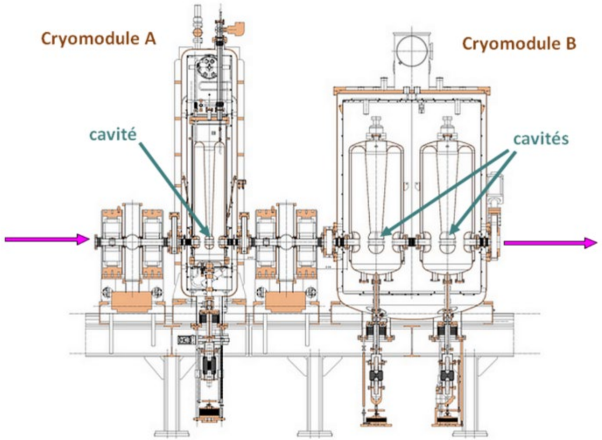
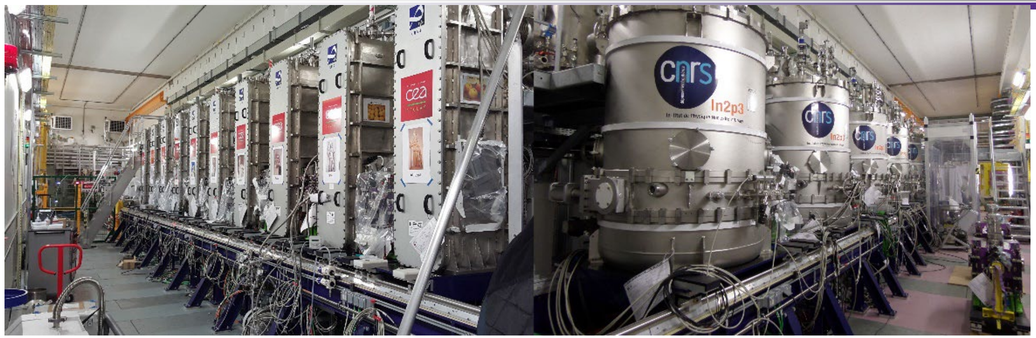
# Synchronous Phases and Transit Time Factors

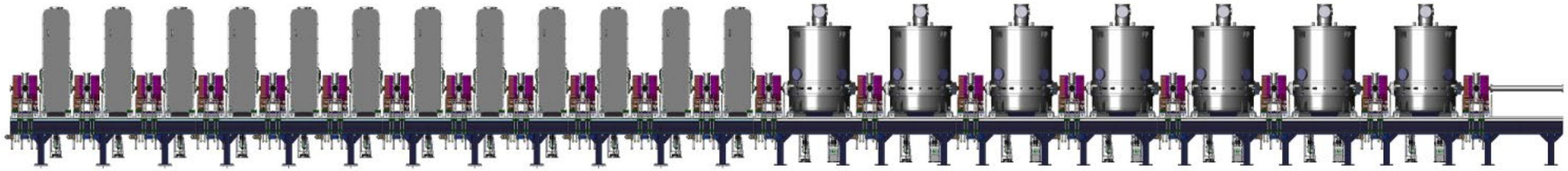
## $\sigma_l$ = 90° resonance

Beam physics - Light and heavy ion superconducting linacs - **High energy gains**

Jean-Michel Lagniel, GANIL





**SPIRAL2 linac commissioning & tunings** Angie Orduz, Tuesday talk WGD

0.7 MeV/A RFQ => 3 bunchers + 12 low β + 2x7 high β SC cavities => 29 cavities to tune

Large variety of Ions: 1 < A/Q < 3 – 7 Intensities: 0 to 5mA (200 kW) Energies: 0.7 to 20 (33) MeV/A
Duty-cycles: 1kHz up to CW + 1/100 bunch selector + Demands for experiments (dp/p …)

Dedicated "**SP2_linac_generator**" code => **TraceWin** input file with **kE** (E = kE Emax) and **Φs**

# MENU

1- Low fields - $\Delta W(z) \ll W$ - Panofsky equation

2- Paramount importance of $\Phi s$ (synchronous phase) and T (TTF)

3- What changes at high accelerating fields

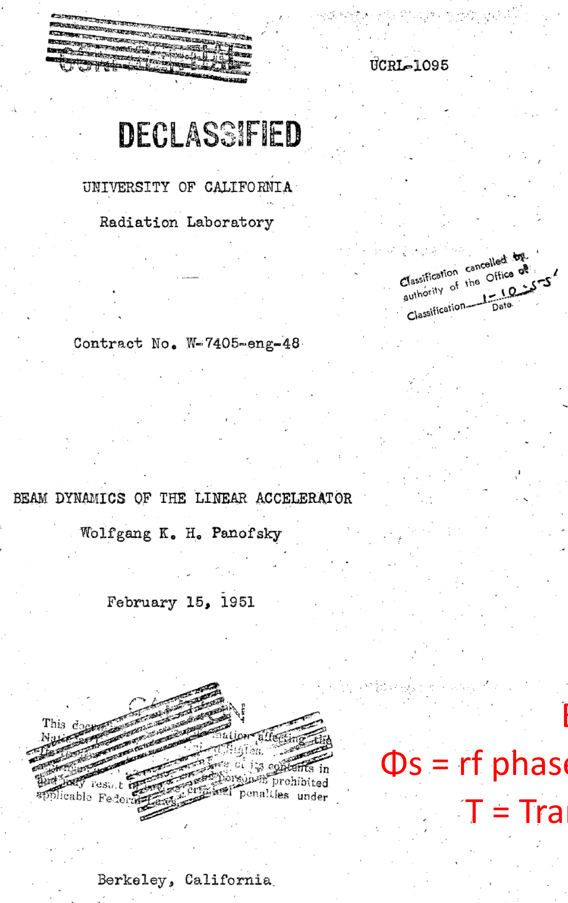
4- $\Phi s$ and TTF computations (to be "right" at high accelerating fields)

5- $\sigma_{|\_|}$ = 90° resonance, space-charge and cavity-field excitation

# 1- Low fields - ΔW(z) << W - Panofsky equation 1/2



72 years ago (1951), Wolfgang Kurt Hermann Panofsky published a paper opening the way to **study and understand the "linear accelerator beam dynamics"** building an **analytical treatment** using **simplifying assumptions**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BEAM DYNAMICS OF THE LINEAR ACCELERATOR

Wolfgang K. H. Panofsky

February 15, 1951

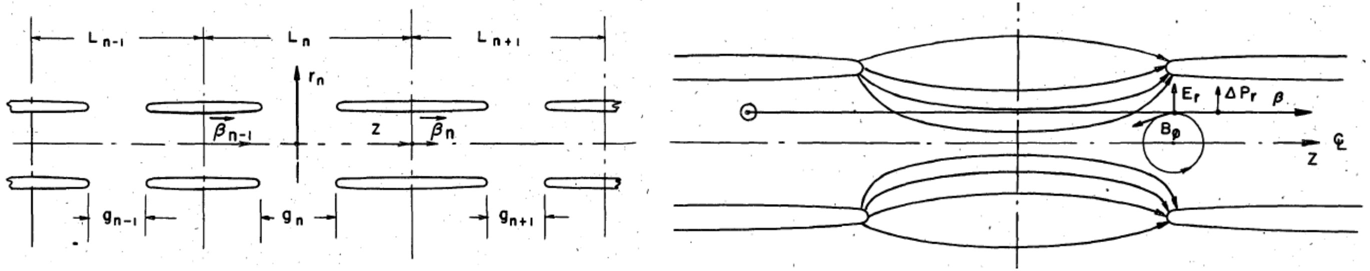
[redacted]

Berkeley, California

Introduction of the main basic concepts still in use in linac beam dynamics studies today

Still taught in accelerator books and accelerator schools

Drift tube linac, accelerating field in the gaps




$$W_{n,s}-W_{n-1,s} = \int e\, E_z^0(z)\cos\left(\frac{\omega z}{V_s}+\phi_s\right)dz$$

$E_o\, L_n$ = Accelerating voltage

$\Phi s$ = rf phase when particle at gap electrical center

T = Transit Time Factor, f(βs) … only at low field

$$\boxed{\Delta W = e\, T\, E_0\, L_n \cos\phi_s}$$

Complexity hidden in T !

# 1- Low fields - ΔW(z) << W - Panofsky equation 2/2

## From Panofsky equation to **longitudinal beam dynamics**

$$\Delta W(\Phi_s) = \int_{-L/2}^{+L/2} q\, E_z(z)\cos[\Phi(z) + \Phi_s]\, dz \qquad \Phi(z) = \int_{-L/2}^{z} \frac{2\pi}{\beta(s)\lambda}\, ds = \boxed{\frac{2\pi z}{\beta_s \lambda}}$$

**Low field ⇒ Low β variation**

field map ⇒ T

$$\boxed{\Delta W_s = q\, E_0\, L\, T\, \cos(\Phi_s)} \qquad E_0\, L = \int_{-L/2}^{+L/2} E_z(z)\, dz \qquad T(\beta_s) = \frac{1}{E_0\, L}\int_{-L/2}^{+L/2} E_z(z)\cos\left(\frac{2\pi z}{\beta_s \lambda}\right) dz$$

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From Panofsky equation to **longitudinal beam dynamics**

$$\Delta W(\Phi_s) = \int_{-L/2}^{+L/2} q\, E_z(z)\, cos[\Phi(z) + \Phi_s]\, dz \qquad \Phi(z) = \int_{-L/2}^{z} \frac{2\pi}{\beta(s)\lambda}\, ds = \boxed{\frac{2\pi z}{\beta_s \lambda}}$$

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Longitudinal beam dynamics around Φs function of $E_o L$ (cavity voltage), Φs and βs

$$\delta W_i = \delta W_{i-1} + q E_0 L_i\, \mathrm{T}_{\beta_{s\,i}} [\cos\,(\Phi_{si} + \delta\varphi_i) - \cos \Phi_{si}]$$

$$\delta\varphi_i = \delta\varphi_{i-1} - \frac{2\pi L_i}{m_0 c^2 \lambda\, \beta_{s\,i-1}^3 \gamma_{s\,i-1}^3}\, \delta W_{i-1}$$

mapping

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## From Panofsky equation to longitudinal beam dynamics

$$\Delta W(\Phi_s) = \int_{-L/2}^{+L/2} q\, E_z(z) \cos[\Phi(z) + \Phi_s]\, dz \qquad \Phi(z) = \int_{-L/2}^{z} \frac{2\pi}{\beta(s)\lambda}\, ds = \boxed{\frac{2\pi z}{\beta_s \lambda}}$$

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$$\boxed{\Delta W_s = q\, E_0\, L\; T\; \cos(\Phi_s)} \qquad E_0\, L = \int_{-L/2}^{+L/2} E_z(z)\, dz \qquad T(\beta_s) = \frac{1}{E_0\, L} \int_{-L/2}^{+L/2} E_z(z) \cos\left(\frac{2\pi z}{\beta_s \lambda}\right) dz$$

Longitudinal beam dynamics (around Φs) function of $E_oL$ (cavity voltage), Φs and βs through T

$$\delta W_i = \delta W_{i-1} + q E_0 L_i\, \mathrm{T}_{\beta_s\, i} \left[\cos\left(\Phi_{si} + \delta\varphi_i\right) - \cos \Phi_{si}\right]$$

$$\delta\varphi_i = \delta\varphi_{i-1} - \frac{2\pi L_i}{m_0 c^2 \lambda\, \beta_{s\, i-1}^3 \gamma_{s\, i-1}^3}\, \delta W_{i-1}$$

mapping

Smooth approximation → large amplitude motions + separatrix shapes

$$\frac{d^2\delta\varphi}{dz^2} + K_{dp} \frac{d\delta\varphi}{dz} + \left[\frac{2\pi q\, E_0\, \mathrm{T}_{\beta_s}}{m_0 c^2 \lambda\, \beta_s^3 \gamma_s^3}\right] \left[\cos(\Phi_s + \delta\varphi) - \cos \Phi_s\right] = 0$$

# 1- Low fields - ΔW(z) << W - Panofsky equation 2/2

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## From Panofsky equation to longitudinal beam dynamics

$$\Delta W(\Phi_s) = \int_{-L/2}^{+L/2} q\, E_z(z) \cos[\Phi(z) + \Phi_s]\, dz \qquad \Phi(z) = \int_{-L/2}^{z} \frac{2\pi}{\beta(s)\lambda}\, ds = \boxed{\frac{2\pi z}{\beta_s \lambda}}$$

Low field ⇒ Low β variation

field map (shape) → T

$$\boxed{\Delta W_s = q\, E_0\, L\, T\, \cos(\Phi_s)} \qquad E_0\, L = \int_{-L/2}^{+L/2} E_z(z)\, dz \qquad T(\beta_s) = \frac{1}{E_0\, L}\int_{-L/2}^{+L/2} E_z(z) \cos\left(\frac{2\pi z}{\beta_s \lambda}\right) dz$$

Longitudinal beam dynamics (around Φs) function of $E_0L$ (cavity voltage), Φs and βs through T

$$\delta W_i = \delta W_{i-1} + q E_0 L_i\, \mathrm{T}_{\beta_{s\,i}}\left[\cos\left(\Phi_{si} + \delta\varphi_i\right) - \cos \Phi_{si}\right]$$

mapping

$$\delta\varphi_i = \delta\varphi_{i-1} - \frac{2\pi L_i}{m_0 c^2 \lambda\, \beta_{s\,i-1}^3 \gamma_{s\,i-1}^3}\, \delta W_{i-1}$$

Smooth approximation → large amplitude motions + separatrix shapes

$$\frac{d^2\delta\varphi}{dz^2} + K_{dp}\frac{d\delta\varphi}{dz} + \left[\frac{2\pi q\, E_0\, \mathrm{T}_{\beta_s}}{m_0 c^2 \lambda\, \beta_s^3 \gamma_s^3}\right]\left[\cos(\Phi_s + \delta\varphi) - \cos\Phi_s\right] = 0$$

$$\frac{d^2\delta\varphi}{dz^2} + \sigma_{0l}^2(z)\, \delta\varphi = 0$$

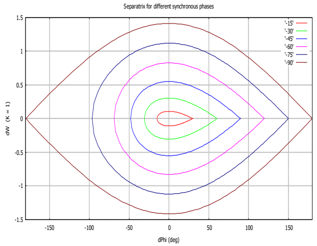
small amplitude motions

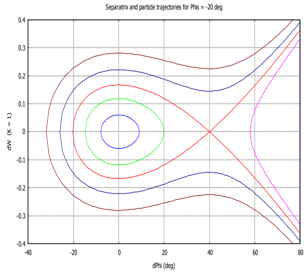
$$\sigma_{0l} = \sqrt{\frac{-2\pi q\, E_0\, T \sin\Phi_s}{m_0 c^2 \lambda\, \beta_s^3 \gamma_s^3}}$$

longitudinal phase advance
= longitudinal focalization

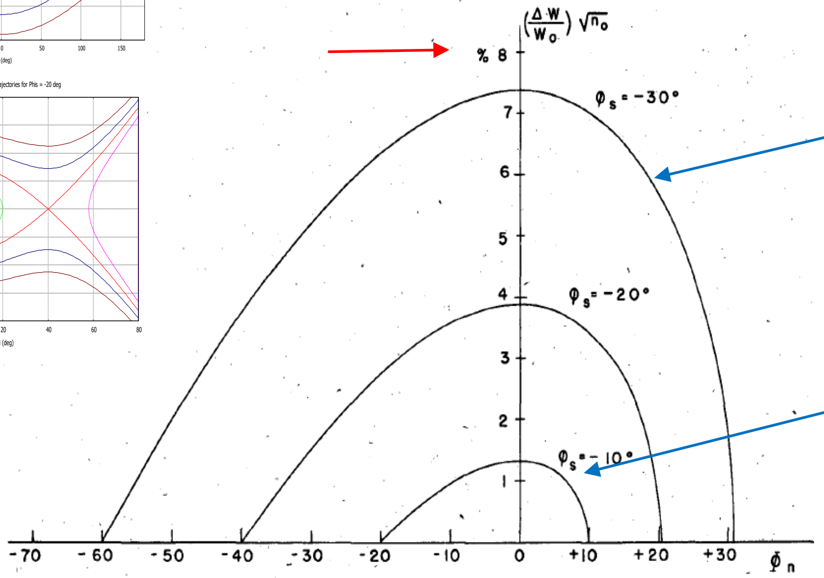
# 2- Paramount importance of Φs and T 1/3






Smooth approximation
without damping




Separatrix shapes f(Φs) = longitudinal acceptances

**Φs = -20° → -15° → acceptance / 2**

## Acceleration and longitudinal focalization

**Choice of Φs for the cavities**

**Low Φs**

↓

large acceptance (low losses)
but low acceleration efficiency (long linac)

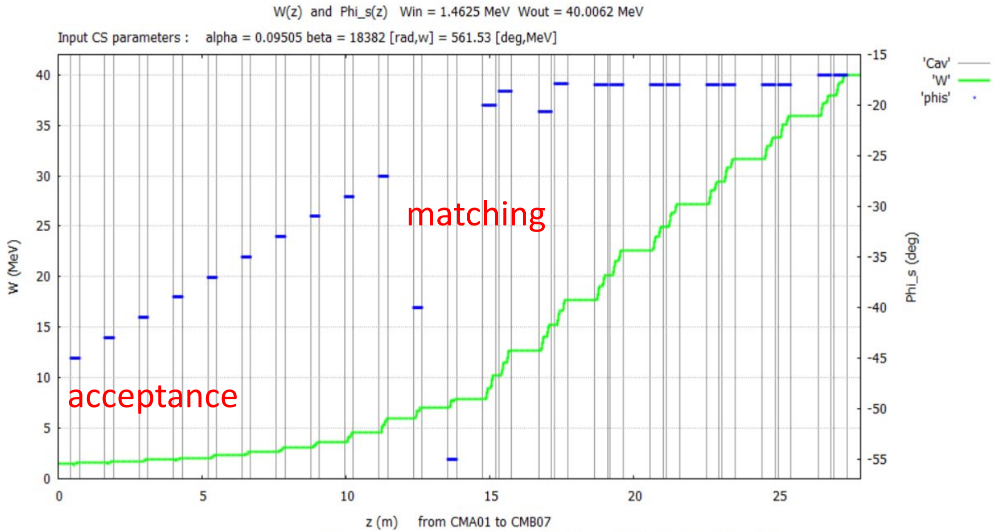
**High Φs**

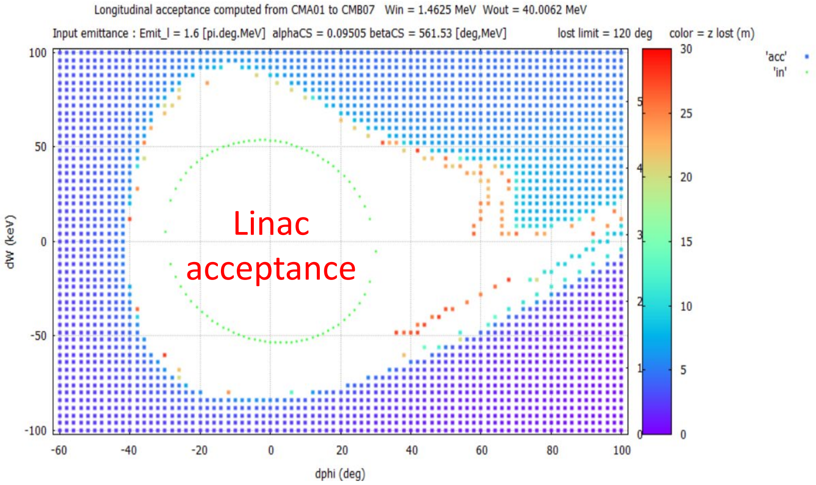
↓

small acceptance (risk for operation @ beam losses)
but high acceleration efficiency (lower cost)

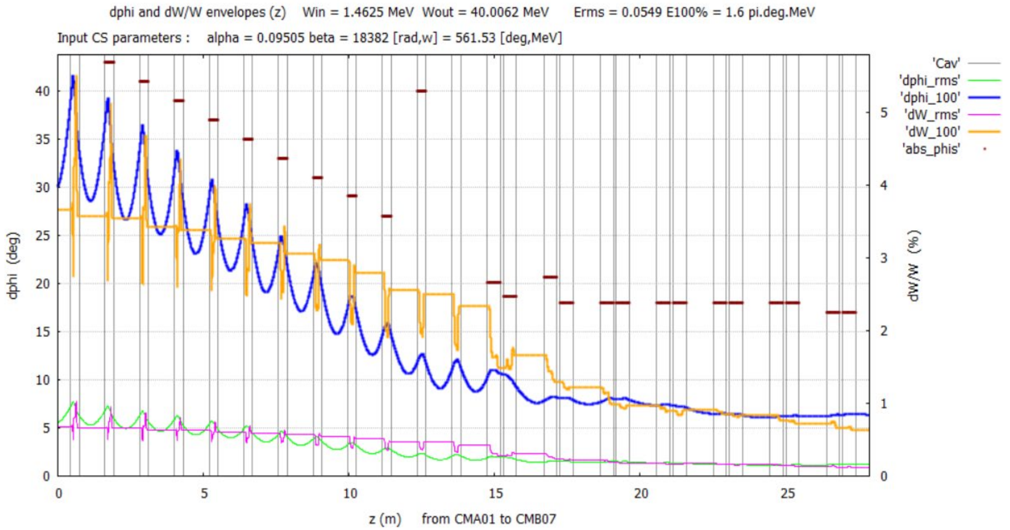
Compromise between
**efficiency** (then construction cost)
and **risk** for operation (beam losses)

# 2- Paramount importance of $\Phi$s and T 2/3




## SPIRAL2 linac



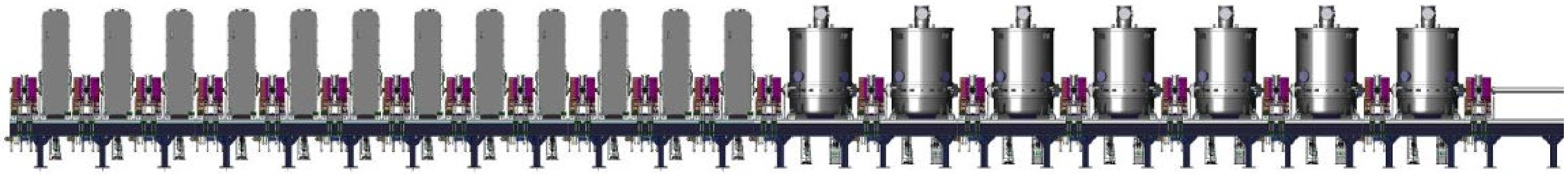

$\Phi$s = Main parameter for linac design

$\Phi$s = Main parameter for linac tuning

$\Phi$s (and E) choice for each cavity

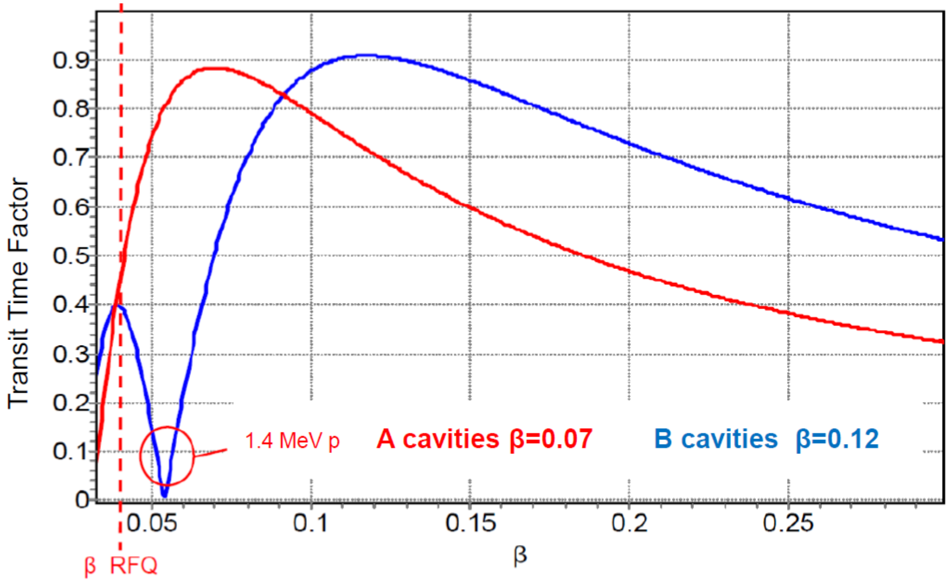
**Important to compute $\Phi$s correctly !!!**

# 2- Paramount importance of Φs and T 3/3



14.5 m
12 A cryomodules (12 cavities)

15.5 m
7 B cryomodules (14 cavities)




$E_O L T$ = cavity effective voltage

→ **T = cavity efficiency**

T = Important parameter for linac design
(choice of the **β families**)

T = Important parameter for cavity design
(geometry => **peak field**)

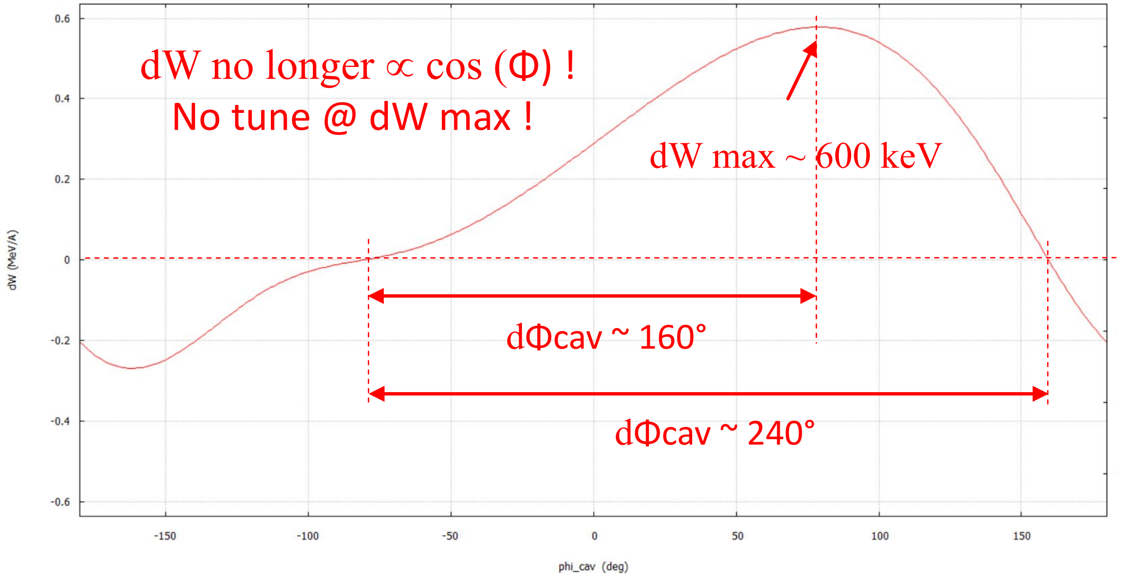
**Important to compute T correctly !!!**

## 3- What changes at high accelerating fields ?

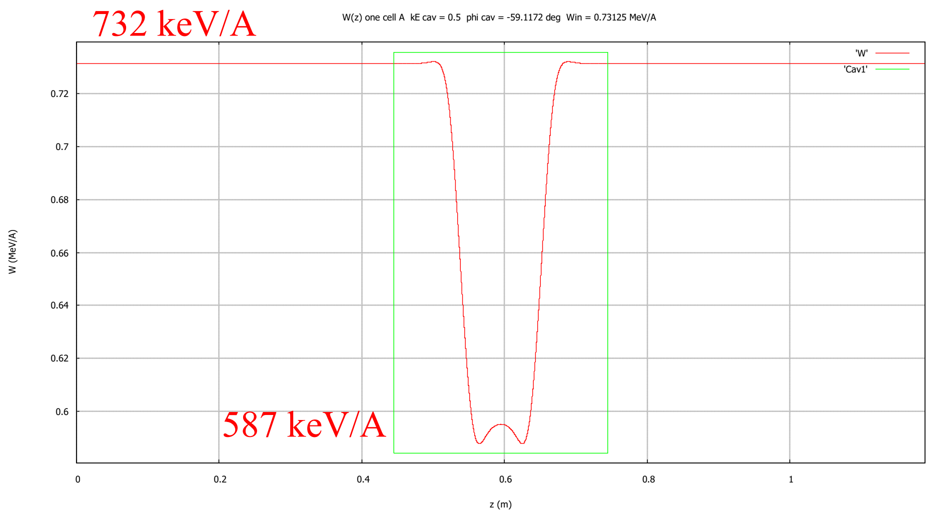
ALL !

(Nearly all !)

# 3- What changes at high accelerating fields 1/1




Energy gain for a -180 / +180° cavity phase scan
SPIRAL2 cavity A at 3.25 MV/m (half nominal)
protons, W_in = 732 keV (RFQ output energy)




Energy evolution in buncher mode
SPIRAL2 cavity A at 3.25 MV/m (half nominal)
deuterons, W_in = 732 keV/A (RFQ output energy)

High accelerating field at low energy → large evolution of β in the cavities

As said by Panofsky in his 1951 paper

in this case the beam dynamics (Φs and T) must be computed from tracking in cavity field maps **How ?**

# 4- Φs and TTF computations 1/3

Compute **1: ΔW** @ tracking **2: Φs** @ Φs definition (?) **3: T** = ΔW / q Eo L cos(Φs)

TraceWin "Historic model"

(rf phase when reference particle at cavity electrical center)

$$tan[\Phi_s] = \frac{\int_{-\infty}^{+\infty} q\ E_z(s)\ sin(\Phi(s))\ ds}{\int_{-\infty}^{+\infty} q\ E_z(s)\ cos(\Phi(s))\ ds}$$

Ok at low accelerating field, several issues at high accelerating field discovered working on SPIRAL2

# 4- Φs and TTF computations 1/3

Compute **1: ΔW** @ tracking **2: Φs** @ Φs definition (?) **3: T** = ΔW / q Eo L cos(Φs)

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$$tan[\Phi_s] = \frac{\int_{-\infty}^{+\infty} q\, E_z(s)\, sin(\Phi(s))\, ds}{\int_{-\infty}^{+\infty} q\, E_z(s)\, cos(\Phi(s))\, ds}$$

Ok at low accelerating field, several issues at high accelerating field discovered working on SPIRAL2

SPIRAL2 codes : (Φs ,T) definition to obtain the correct $\Delta W_s$ **and** longitudinal focalization around Φs

$$\Delta W_s = qE_0L\ T\ \cos(\Phi_s) \longrightarrow T\cos(\Phi_s) = \frac{\Delta W_s}{qE_0L}$$

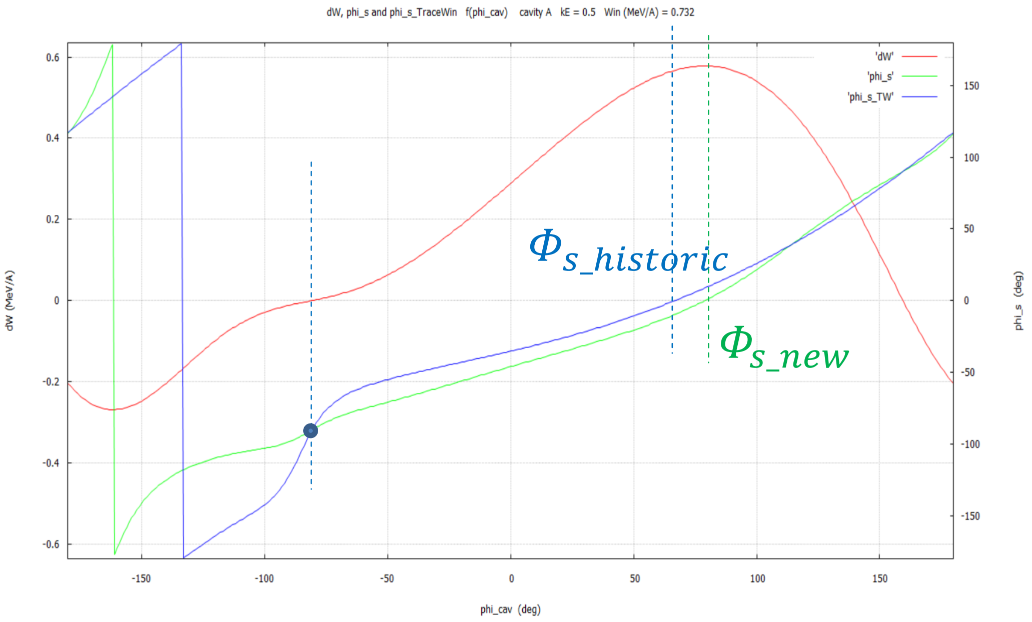
$$\left.\frac{d\Delta W}{d\Phi}\right|_{\Phi_s} = -qE_0L\ T\sin(\Phi_s) = fm_{21} \longrightarrow T\sin(\Phi_s) = \frac{-fm_{21}}{qE_0\mathrm{L}}$$

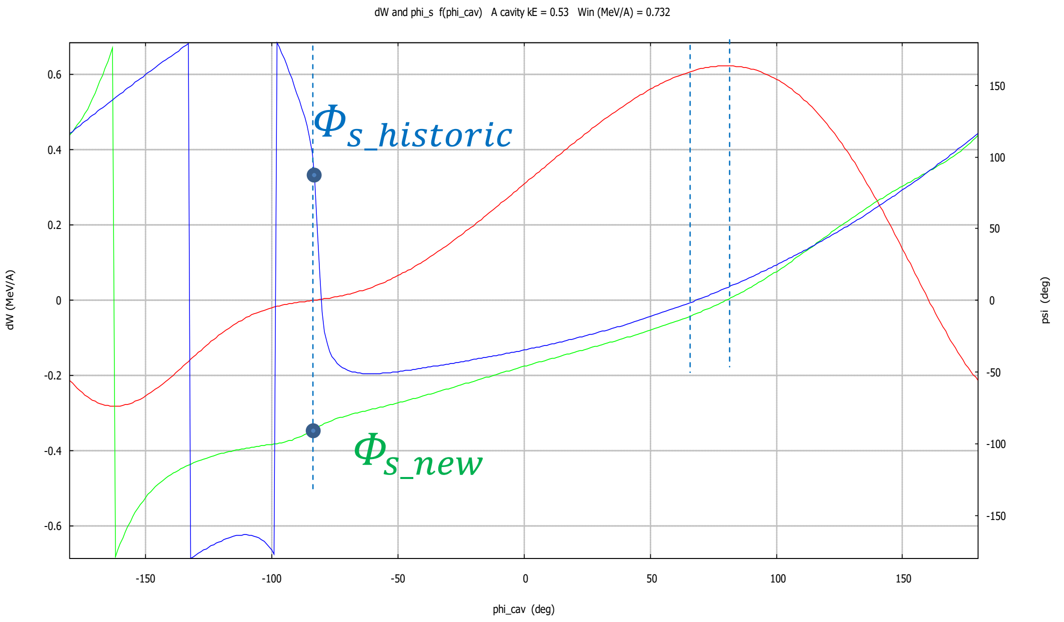
$$\mathrm{tg}(\Phi_s) = \frac{-fm_{21}}{\Delta W_s}$$

TraceWin "new model"

$fm_{21}$ = '21' coefficient of the "cavity field-map transfer matrix"
$\Delta W$ and $fm_{21}$ computed tracking the reference particle in field map (very fast, fm transfer matrix for $\sigma_{0l}$)

# 4- Φs and TTF computations 2/3




SPIRAL2 cavity A -180 / +180° phase scan
red = energy gain $\Delta W(\varphi_{cav})$, green = $\Phi_s$ SP2, blue = $\Phi_{s\_TW}$
protons, W_in = 732 keV (RFQ energy), 3.25 MV/m (half nominal)

$\Delta W$ max not at $\Phi_{s\_historic}$ = 0° (17° shift)

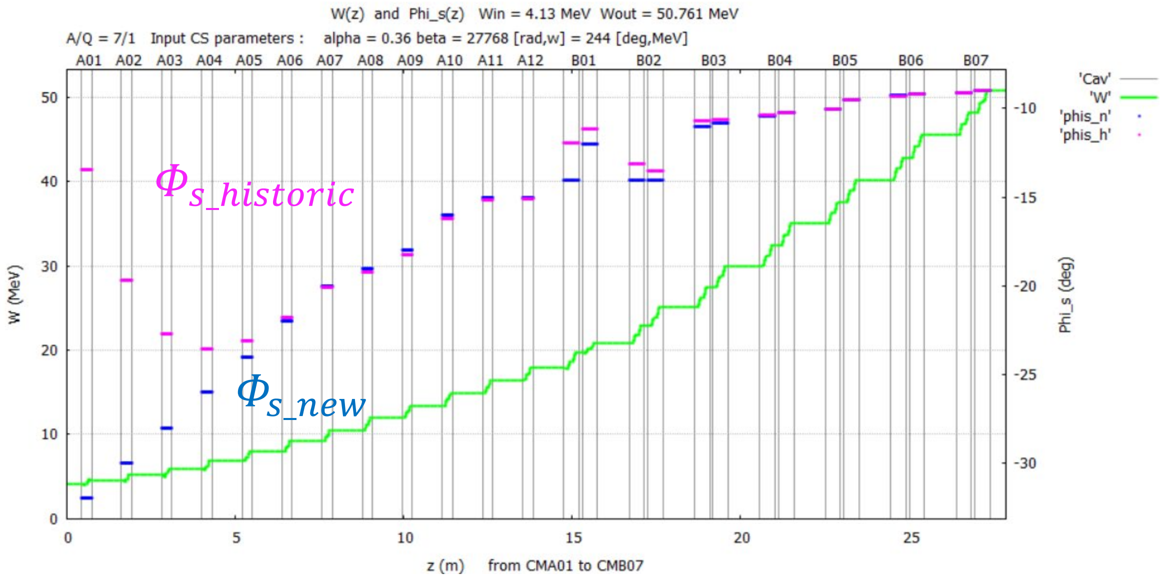
kE = 0.53

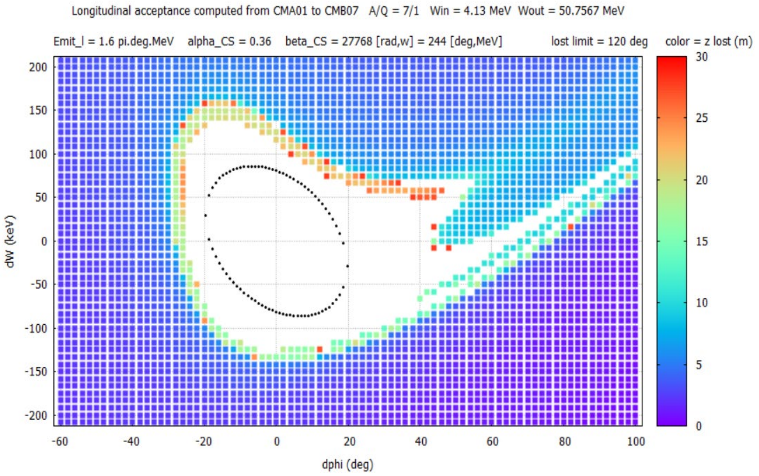
$\Phi_{s\_historic}$ = 0 not at $\Delta W$ max and never = -90°
Buncher mode : $\Delta W$ = 0 $\Phi_{s\_historic}$ = +90°

Large phi_cav shifts (up to ~30°) between $\Phi_{s\_historic}$ and $\Phi_{s\_new}$ → Not the same linac tuning

## A/Q = 7 (U238) SPIRAL2 linac tuning (NewGAIN studies)

Two different beam dynamics for a linac tuned using $\Phi_{s\_historic}$ or $\Phi_{s\_new}$




Longitudinal acceptance
consistent with $\Phi_{s\_new}$ **not** with $\Phi_{s\_historic}$

$\Phi_{s\_new}$ definition = "New model" in TraceWin

"Use new synchronous phase definition" option *Thank you Didier !*

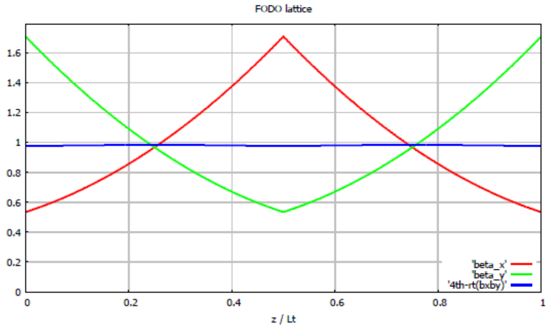
# 5- $\sigma_l$ = 90°, space-charge and cavity-field excitation

JM. Lagniel, "Zero-current longitudinal beam dynamics", LINAC14
JM. Lagniel, "Excitation of the σl_l = 90° resonanceby the cavity RF field",
IPAC22 and Institute of Physics Journal of Physics: Conference Series

## EXCITATION BY THE SPACE-CHARGE FIELD

Fourth-order parametric resonance … and the other longitudinal parametric resonances …

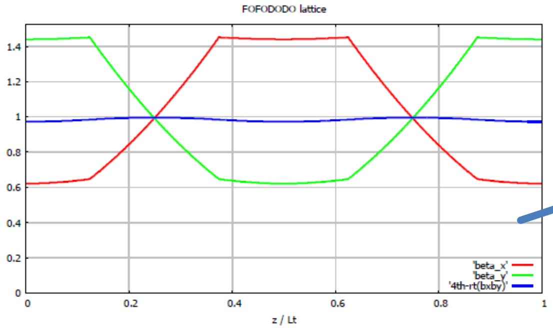
Longitudinal phase advance $\sigma_{l\_l}$ = per longitudinal period $\sigma_{l\_t}$ = per transverse period




**FDO (doublet, solenoid) $\sigma_{l\_t}$ = $\sigma_{l\_l}$ => excitation in phase opposition with the longitudinal envelope oscillation**

(lower space-charge effect vs constant $a_x.a_y$)

longitudinal space charge force (via the Poisson equation) $\propto \dfrac{1}{\sqrt{a_x\, a_y}\, b^2}$




**FODO $\sigma_{l\_t}$ = 2 $\sigma_{l\_l}$ NO excitation by the transverse plane**

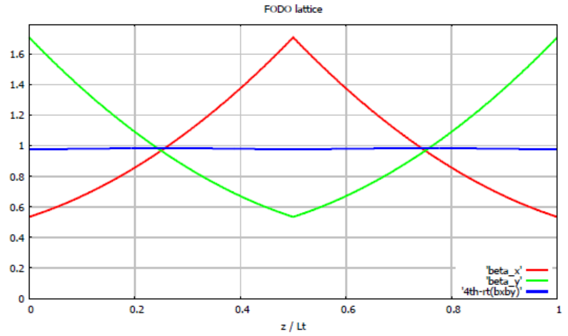
**FOFODODO $\sigma_{l\_t}$ = 4 $\sigma_{l\_l}$** nearly **NO excitation by the transverse plane**

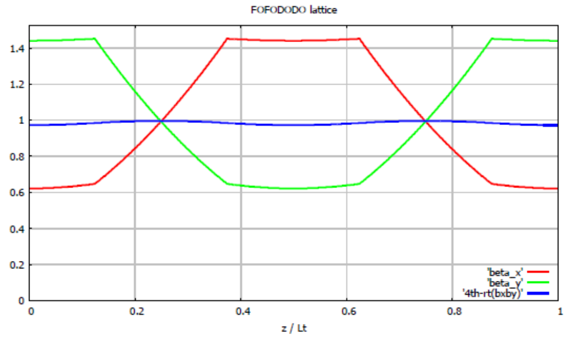
# 5- $\sigma_l$ = 90°, space-charge and cavity-field excitation

JM. Lagniel, “Zero-current longitudinal beam dynamics”, LINAC14
JM. Lagniel, “Excitation of the σl_l = 90° resonanceby the cavity RF field”, IPAC22 and Institute of Physics Journal of Physics: Conference Series

## EXCITATION BY THE SPACE-CHARGE FIELD

**longitudinal phase advance** **$\sigma_{l\_l}$ = per longitudinal period** **$\sigma_{l\_t}$ = per transverse period**




**FDO (doublet, solenoid) $\sigma_{l\_t}$ = $\sigma_{l\_l}$ => excitation in phase opposition with the longitudinal envelope oscillation**

**FODO $\sigma_{l\_t}$ = 2 $\sigma_{l\_l}$ NO excitation by the transverse plane**

**FOFODODO $\sigma_{l\_t}$ = 4 $\sigma_{l\_l}$** nearly **NO excitation by the transverse plane**

It is a mistake to consider $\sigma_{l\_t}$ studying the 90° resonance in the longitudinal plane

It is a mistake to design a linac with $\sigma_{l\_t}$ < 90° (FODO, FOFODODO)

# 5- $\sigma_l$ = 90°, space-charge and cavity-field excitation

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IPAC22 and Institute of Physics Journal of Physics: Conference Series

## EXCITATION BY THE CAVITY RF-FIELD

**Cavity rf-field**

**= nonlinear longitudinal focusing force**

$$= [\cos(\Phi_s + \delta\varphi) - \cos\Phi_s] =$$

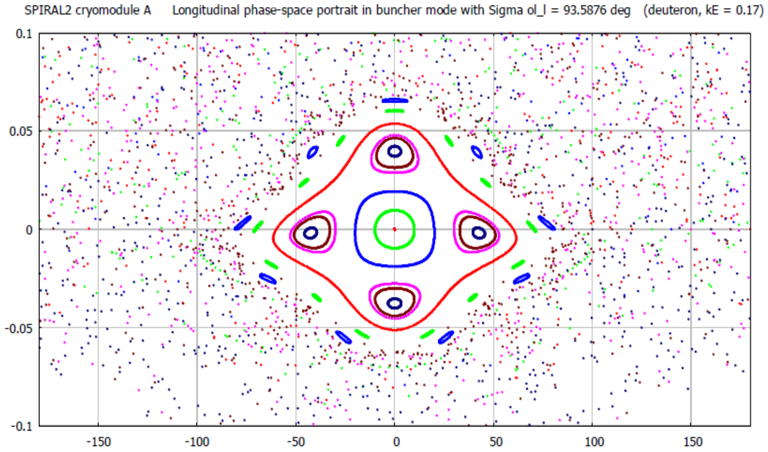
$- [\sin\Phi_s]\,\delta\varphi$ "quadrupole" (linear focusing)

$- [\cos\Phi_s/2!]\,\delta\varphi^2$ "sextupole"

$- [\sin\Phi_s/3!]\,\delta\varphi^3$ "octupole"

$+ [\cos\Phi_s/4!]\,\delta\varphi^4$ "decapole" + ...

Taylor series




Phase-space portrait, particle tracking in the SPIRAL2 low beta cavities
Buncher mode, $\sigma_{0l\_l}$ = 93.6° (kE = 0.17), RFQ output energy

NO SPACE-CHARGE

$\boldsymbol{\sigma_{0l\_l} = 93.6°}$ => excitation of 1/4 = 90° and all lowest order parametric resonances (1/6 = 60°, 1/8 = 45° …)

Low-order resonance overlap => chaotic sea, separatrix destruction

Huge longitudinal acceptance reduction

# 5- $\sigma_l$ = 90°, space-charge and cavity-field excitation

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## EXCITATION BY THE CAVITY RF-FIELD

**Cavity rf-field**
**= nonlinear longitudinal focusing force**

$$= \quad [\cos(\Phi_s + \delta\varphi) - \cos\Phi_s] =$$

$- [\sin\Phi_s]\,\delta\varphi$ "quadrupole" (linear focusing)

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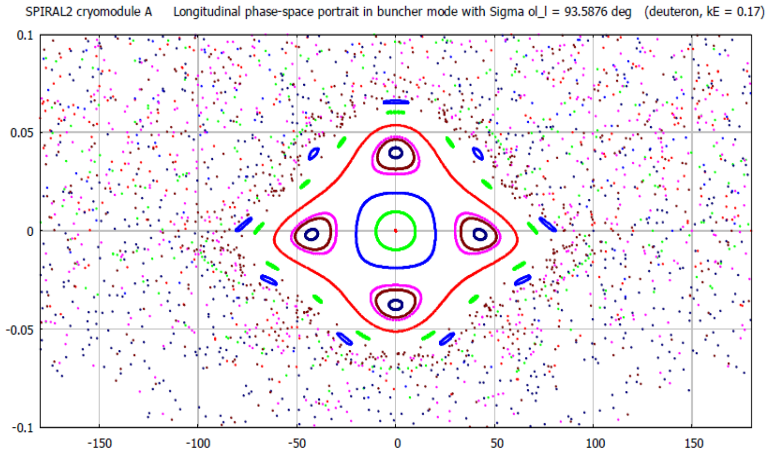
$+ [\cos\Phi_s/4!]\,\delta\varphi^4$ "decapole" + ...




The $\sigma_l$ = 90° resonance main source of excitation (as well as the other parametric resonances in the longitudinal plane !)
is the cavity rf-field, Not space-charge !
Excitation period = cavity period => consider $\sigma_{l\_l}$ not $\sigma_{l\_t}$ ... again

# THANK YOU FOR YOUR ATTENTION!

## MERCI !

My last HB

## VIVE HB !

# $\sigma_l > 90°$ resonance experiment at SPIRAL2 ?

DIFFICULT !

1- Diagnostics : only BPM

2- Matching to the linac : buncher #3 far from cavity #1

3- Low-beta to high-beta matching

4- Which linac tuning ? $\Phi_s$ = -90° all along the linac ?

Phase acceptance estimation shifting RFQ and buncher phases ?

Comparison "$\sigma_l < 90°$" vs "$\sigma_l > 90°$" with good matchings in both cases