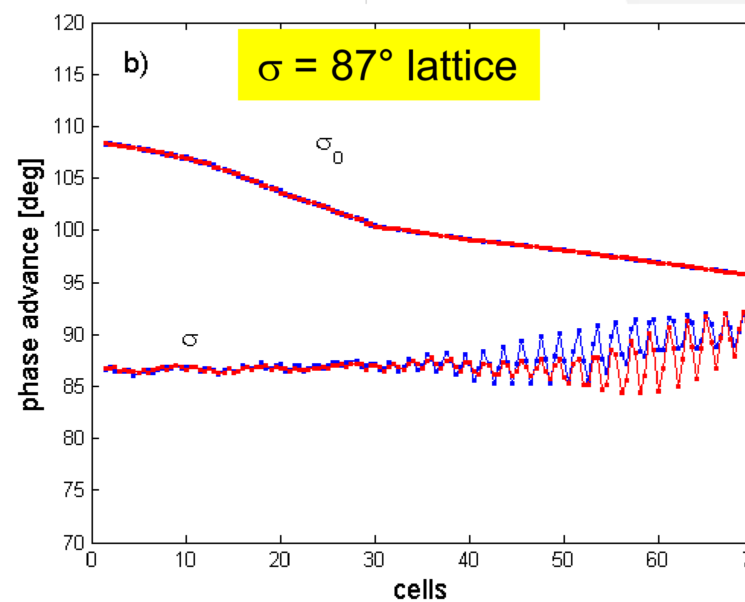


4th order instability and envelope instability









4^{th} order particle resonance dominates over 4^{th} order parametric instability

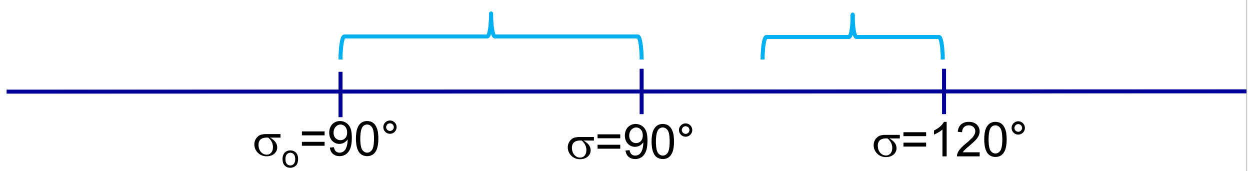
- **4^{th} order particle resonance:**
  - dominates for non-KV distribution,
  - verified by two experiments.
  - the stopband is (90° < σ_o & σ < 90°),

- **4^{th} order parametric instabilities:**
  - $4(\sigma_o - \Delta\sigma_{4,coh}) = 360°$ instability only for KV distribution,
  - dominates for KV distribution,
  - overlapping with the 4^{th} order resonance

  - $4(\sigma_o - \Delta\sigma_{4,coh}) = 180°$ instability for waterbag and KV distribution, manifested away from particle resonances (at $\sigma \sim 35°$),
  - No overlapping particle resonance exists.

Operating in σ_o > 90°

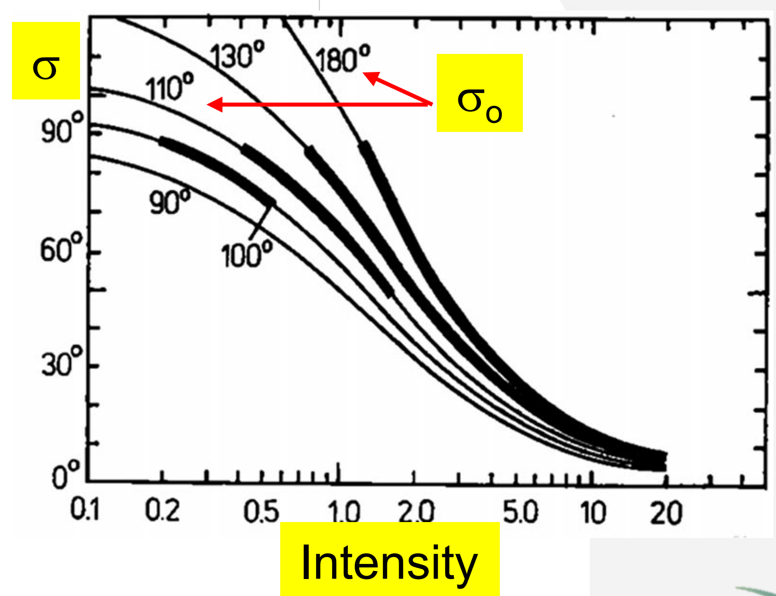
operating linacs in σ > 90°



- σ_o > 90° region has been avoided due to the envelope instability.

- No 4th order resonance effect in σ > 90° (Hamiltonian property).
- No envelope instability in σ > 90°.
- 6th order resonance is weak.

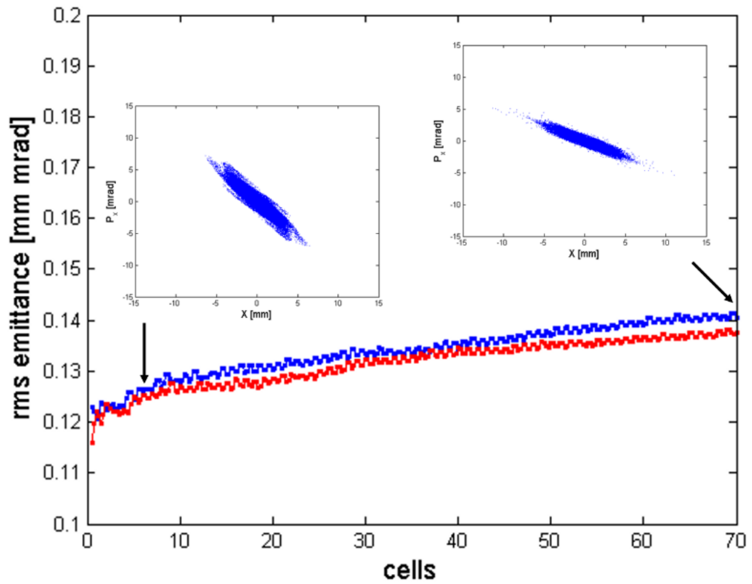
- Operation in σ > 90° seems feasible.

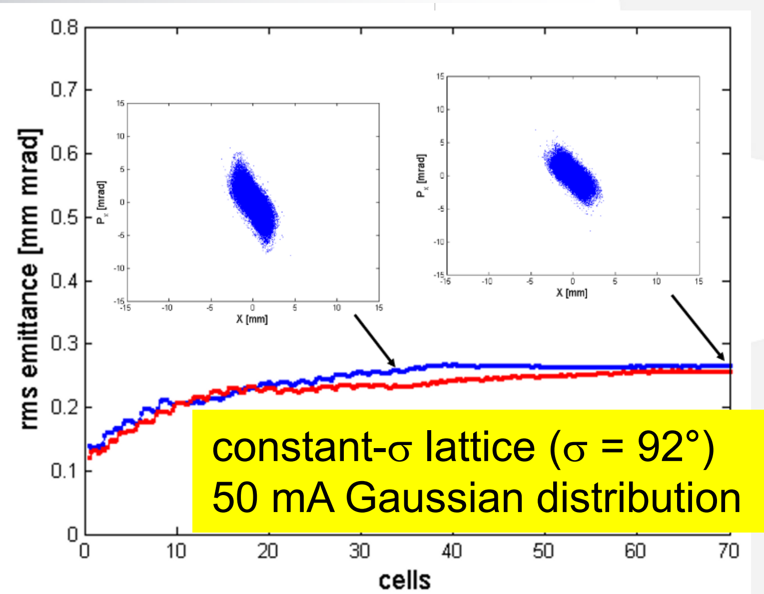


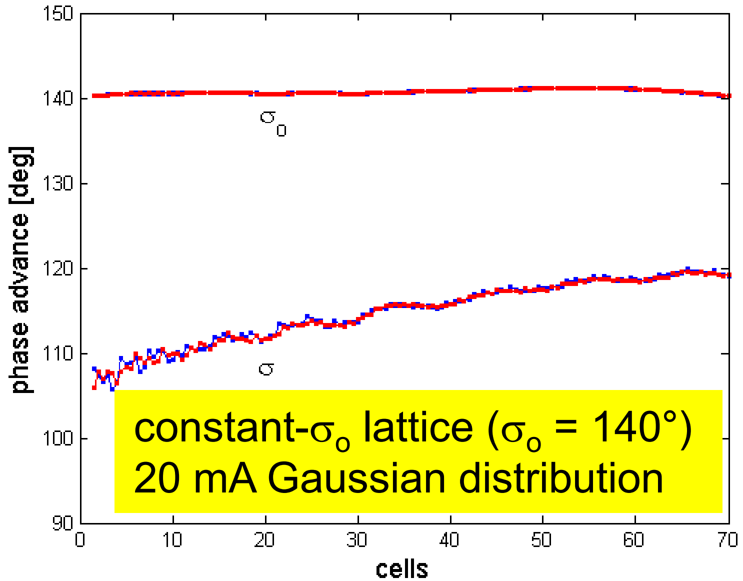
Hofmann et al, PRAB 20, 014202 (2017)

Operating in σ_o > 90°

operating linacs in σ > 90°







Operation in σ > 90° is feasible and has advantages:

- avoids the 4th order resonance and the envelope instability completely,
- emittance growth is minimal,
- does not require additional hardware,
- good when small beam size is needed.

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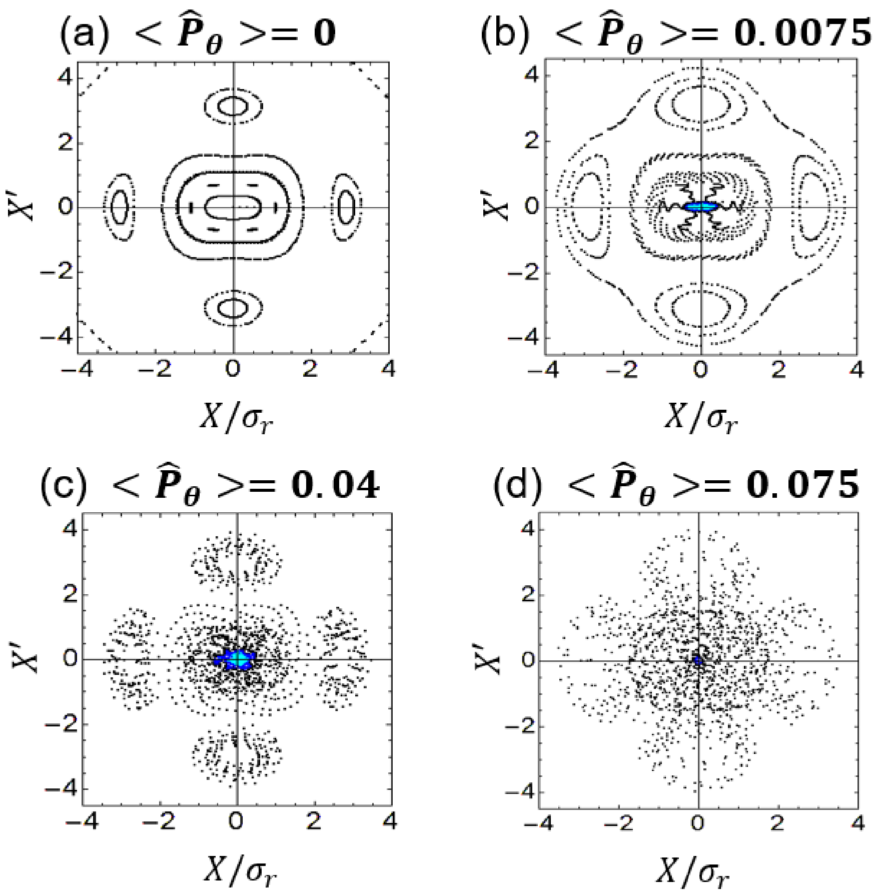
RISP

Operating in σ_o > 90°

beam spinning

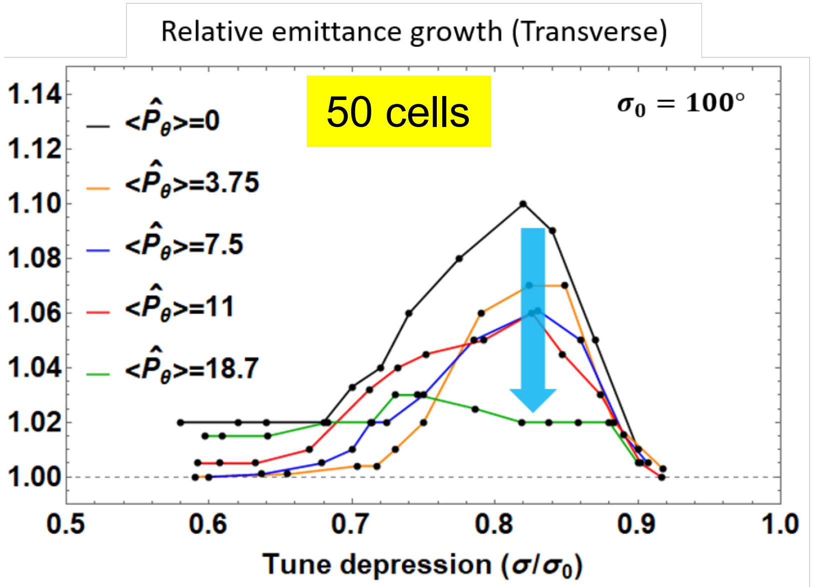
Beam spinning can mitigate the 4th order resonance & envelope instability.

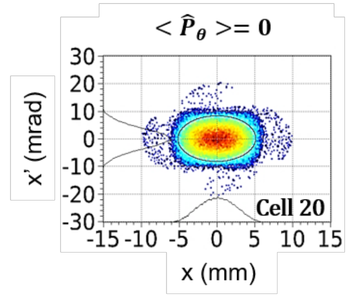


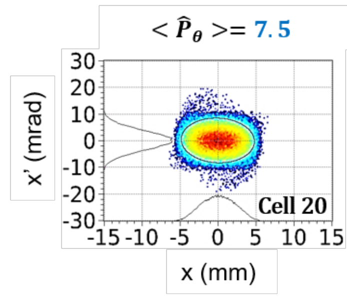
As angular momentum $<P_\theta>$ increases, resonance island structures get blurred (through coupling).

Cheon et al, NIM A 1013, 165647 (2021)
Cheon el al, PRAB 25, 064002 (2022)

Operating in $\sigma_o > 90°$
beam spinning

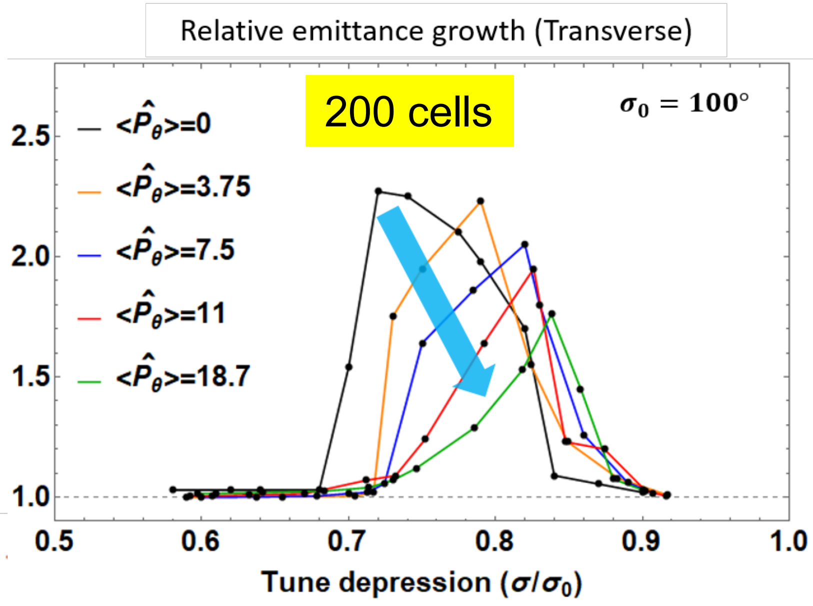


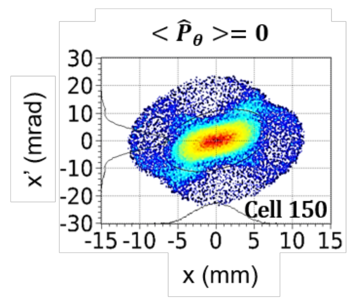


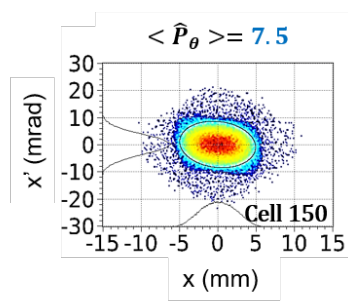


Beam spinning mitigates the 4th order resonance and envelope instability.

4th order resonance only.







4th order resonance + envelope instability.

Mitigation of envelope instability

- Operating linacs in $\sigma > 90°$ avoids the 4th order resonance and the envelope instability.
- Adopting constant-σ lattice (when $\sigma < 90°$) suppresses the envelope instability (for non-KV distribution).

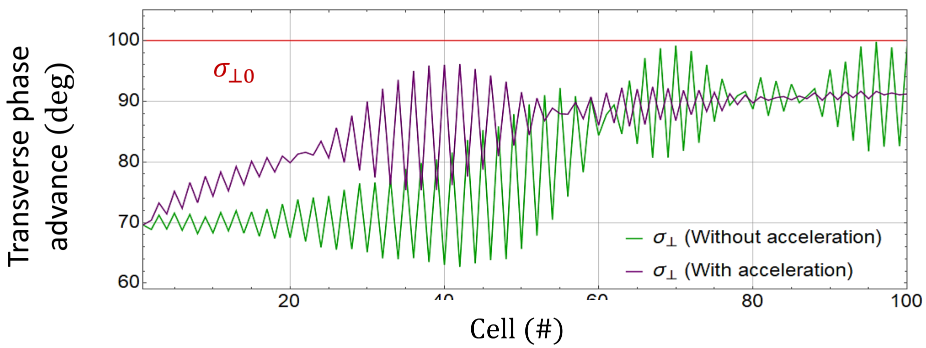
Jeon et al, NIM A 832, 43 (2016)

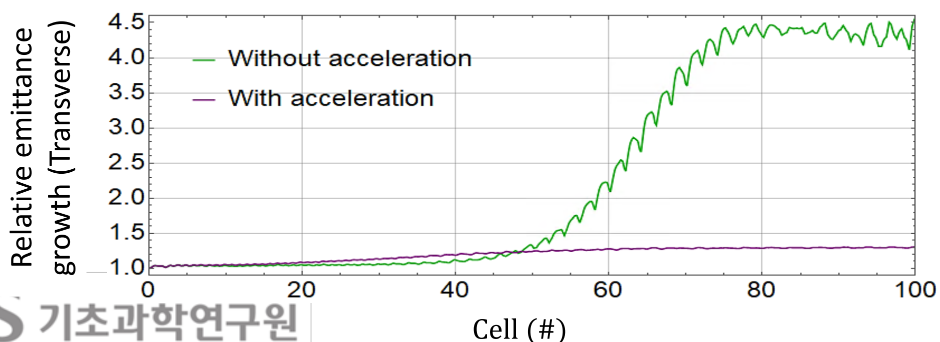
- Beam spinning mitigates the 4th order resonance and the envelope instability.

Cheon et al, NIM A 1013, 165647 (2021); PRAB 25, 064002 (2022)

- Fast acceleration mitigates the envelope instability.

Qiang, PRAB 21, 114201 (2018)





By fast acceleration, beam passes through the instability stopband fast
-> **Envelope instability is mitigated.**

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Conclusion

- Particle resonances dominate over parametric instabilities in their stopbands.

- Parametric instabilities can be disregarded in actual linacs except for the envelope instability, unless waterbag or KV distribution is generated.
- Even the envelope instability can be disregarded when constant-σ lattices are used (4^{th} order particle resonance is maintained).

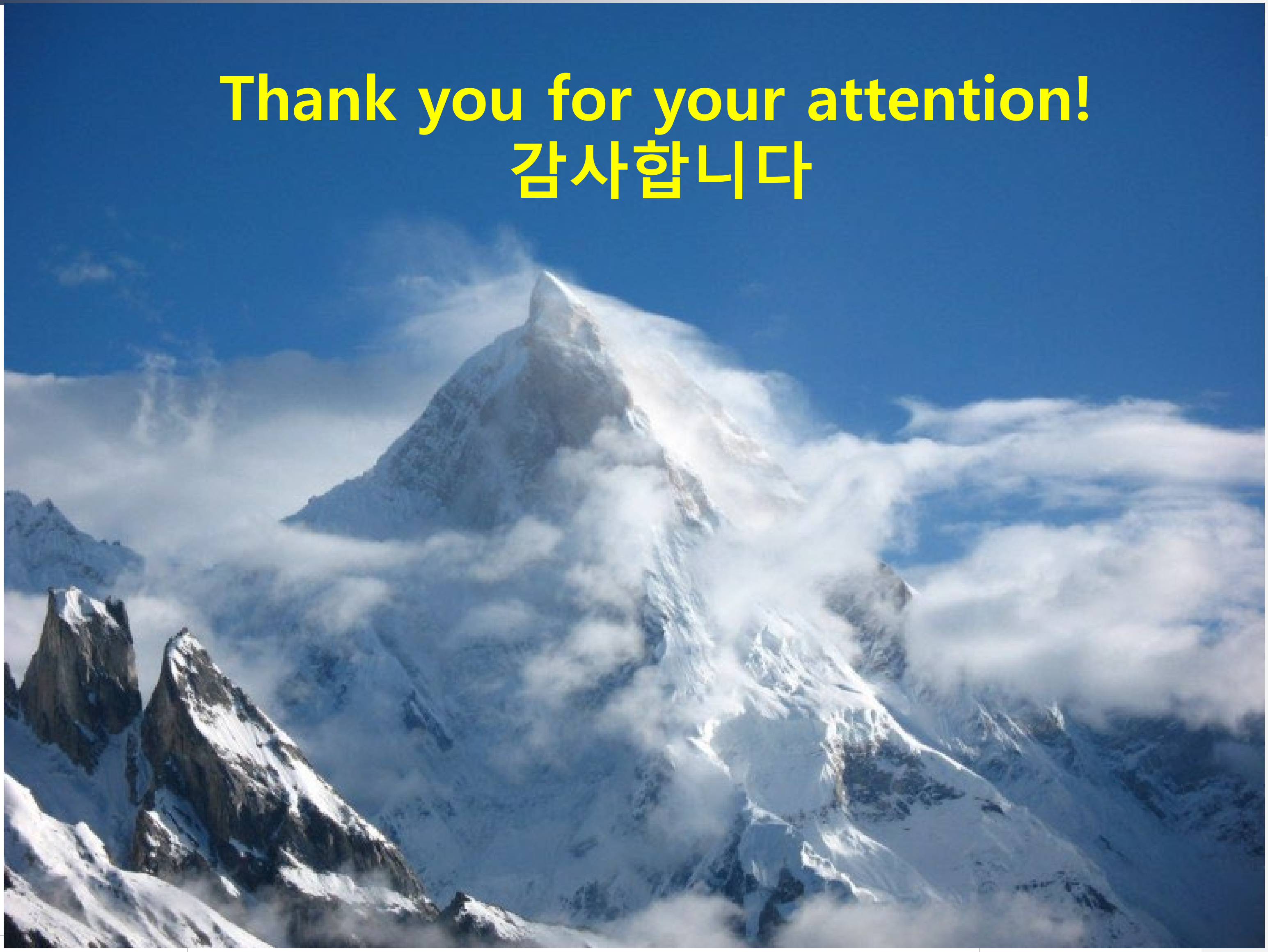
- Operating linacs in $\sigma > 90°$ can avoid the 4^{th} order particle resonance and the envelope instability completely.

- The envelope instability can be suppressed or mitigated by:
  - operating a linac in $\sigma > 90°$;
  - operating a constant-σ linac in $\sigma < 90°$;
  - beam spinning;
  - fast acceleration.

Terminology Suggestion

- Parametric instabilities of beam eigenmodes:
  - a.k.a. parametric resonances, instabilities, coherent resonances, structure resonances…
  - better be called parametric instabilities to distinguish them from “particle” parametric resonances.
  - 2nd order instability is called the envelope instability rather than the envelope resonance.

- Particle resonances are resonances of the beam particle, as widely known in circular accelerators:
  - a.k.a. resonances or incoherent resonances.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