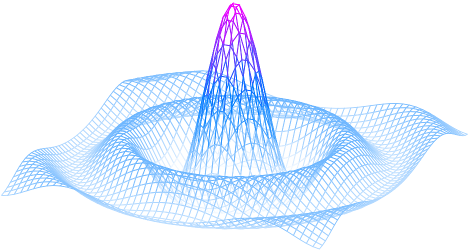

Transverse measurements of statistical dependence in the PSB

68^{th} ICFA Advanced Beam Dynamics Workshop on High-Intensity and High-Brightness Hadron Beams
CERN, Geneva (CH), October 9-13, 2023

E. Lamb, F. Asvesta, H. Bartosik, M. Seidel, G. Sterbini
Thanks to K. Paraschou, T. Prebibaj, A. Fornara

Motivation

- We observe **non-Gaussian like beam profiles**, specifically heavy-tailed, throughout the CERN accelerator complex, in the LHC and the injectors [1][2].

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- In the context of high energy, and high intensity accelerators, it is **important to address the tails of the distribution** in detail. The tails can cause a problem for losses in machines and consequent performance **limitation and availability**, for example during LHC injection.
- Ideally, we would like to know the **normalised 4D phase space reconstruction** given a matched heavy-tail beam, allowing for investigations of loss mechanisms, luminosity, and lifetime.

Distribution reconstruction

We define a **normalised phase space** from the physical space for **linear machines** via a transformation, yielding a rotationally symmetric x-p_x phase space. α_x, β_x are the machine optic functions:

$$\begin{bmatrix} 1/\sqrt{\beta}_x & 0 \\ \alpha_x/\sqrt{\beta}_x & \sqrt{\beta}_x \end{bmatrix} \begin{bmatrix} X_1 & P_{x_1} \\ X_2 & P_{x_2} \\ \dots & \dots \end{bmatrix}^T$$

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We can define an observed profile, as the integration of a 4D transverse distribution [3], (neglecting 6D),

$$f_{\mathrm{1D}}(x) = \iiint f_{\mathrm{4D}}(x, p_x, y, p_y)\ \mathrm{d}p_x\ \mathrm{d}y\ \mathrm{d}p_y.$$

We normalise the distribution to the intensity,

$$\iiiint_{-\infty}^{\infty} f_{\mathrm{4D}}\left(x, p_x, y, p_y\right) \mathrm{d}x\ \mathrm{d}p_x \mathrm{d}y\ \mathrm{d}p_y = 1.$$

Distribution reconstruction

There are constraints on the normalised phase space x-p_x and y-p_y, that the distribution is circularly symmetric, so we can find the 2D distribution via an inverse Abel transform [4][5].

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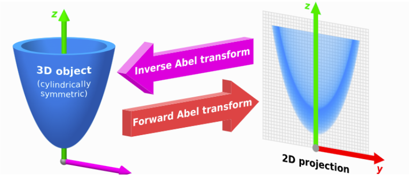
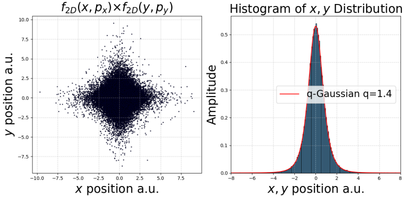


[6]

Addressing the full 4D phase space, there are no constraints on the x-y projection. The inversion from a profile, to a 4D distribution, **does not have a unique solution** for heavy-tailed beams, in a **linear** machine.

Non-unique distribution reconstruction

a: **factorizable** in x–y



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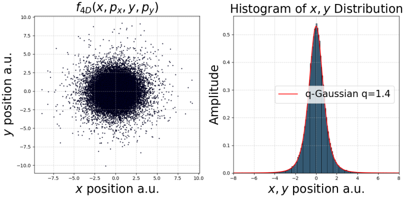

- We show the example of a **q-Gaussian [7] profile with a q of 1.4**, a heavy-tailed beam. Reconstructed under two scenarios, factorizable 2D distributions, a), and circularly symmetric x-y projection, b).

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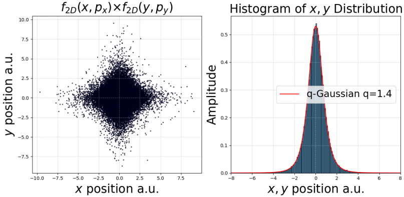

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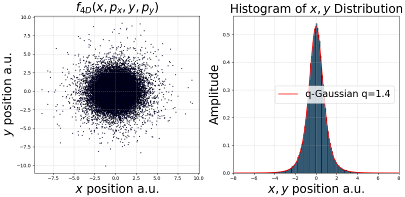

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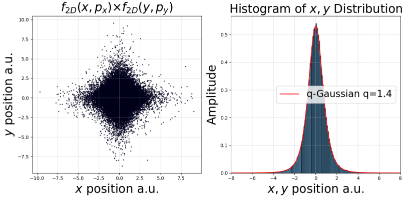

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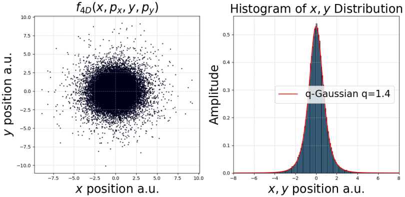

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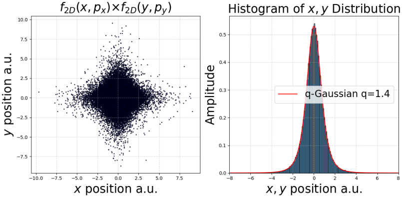

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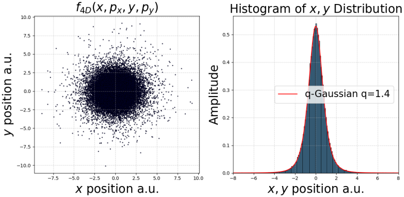

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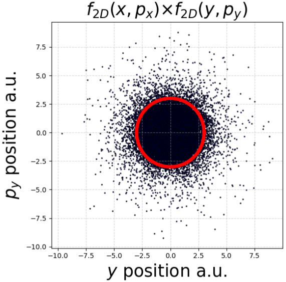
- Both distributions (**a** and **b**) are **matched** and give the same x and y projections.

Dependence for non-factorizable distributions through losses

Taking the two distributions from the previous slide:



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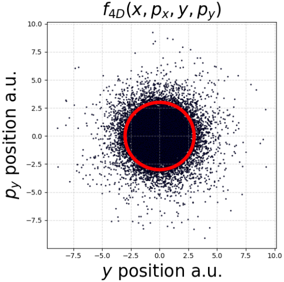

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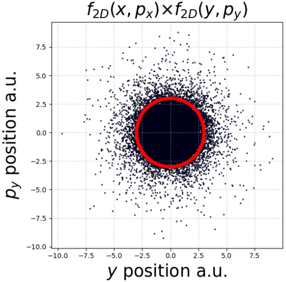

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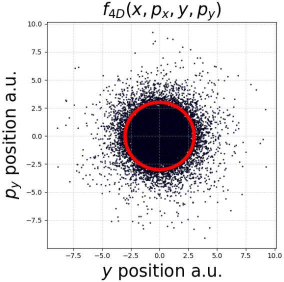

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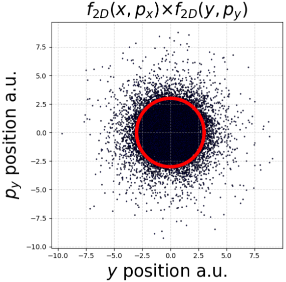

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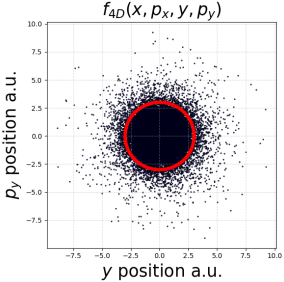

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The beam sees an aperture at 3σ in both cases $\rightarrow$

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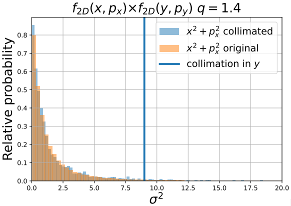

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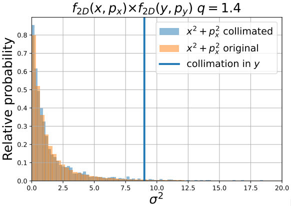

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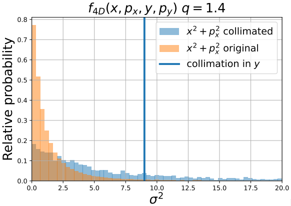

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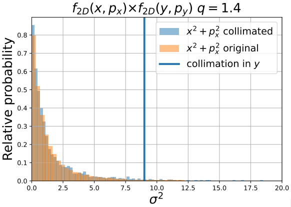

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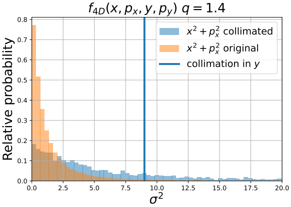

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- For the rotationally symmetric x-y plane, the profile is changed in x when the beam is shaved in y. This is a result of the distributions being non-factorizable in x-y.
- In a linear machine, both distributions, or a combination, can match the profiles seen.

Luminosity considerations for non-factorizable beams

- In general, we focus on the x-p_x, y-p_y planes. However, finding the full 4D distribution is important for Luminosity. In general, the luminosity integral is calculated using factorizable beam distributions for both Gaussian and non-Gaussian beams [9] [1].
- It was found during Van Der Meer scans for luminosity calibration that the 'non-factorizable' x-y distribution of the real beam contributes to an error in calibration in the LHC [10].

$$\mathcal{L} \propto A \iiiint_{-\infty}^{\infty} \rho_{1x}(x)\rho_{1y}(y)\rho_{1s}\left(s-s_0\right)\rho_{2x}(x)\rho_{2y}(y)\rho_{2s}\left(s+s_0\right)\ \mathrm{d}x\ \mathrm{d}y\ \mathrm{d}s\ \mathrm{d}s_0,$$

$A = 2N_1N_2N_bf_{\mathrm{rev}}$, where $N_{1,2}$ is the particle number, f_{rev} the revolution frequency, ρ the particle density functions.

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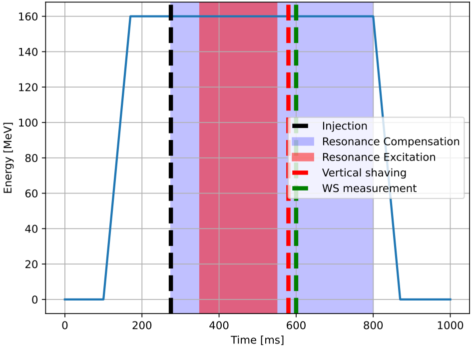
- It is hypothesized the **dependence is preserved** after the resonance excitation is removed, as all that is remaining is a 'linear' machine.

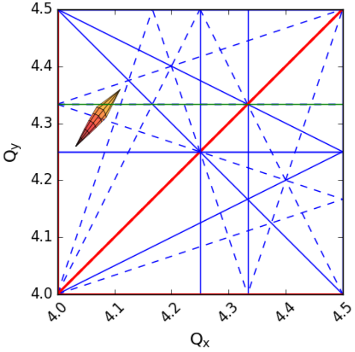
Experimental procedure

- A working point near the resonance is chosen (to give blow up but **no losses** when excited), and the resonances in the region are corrected [13].
- A low intensity beam of 50×10^{10} protons is injected in Ring 1, and the particular resonance is excited or not excited for a period. **Then the excitation is removed, and then the beam is shaved using the vertical shaver**.
- The beam is observed both on the Horizontal and Vertical planes.

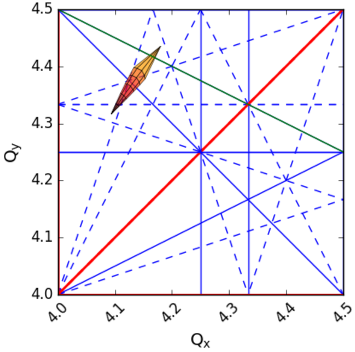


Working point and resonance selection

Two resonances were selected, a 1D $3Q_y = 13$, and 2D coupled $Q_x + 2Q_y = 13$. The two working point diagrams show the resonance in green and the space charge tune-spread. The tune spread is calculated with PySCRDT [14][15].



$3Q_y = 13$

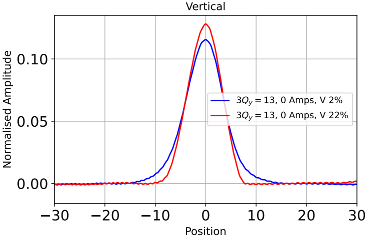


$Q_x + 2Q_y = 13$

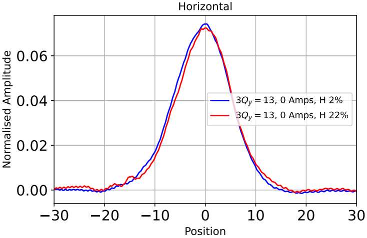
Results - Transverse Profiles near $3Q_y = 13$

- The profiles are normalised to the area, to account for the intensity reduction by the shaving.

V - Resonance excitation OFF



H - Resonance excitation OFF



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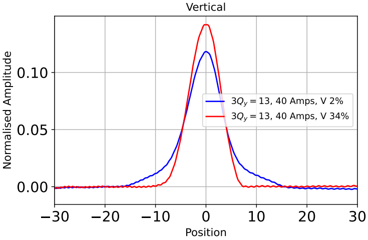

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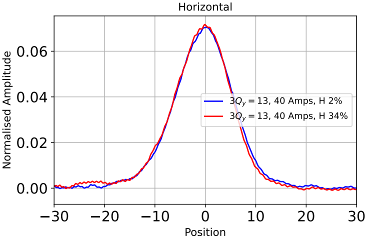
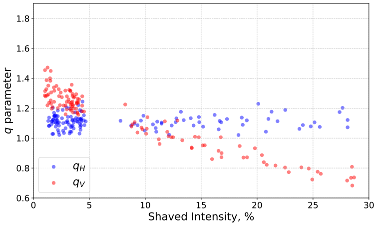


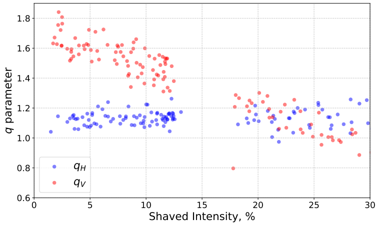
Results - Fitted data near $3Q_y = 13$

- The profiles are fitted with a q-Gaussian, and the *q*-parameter is plotted as function of the shaved intensity (via a vertical shaving).
- The gap seen is due to limitations of the shaver orbit bump.

Resonance excitation OFF

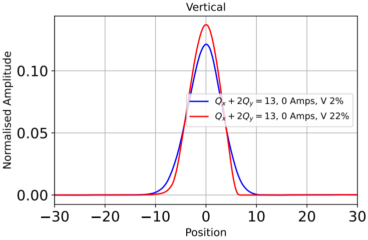


Resonance excitation ON

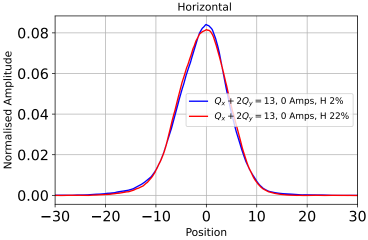


Results - Transverse Profiles near $Q_x + 2Q_y = 13$

V - Resonance excitation OFF



H - Resonance excitation OFF



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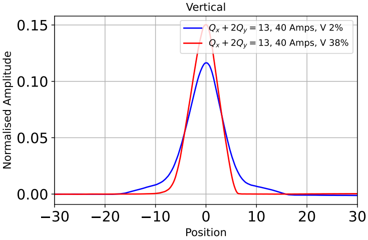

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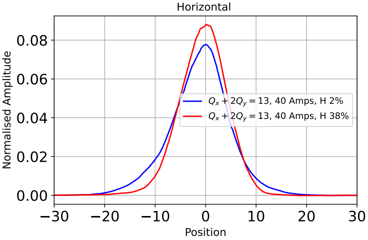
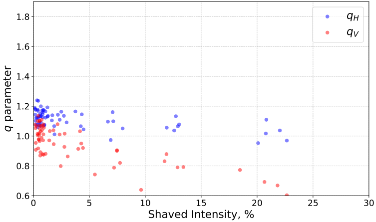


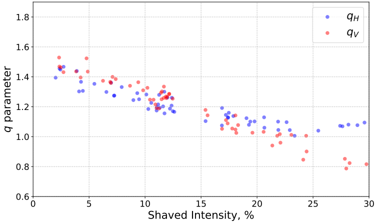
Results - Fitted data near $Q_x + 2Q_y = 13$

- The q_H changes as a function of q_V when the resonance is excited.
- After the halo has been removed, the q_H levels out, pointing to a non-linear source (dependence), and not a linear coupling effect.

Resonance excitation OFF



Resonance excitation ON



Summary and discussion

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- We have measured dependence being introduced into the distribution in the PSB via **coupled resonances**. This mechanism is not so far from a **potential operational mechanism** (high space charge and crossing coupled resonances).
- Further **simulation** and experiments can be performed to assess the level of dependence in operational beams (if any) and if dependence is **transferred** from machine to machine.

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[14] Pyscrdt.

[15] Tune spread.

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