ANALYTICAL AND NUMERICAL STUDIES ON KICKED BEAMS IN THE CONTEXT OF HALF-INTEGER STUDIES

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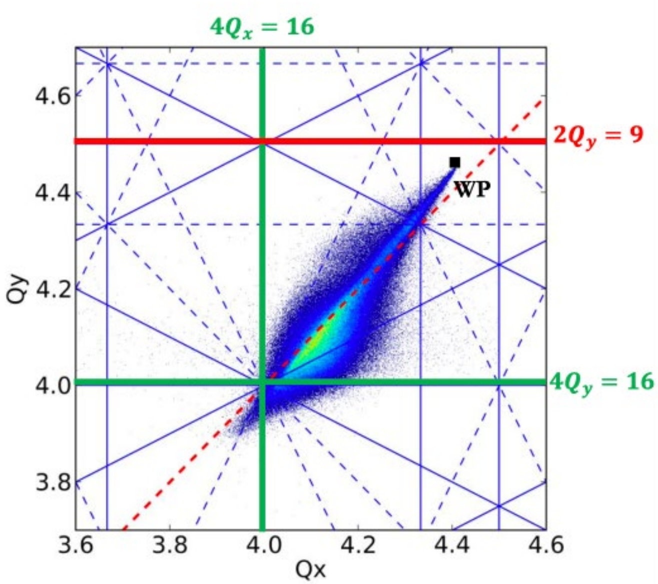
11.10.2023

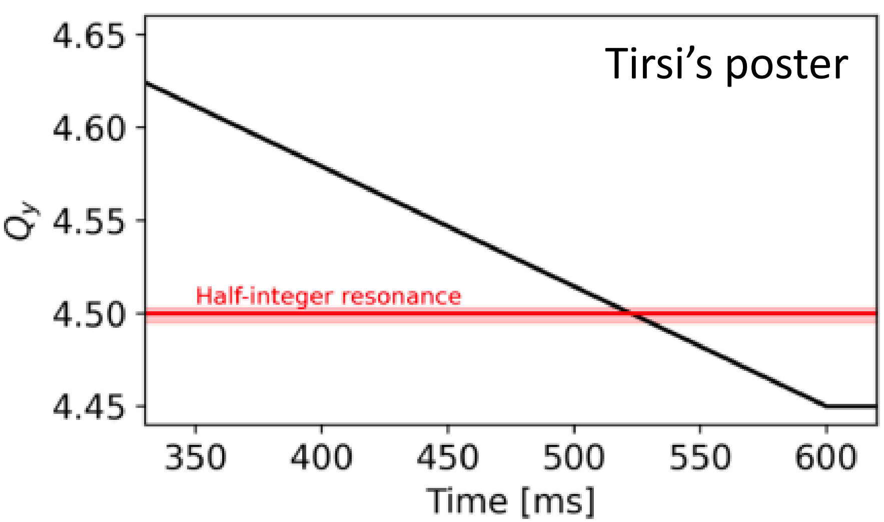
HB 2023

Context →
half-integer studies at the CERN-PSB

Studies at the PSB on half-integer resonance



Studies on the effect of space charge on the half-integer



The dynamics of resonance crossing has brought to the attention the interplay of

Chromaticity ↔ **Space charge** ↔ **Coherent oscillations**

in a Coasting Beam

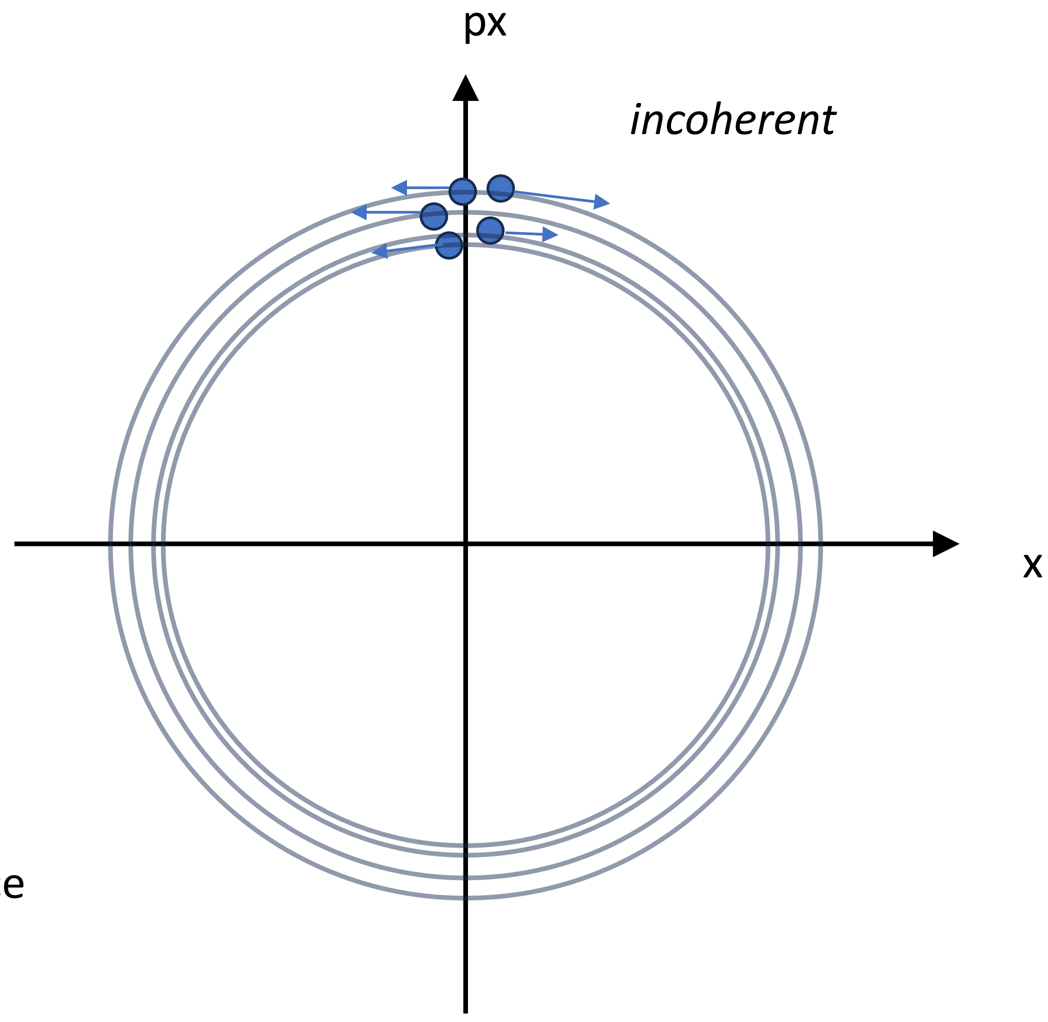
Chromaticity

$$\langle \delta p \rangle = 0$$
$$\langle \delta p^2 \rangle = \sigma_p^2$$

Each particle has a tune-shift as

$$\Delta Q_x = Q_{x0} \xi \delta p$$

ξ Is taken positive for convenience



Betatron amplitude modulated by

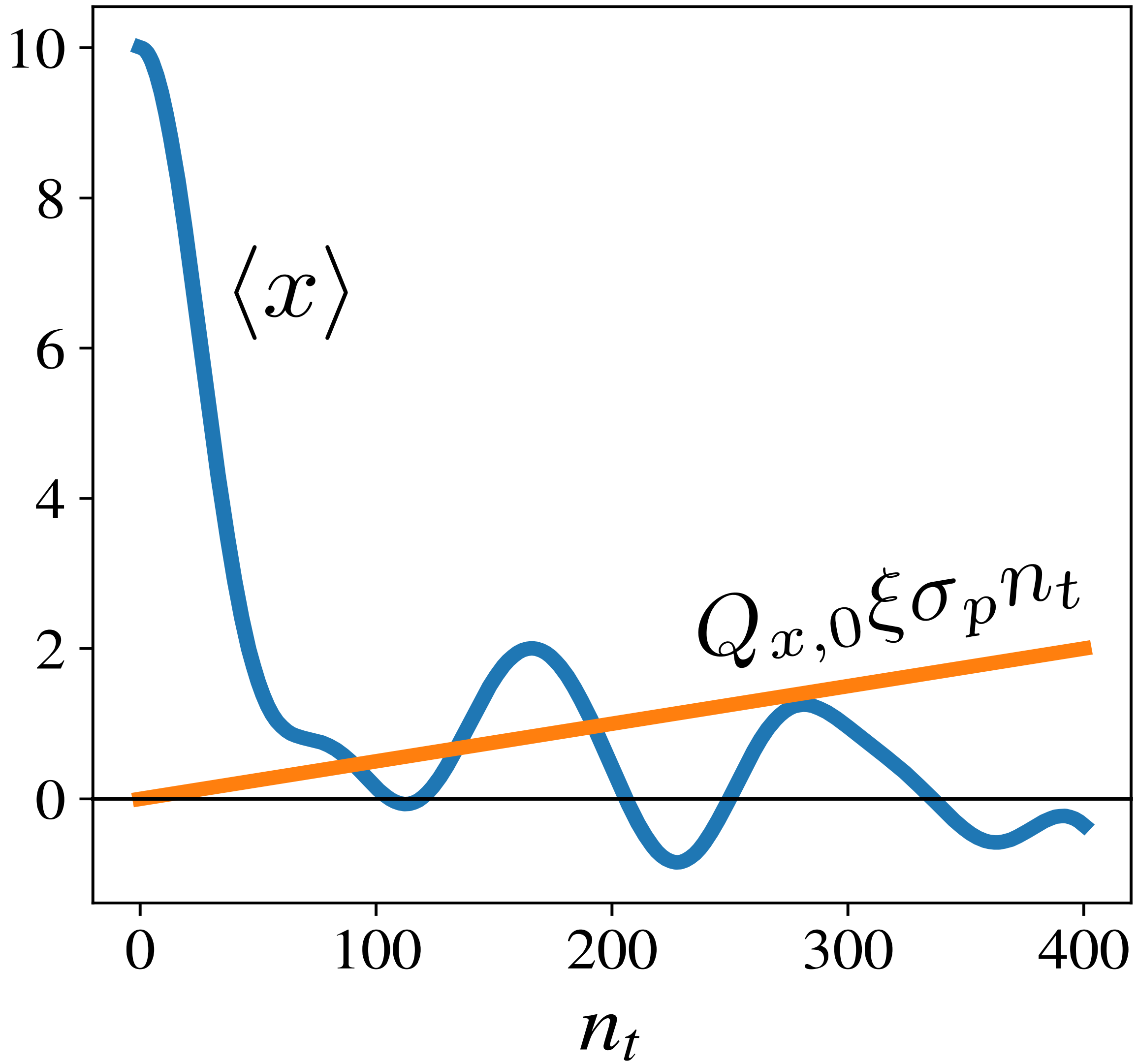
$$\Lambda\left(\frac{Q_{x0}\xi_x\sigma_p}{R}s\right)$$

$$\Lambda(u) = \int \cos(u\lambda) g(\lambda) d\lambda$$

For a Gaussian distribution

$$\Lambda(u) = \exp\left(-\frac{1}{2}u^2\right)$$

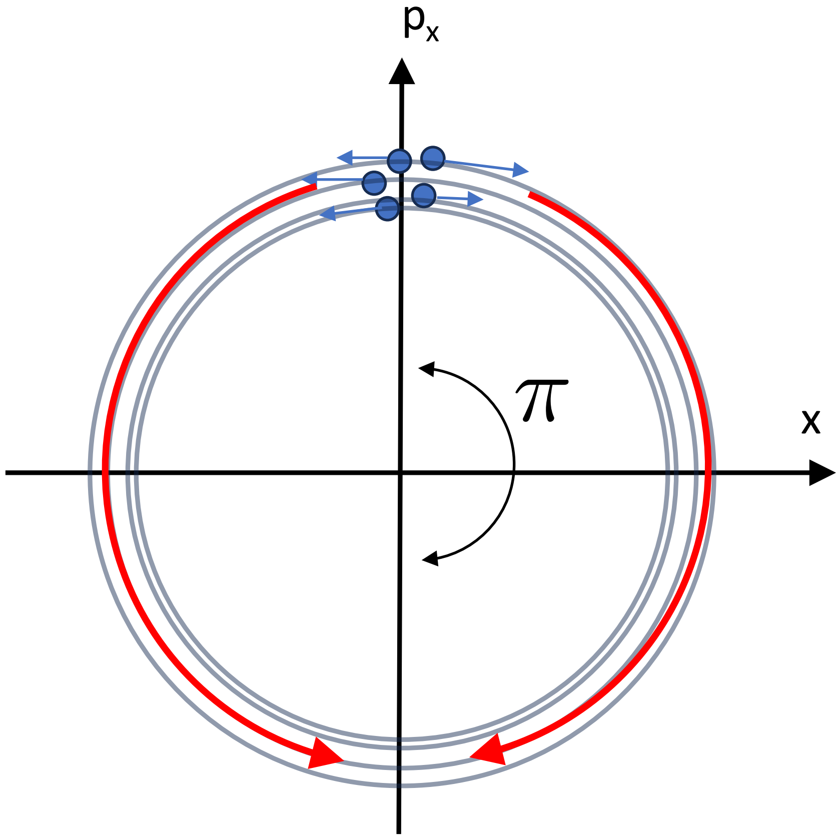
Characteristic scaling



Characteristic scaling

Decoherence has a characteristic scale proportional to

$$Q_{x,0}\xi\sigma_p n_t \simeq 0.5$$



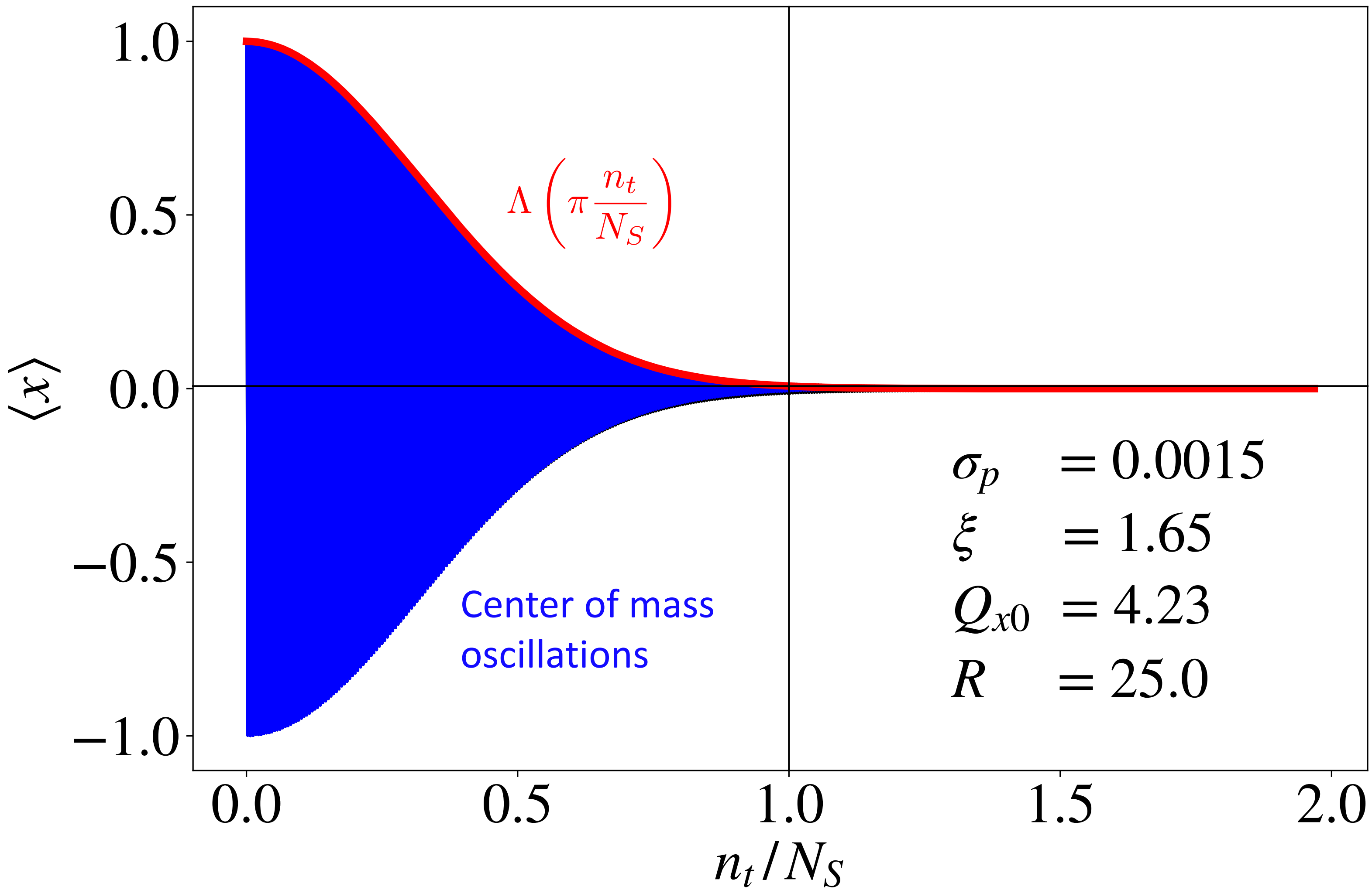
Define the rms chromatic detuning

$$\delta Q_\xi = Q_{x,0}\xi\sigma_p$$

Define the reference turn number

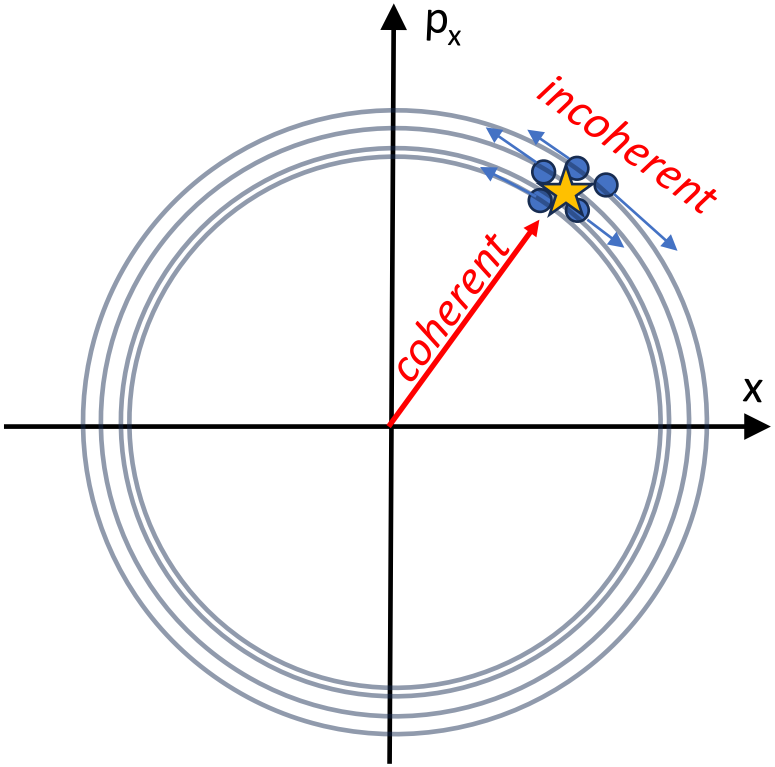
$$N_S = \frac{1}{2\delta Q_\xi}$$

For a Gaussian distribution in δp/p

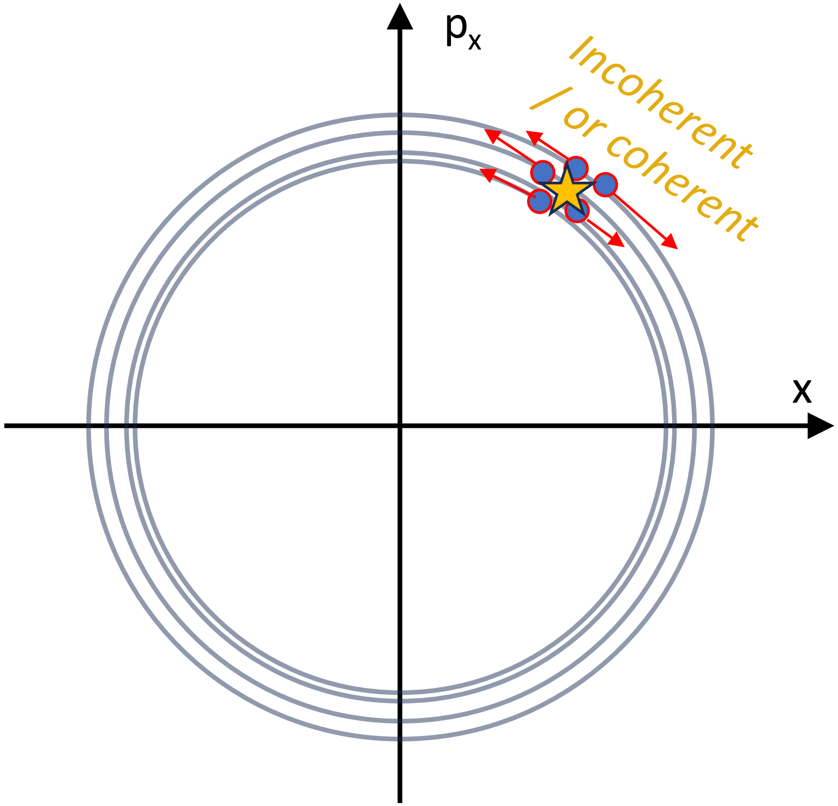


Space charge and chromaticity

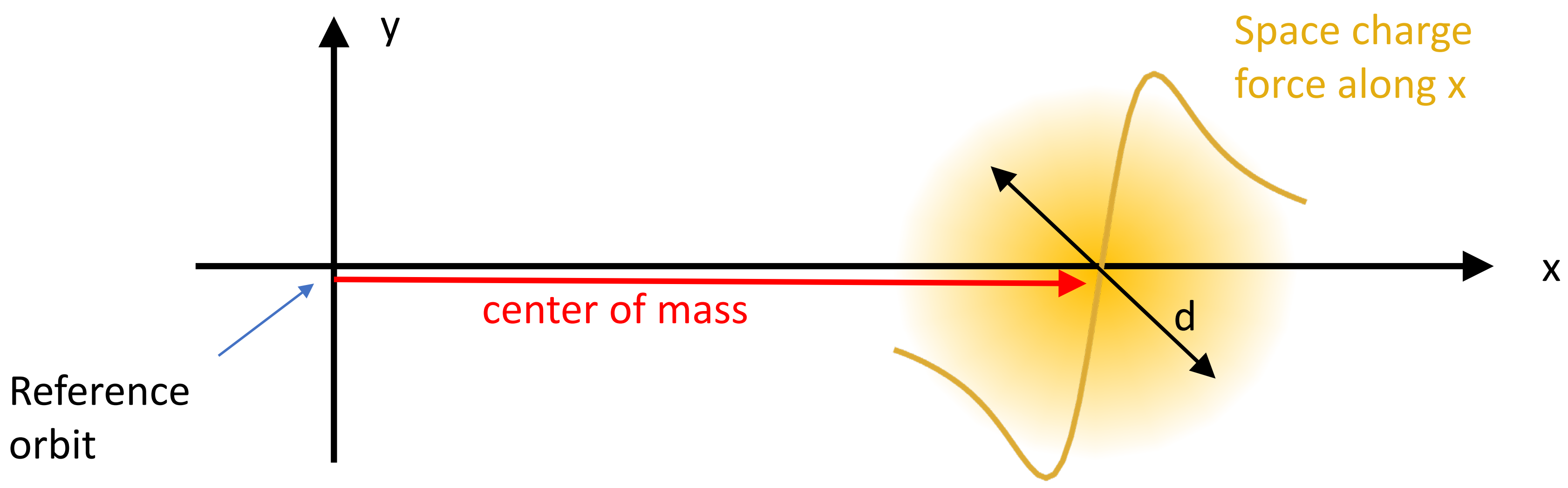
Effect of chromaticity →
Damping by decoherence



The effect of space charge is a defocusing force.
It is zero on the center of mass.

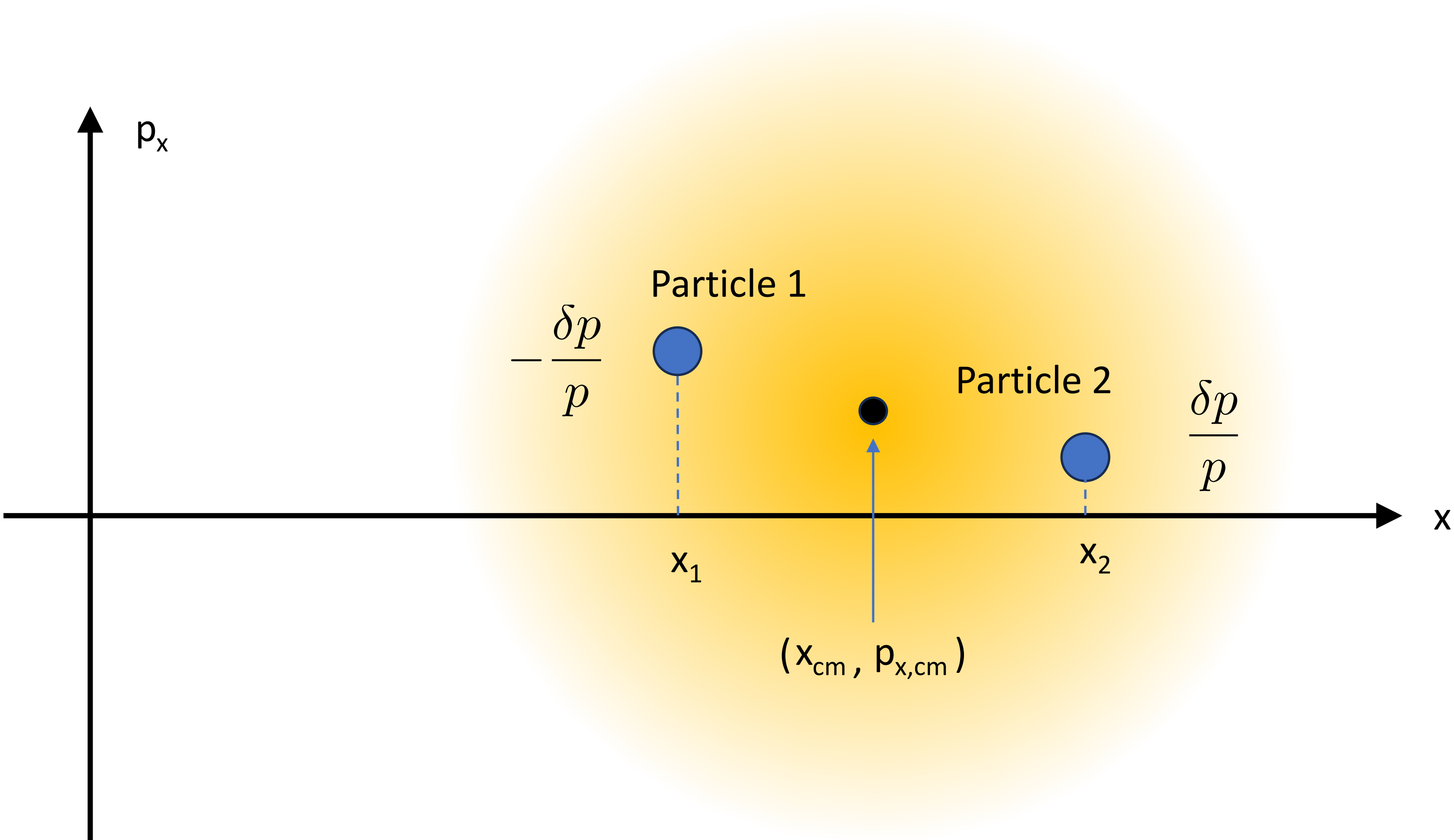


Space charge forces



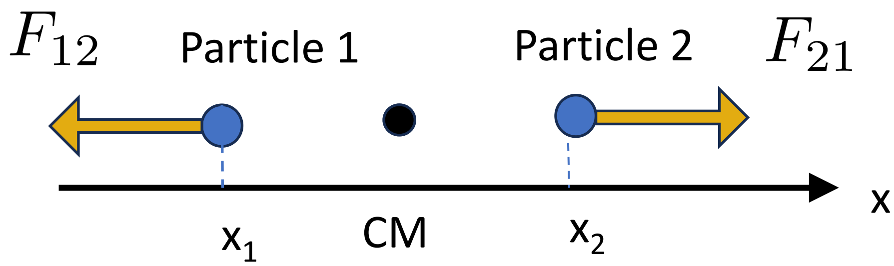
1) A particle *close* to the beam center feels a linear force $\propto r$
2) If it is *away* from it, it decays as $\propto 1/r$
3) "*close*" and "*away*" are is relative to the center of mass and "d"

Two-Particle Model



→ Two particles represent the beam

Two-Particle Model: forces



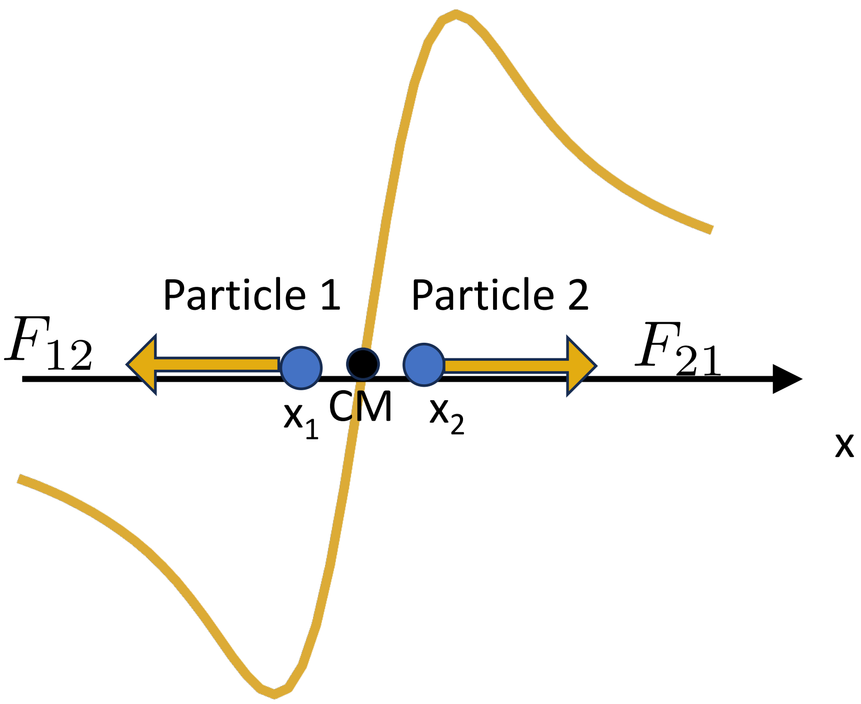
$$F_{12} = \frac{x_1 - x_2}{|x_1 - x_2|} f(x_{12})$$

$$f(x_{12}) = \lambda \frac{|x_1 - x_2|}{d^2 + (x_1 - x_2)^2}$$

d = is a characteristic length
λ = coulomb strength

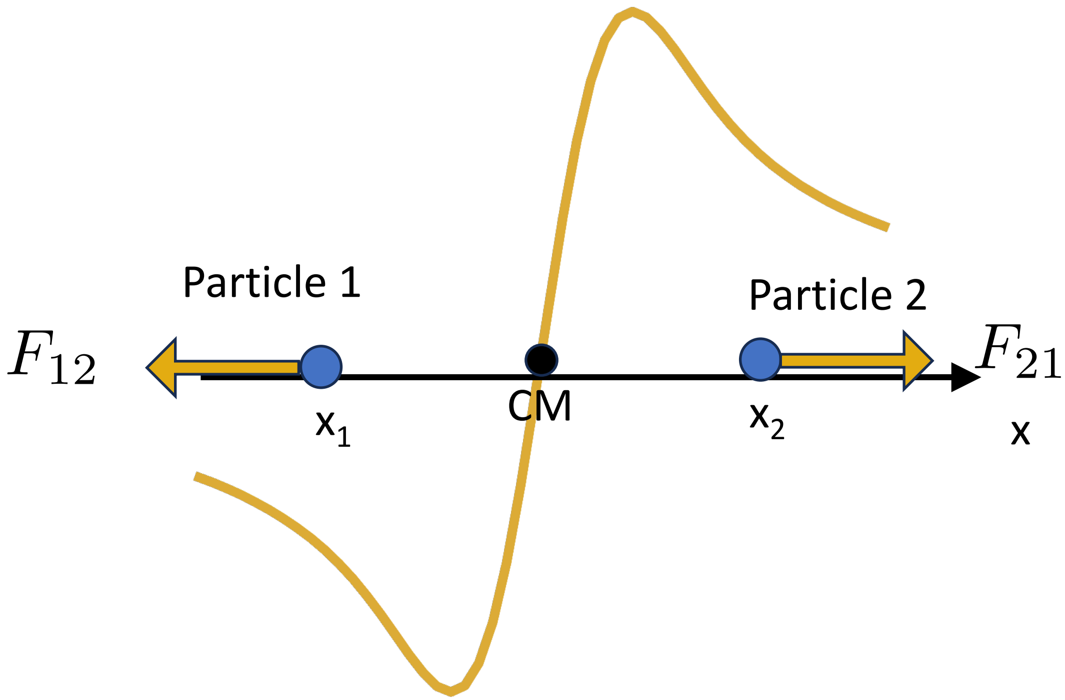
Properties → $$F_{12} + F_{21} = 0$$

“close” particles



$$f(x_{12}) \simeq \frac{\lambda}{d^2}|x_1 - x_2|$$

“away” particles



$$f(x_{12}) \simeq \lambda \frac{1}{|x_1 - x_2|}$$

Equations of motion

In the reference closed orbit we have

$$\begin{cases} x_1'' + \dfrac{(Q_{x,0}+Q_{x,0}\xi\delta p/p)^2}{R^2}x_1 & = & F_{12} \\ x_2'' + \dfrac{(Q_{x,0}-Q_{x,0}\xi\delta p/p)^2}{R^2}x_2 & = & F_{21} \end{cases}$$

Scaling

$$z = \frac{2}{d}x$$

$$z_{cm} = \frac{z_1 + z_2}{2}$$

This coordinate is the "scaled center of mass"

$$z = z_1 - z_{cm}$$

This coordinate is the "scaled the beam size"

Equations of motion in the scaled coordinates

$$\begin{cases} \ddot{z}_{cm} + k z_{cm} + \Delta k z = 0 \\ \ddot{z} + k z + \Delta k z_{cm} = 2\frac{\lambda}{d^2}\frac{z}{1+z^2} \end{cases}$$

Center of mass

One particle

Focusing of the lattice

Effect of the chromaticity + **δ**p/p

Effect of "space charge"

$$\Delta k = 2k\delta Q_\xi / Q_{x,0}$$

No space charge case

$$\begin{cases} \ddot{z}_{cm} + kz_{cm} + \Delta k z = 0 \\ \ddot{z} + kz + \Delta k z_{cm} = 0 \end{cases}$$

Focusing in z, z_{cm} is on the linear coupling resonance

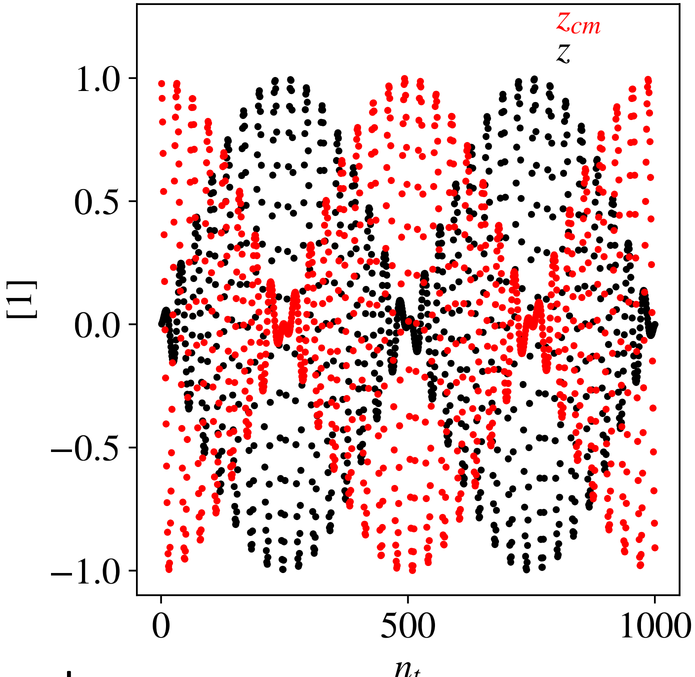
Linear coupling with strength **Δ**k

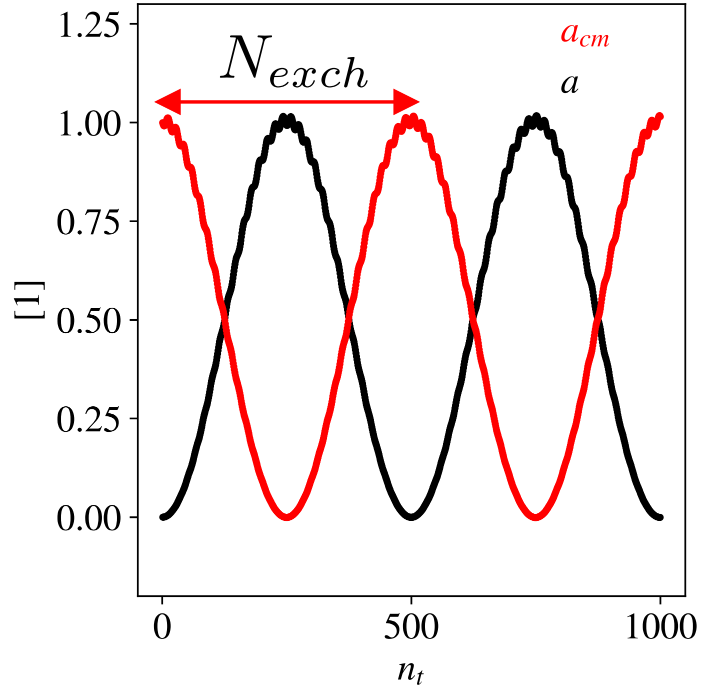
Emittance exchange between the dynamical variables z and z_{cm}

Emittance exchange

Call a, a_{cm} the emittances of z, z_{cm}.

Example $\delta Q_\xi = 10^{-3}$ $N_{exch} = 500$





In general

$$N_{exch} = \frac{1}{2\delta Q_\xi} \quad \Rightarrow \quad N_{exch} = N_S$$

Including space charge

$$\begin{cases} \ddot{z}_{cm} + k z_{cm} + \Delta k z = 0, \\ \ddot{z} + k_d z + \Delta k z_{cm} = -2\frac{\lambda}{d^2}\frac{z^3}{1+z^2}, \end{cases}$$

with k_d a new focusing strength associated with the incoherent tune-shift

$$k_d = k - 2\frac{\lambda}{d^2} = \frac{(Q_{x,0} + \Delta Q_{x,sc})^2}{R^2}$$

$$\Delta Q_z = \Delta Q_{x,sc} = -\frac{\lambda R^2}{Q_{x,0} d^2}$$

Therefore the linear space charge "detunes" the system from the linear coupling resonance

Partial emittance exchange

Amount of emittance exchange

$$\frac{a_{\max}}{a_{cm,0}} = \left[1 + \left(\frac{k_-}{\Delta k}\right)^2\right]^{-1}$$

Scaled exchange

Number of turns of the exchange periodicity

$$\sqrt{1 + \left(\frac{k_-}{\Delta k}\right)^2} \frac{N_{\text{exch}}}{N_S} = 1$$

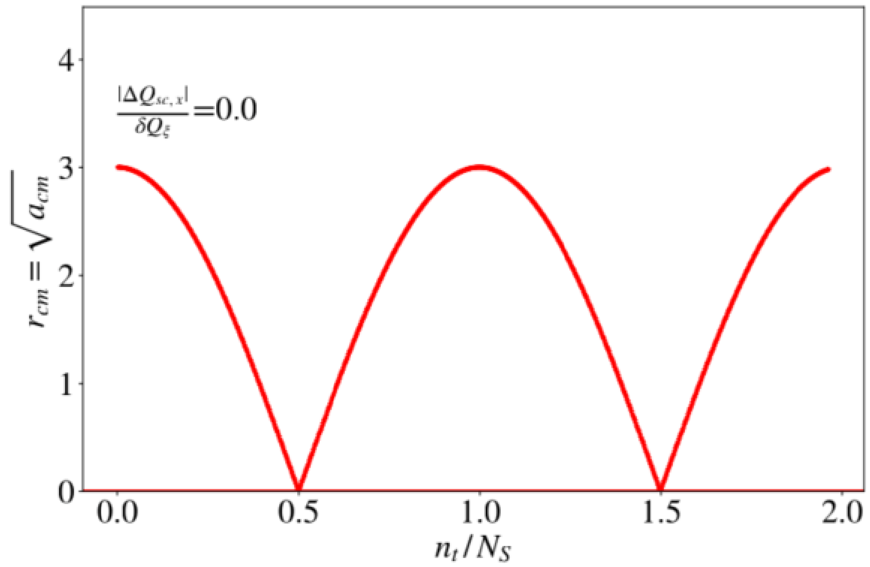
Scaled periodicity

With

$$k_- = \frac{k - k_d}{2} \quad \Rightarrow \quad \left|\frac{k_-}{\Delta k}\right| = \frac{1}{2} \frac{|\Delta Q_{sc}|}{\delta Q_\xi}$$

General parameter controlling the process

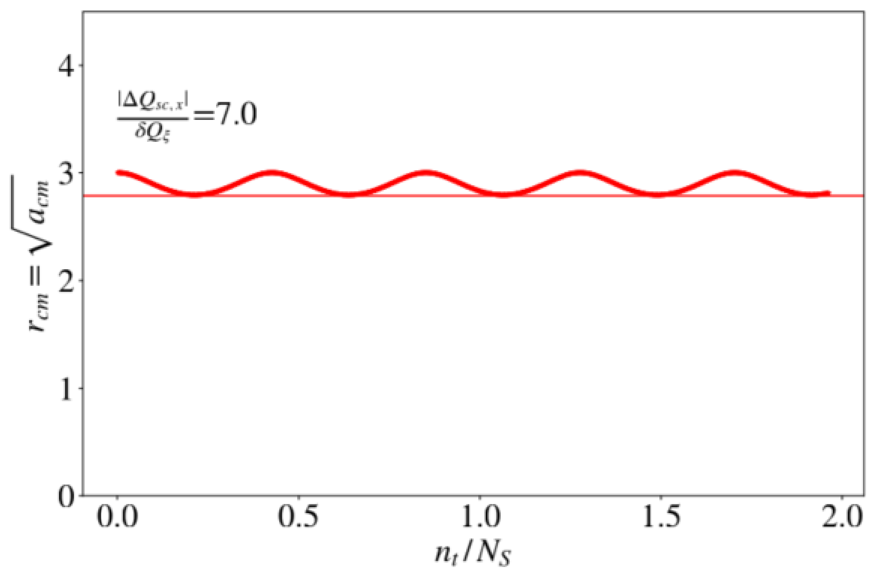
Summary



No space charge →

1) Full emittance exchange
2) Periodicity $N_{exch} = N_S$

Full dechoerence and re-choerence due to the linear coupling
In a beam → irreversible decoherence



With enough space charge →

1) Partial emittance exchange
2) It depends on $|\Delta Q_{sc}|/\delta Q_\xi$
3) The periodicity scales with N_S

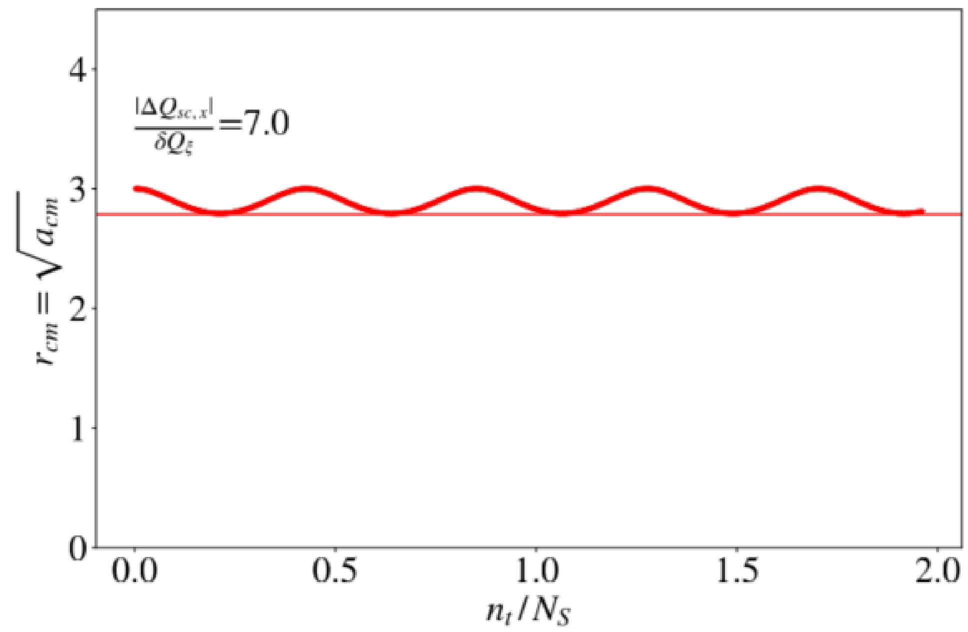
No decoherence:
space charge "detunes from the linear coupling resonance", → it prevents the decoherence from the chromaticity

Intriguing prospects

If the center of mass oscillates,
it is a measurable quantity

The motion of the center of mass
is of a "coherent" dynamics

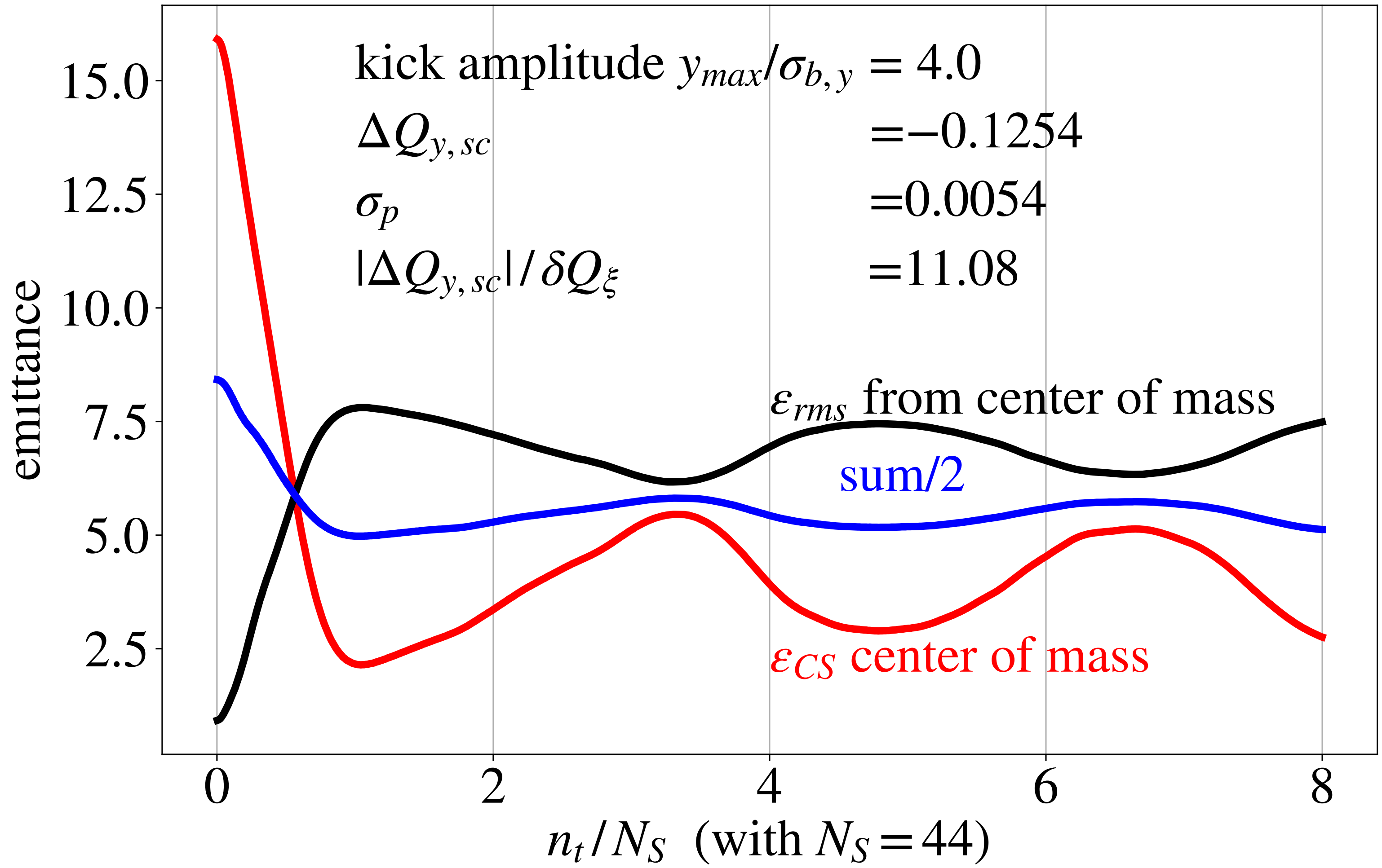
The motion with respect to the
center of mass "may be an
incoherent dynamics"



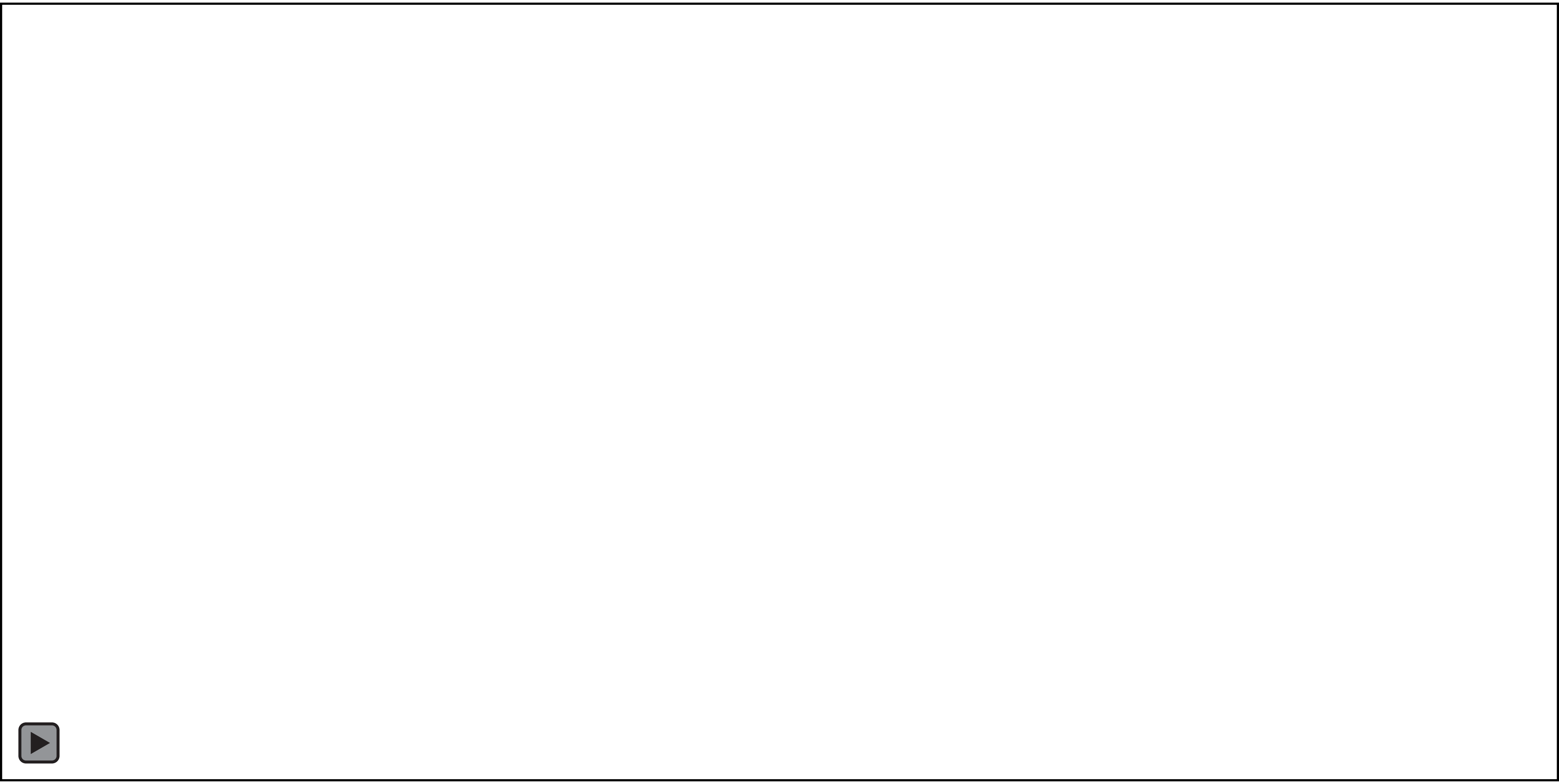
Periodic exchange of energy between two fundamentally different ``modes" of the dynamics.

Particle in Cell Simulations of a coasting beam

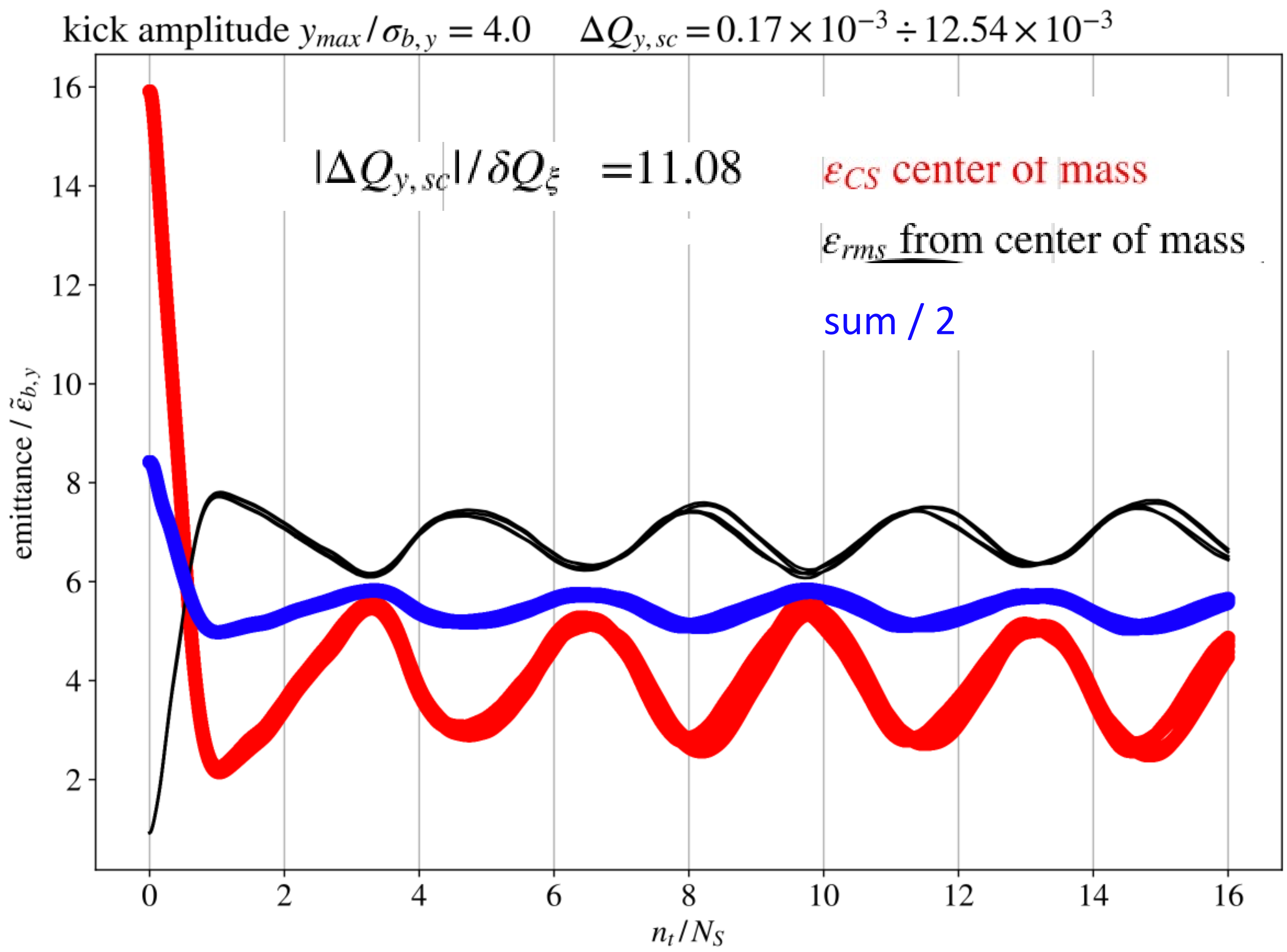
No impedance or collective effect. Pure direct field with image charge.



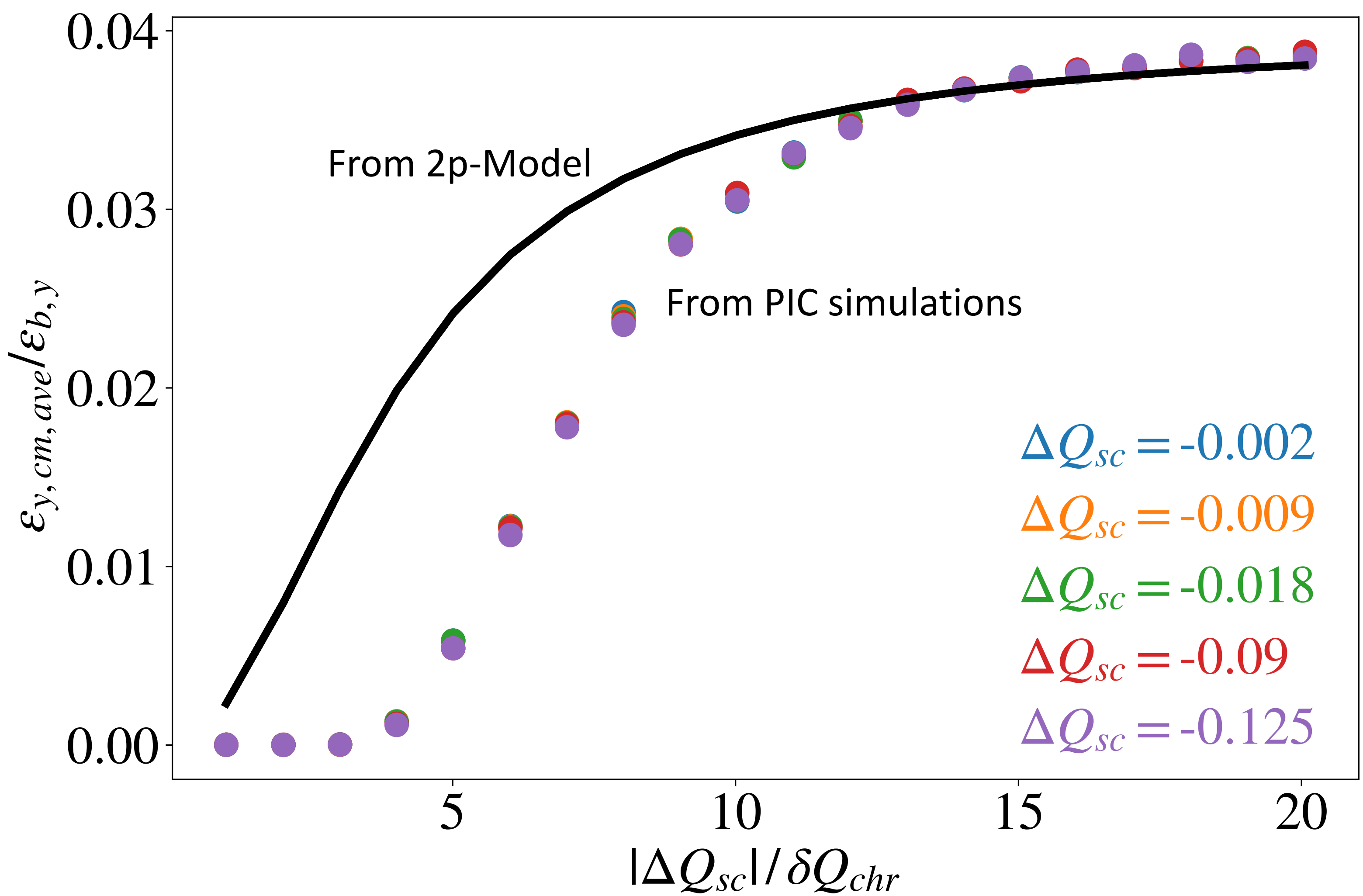
A deeper complexity…



However, the scaling works very well …



Another example: kick amplitude $y_{max} / \boldsymbol{\sigma}_{y,b} = 0.2$



Search of this effect in the CERN-PSB

Make use of a sp… center of mass

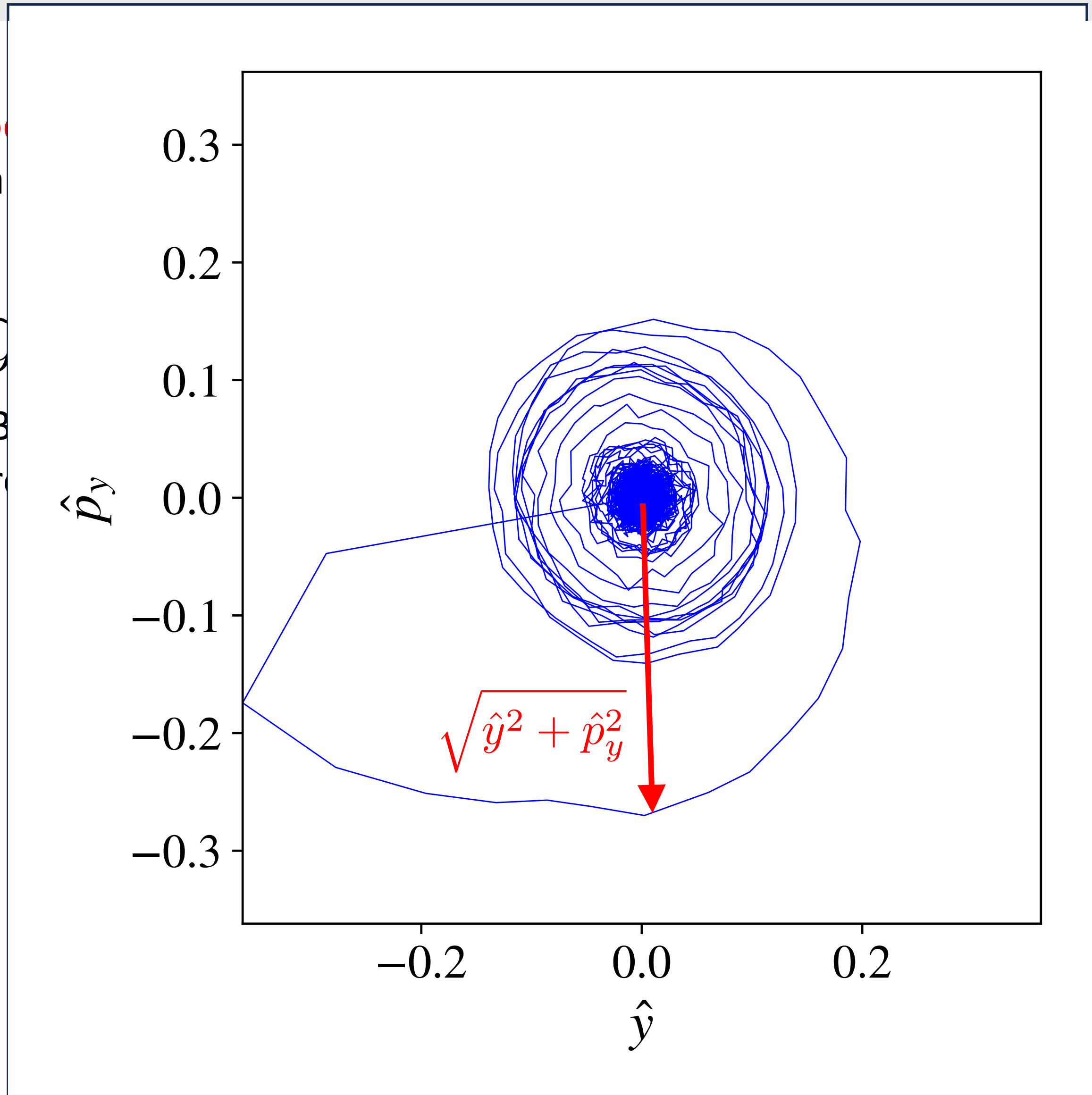
of a coasting bea… RF activated

$n_t \rightarrow \hat{y}$ … $_t)$

turn B…

r…

… $n_t)$



Another example of measurement results

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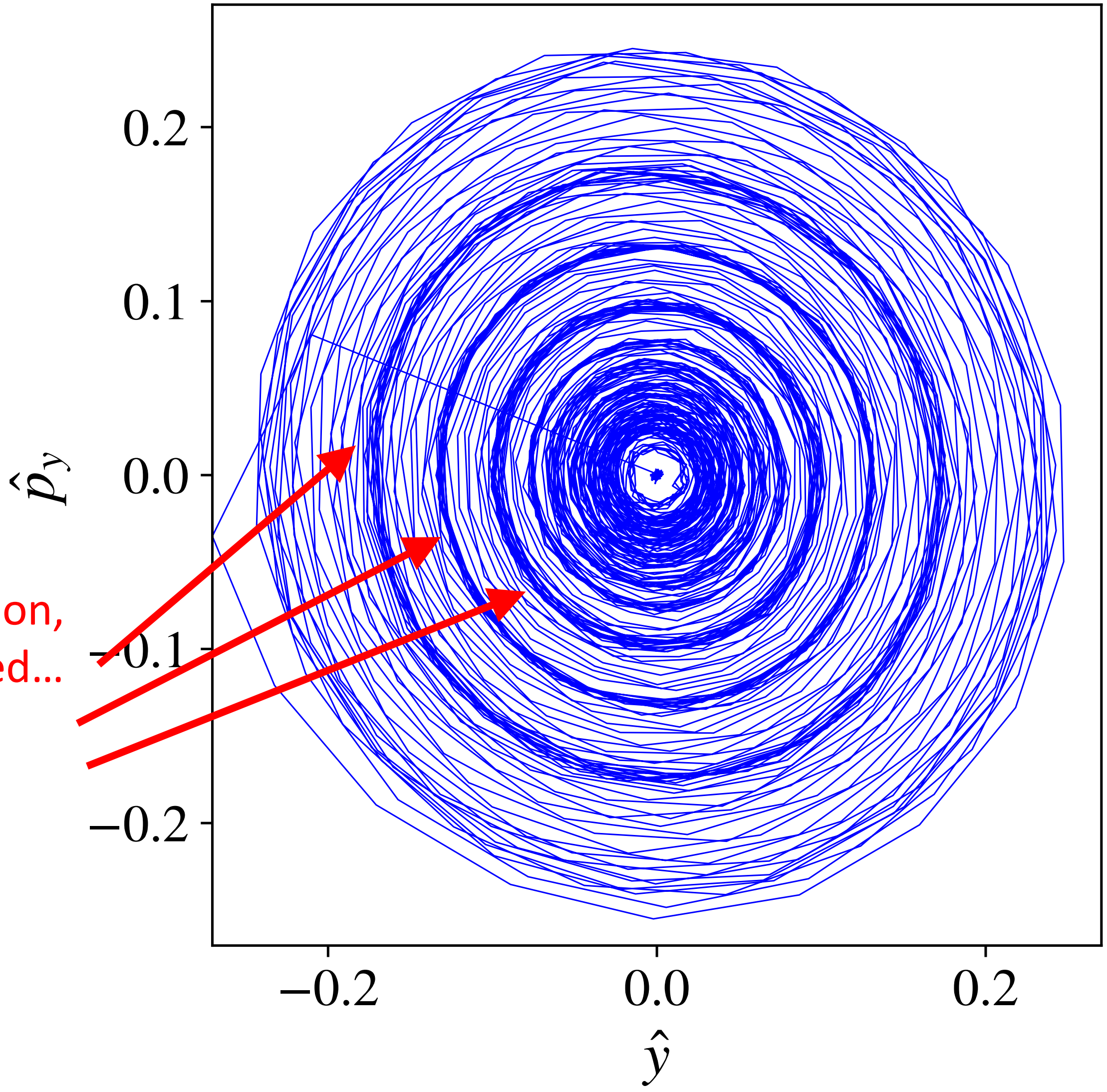

No optical illusion,
you are not tired…
You are seeing
3 distinct rings

PSB Measurements

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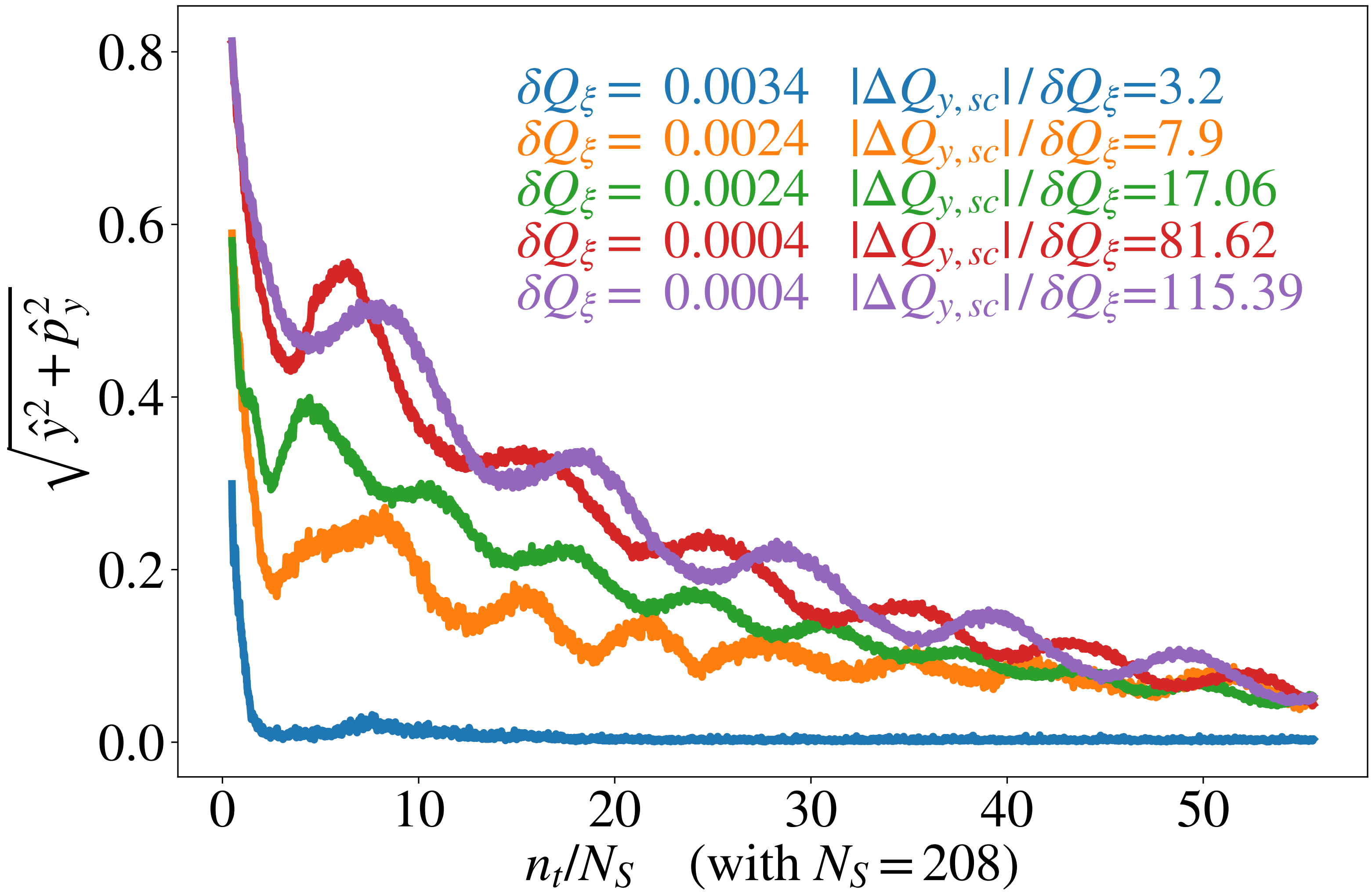

Summary & Outlook

- The joint effect of space charge and chromaticity on a kicked coasting beam has been investigated.
- A two-particle model suggests a linear coupling mechanism between coherent and incoherent dynamics of the kicked system
- This model allows us to retrieve the relevant scaling parameters of the dynamics
- Particle In Cell simulations of a kicked coasting beam follow the same scaling !!
- An experimental campaign at the CERN-PSB has confirmed that the center of mass exhibits ``beating oscillations'' !!
- More studies have to follow to fully interpret the experimental results.

Thank you for your attention