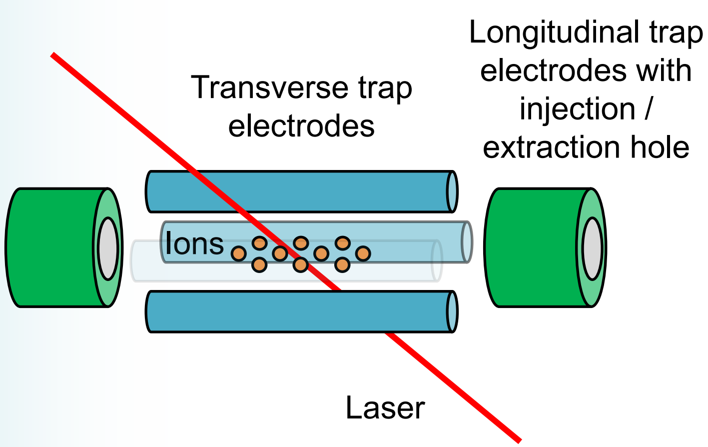
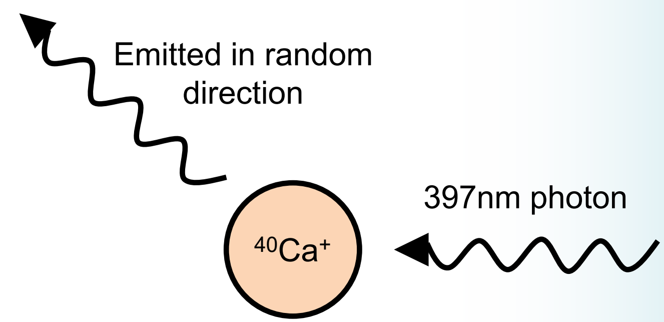
Ultra-low Emittance Bunches from Laser Cooled Ion Traps for Intense Focal Points

Ion Traps & Laser Doppler Cooling

- Paul trap configuration has four AC transverse electrodes (like alternating gradient focussing) and DC longitudinal end caps

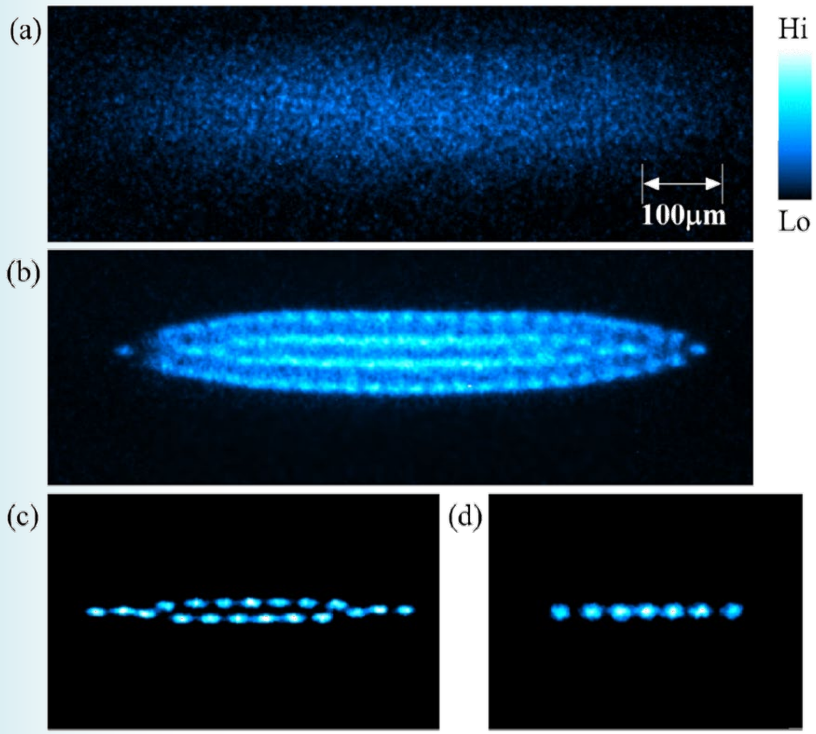




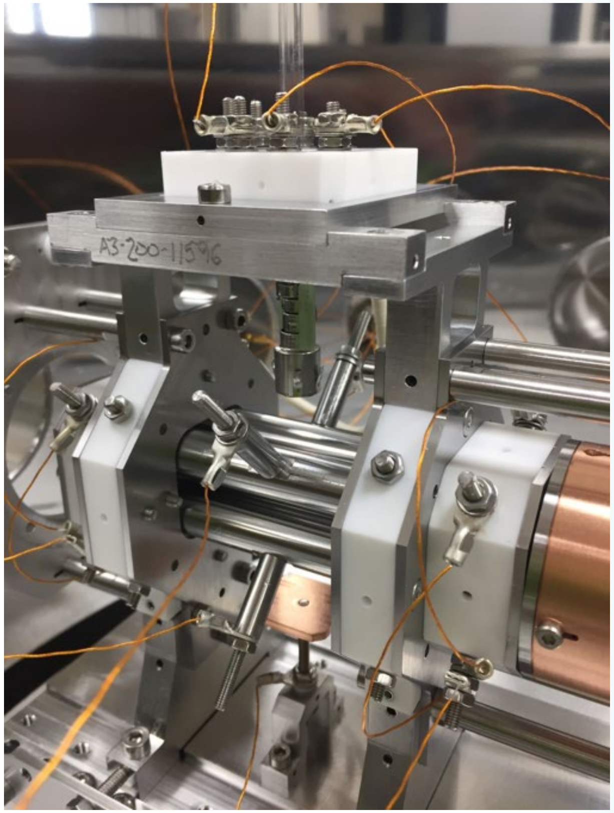
Linewidth $\Gamma = 2\pi$*23MHz, frequency f = c/397e-9 = 755THz
Active over velocity range ~ c(23MHz/755THz) = 9.1m/s
Tune so that force pushes more on ions moving towards laser

- Temperature limit $T_D = \hbar\Gamma/2k_B = 0.552$mK

S-POD, IBEX experiments

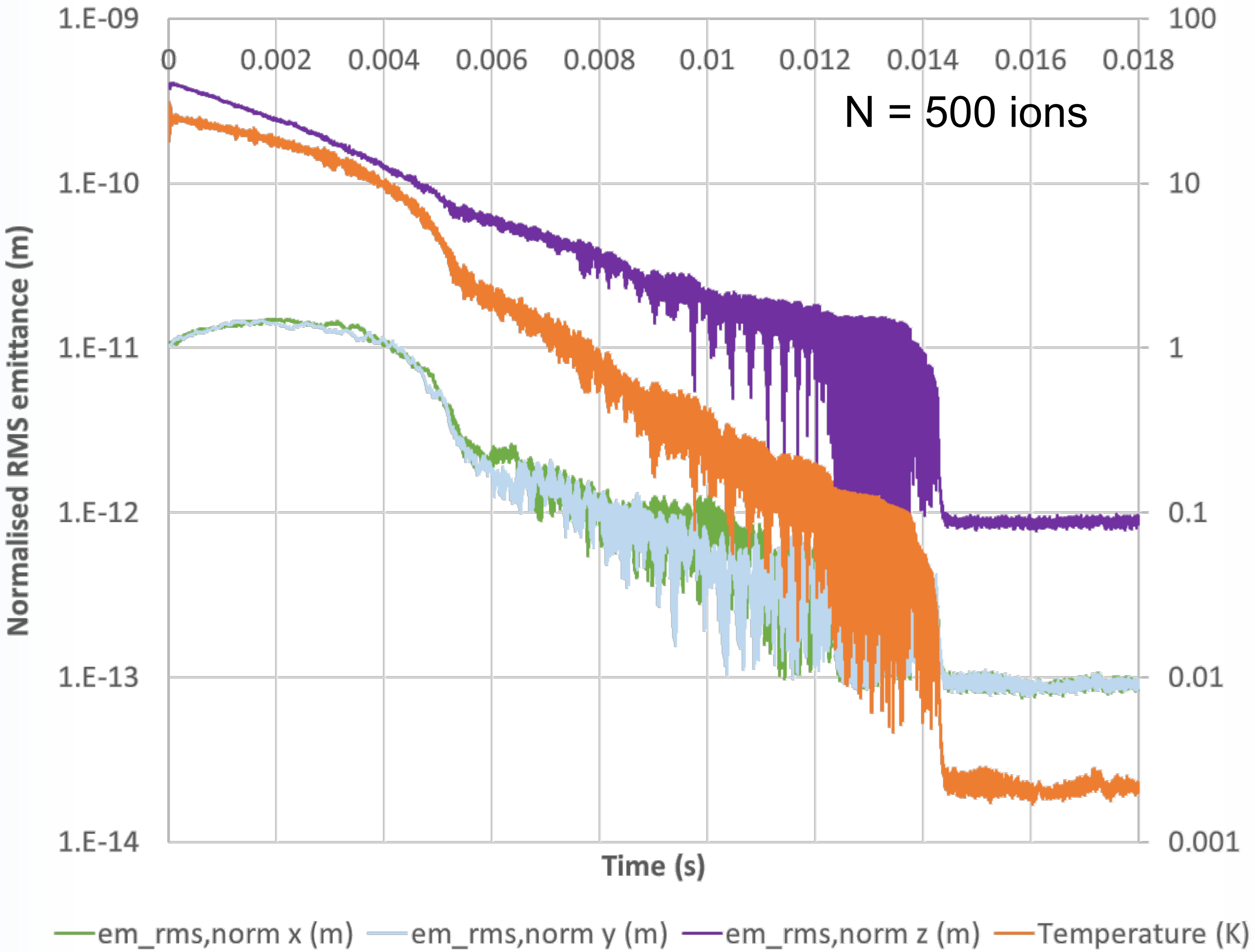


Fluorescence of Coulomb crystal in S-POD trap, from K. Izawa et al., J. Phys. Soc. Jpn., Vol. 79, No. 12 (U.Hiroshima)



The IBEX Paul trap (RAL) with vacuum chamber opened. The four rods in the center of the image are the transverse trap electrodes.

Laser Doppler Cooling Simulation



Ion Trap Flexibility

- Flexible source with variable parameters over wide range:
  - Bunch charge (1 to 10^7 ions)
    - Via gas pressure, trap voltage
  - Bunch size, aspect ratio, shape
    - Via trap voltages, trap geometry, collimation
  - Emittance/temperature
    - Stop cooling part way, or tighten trap (→ quantum limit!)
  - Ion species
    - Use sympathetic cooling - mix coolable ion e.g. $^{40}Ca^+$ ion in contact with desired species
  - Easily exchangeable electrode configurations

Martin *et al.*, New J. Phys. 21 (2019) 053023

IBEX 10^6-10^7

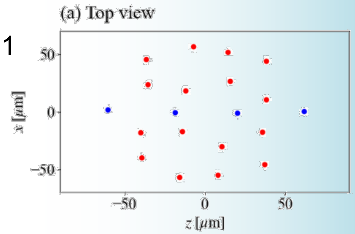
Takai *et al.*, Japan. J. Appl. Phys. 45, No. 6A (2006) 5332-5343

S-POD 1-10^7

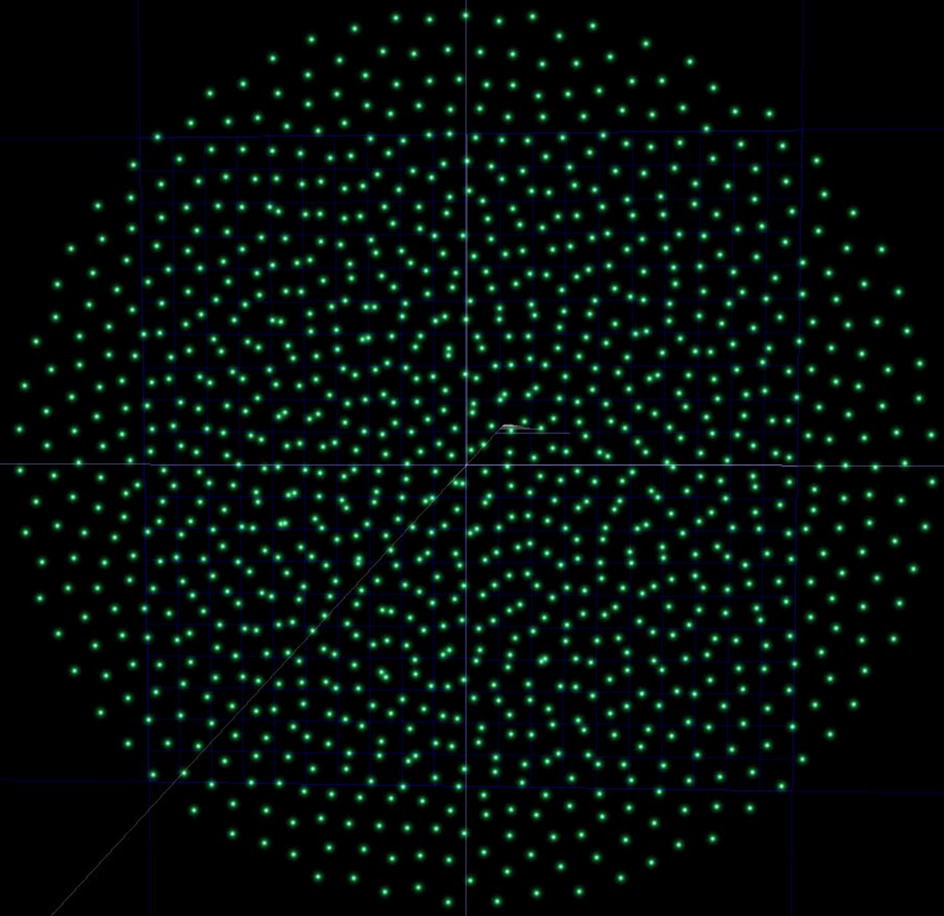
$$\varepsilon_{\text{norm,rms}} \cong \frac{\sigma_x \sigma_v}{c} = \frac{\sigma_x}{c}\sqrt{\frac{k_B T_D}{m}} = \frac{\sigma_x}{c}\sqrt{\frac{\hbar\Gamma}{2m}},$$

$$\varepsilon_{\text{norm,rms}} \geq \hbar/2mc$$

Muroo *et al.*, PTEP2023 063G01

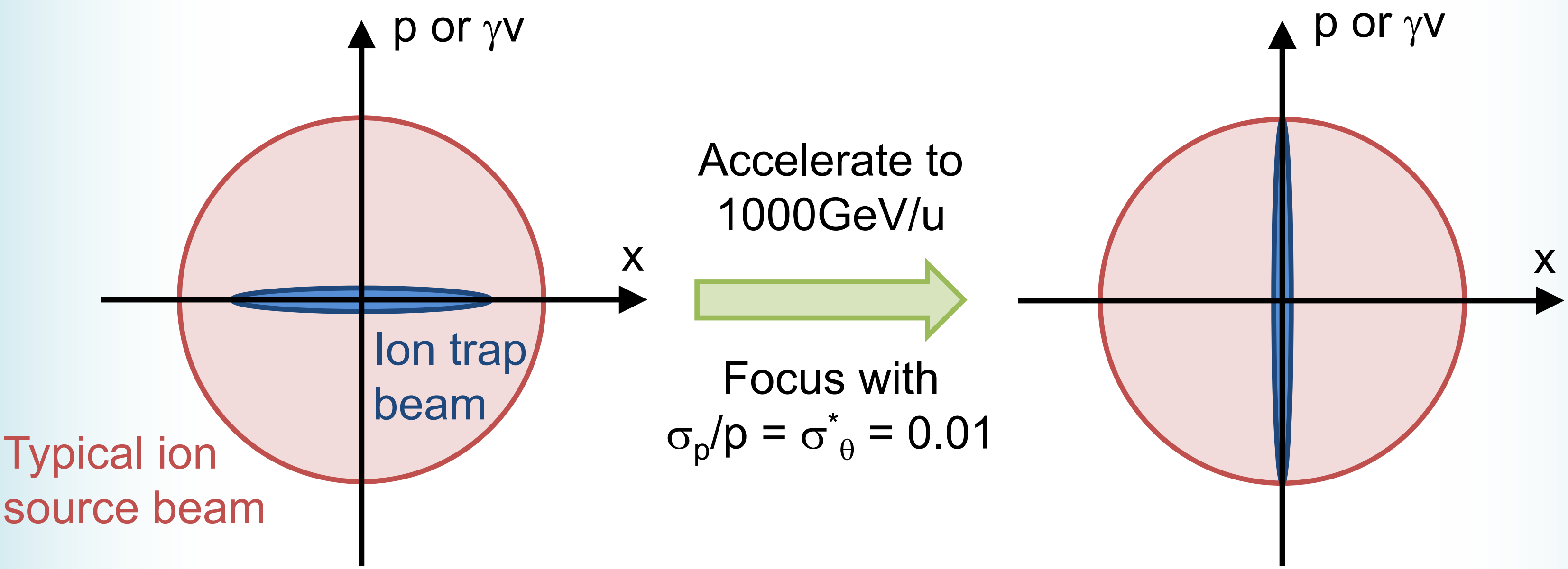


Flexibility e.g.: 2D Coulomb Crystal



Made the transverse trapping voltages much smaller than the longitudinal

What do we do with this beam?



Parameter	Ion source	Ion trap
N	10^9	500
$\varepsilon_{norm,rms}$	10^{-7}m	2×10^{-13}m
σ_x	1mm	90μm
σ_v	30km/s	0.65m/s
T	4.3MK	2mK

Parameter	Ion source	Ion trap
N	10^9	500
$\varepsilon_{norm,rms}$	10^{-7}m	2×10^{-13}m
σ^*_x	9.4nm	18fm
σ^*_θ	10mrad	10mrad
L/bunch	9×10^{28}cm^{-2}	6×10^{27}cm^{-2}

High Specific Luminosity Scaling

- Luminosity is held constant if $N \propto \sigma^*$ $$\mathcal{L} = \frac{N_1 N_2 f N_b}{4\pi\sigma_x\sigma_y}$$
  - If σ^*_θ is also held constant, that's $N \propto \sigma^* \propto \varepsilon$
- But luminosity per ion increases at smaller N
  - 6×10^{27}cm^{-2} from 2×500 ions is 130,000 times more efficient than 9×10^{28}cm^{-2} from 2×10^9 ions
- In fact the ion trap saturated the cross section
  - If σ_{tot} > 84mbarn, then >500 collisions expected
  - One pass and it's gone!

What's going on in the longitudinal plane? and Other Questions

- Need short bunch length for good collisions
  - The ion trap's emittance is small in all three planes
- The bunch seems pretty "opaque", does it self-interact?
  - Space charge repulsion prior to the focus?
- Why don't collider interaction points create a sharp focus all three space planes?
  - Is that even possible?
    - Yes, but relativity reduces longitudinal $\Delta v_{\text{bunch rest frame}}$

Bunch Implosion Radius Limits

- Model: imploding uniform sphere of charge in the bunch's rest frame
- Space charge dictates $r \geq 1/(4\pi\varepsilon_0)\ Nq^2/E_{k,in}$
  - $E_{k,in}$ is inward kinetic energy at sphere surface
  - Density $\rho \propto N/r^3 \propto E_{k,in}{}^3/N^2$
- Emittance dictates $\sigma_x \geq \varepsilon_{norm,rms}/\sigma_{\beta\gamma}$
  - $\sigma_x = r/\mathrm{sqrt}(5)$, $\sigma_{\beta\gamma} = (\beta\gamma)_{in}/\mathrm{sqrt}(5)$
- $\sigma_{\beta\gamma,Transv.} = (\beta\gamma)_{beam}\sigma^*_\theta$, $\sigma_{\beta\gamma,Long.} = \beta_{beam}\sigma_p/p$

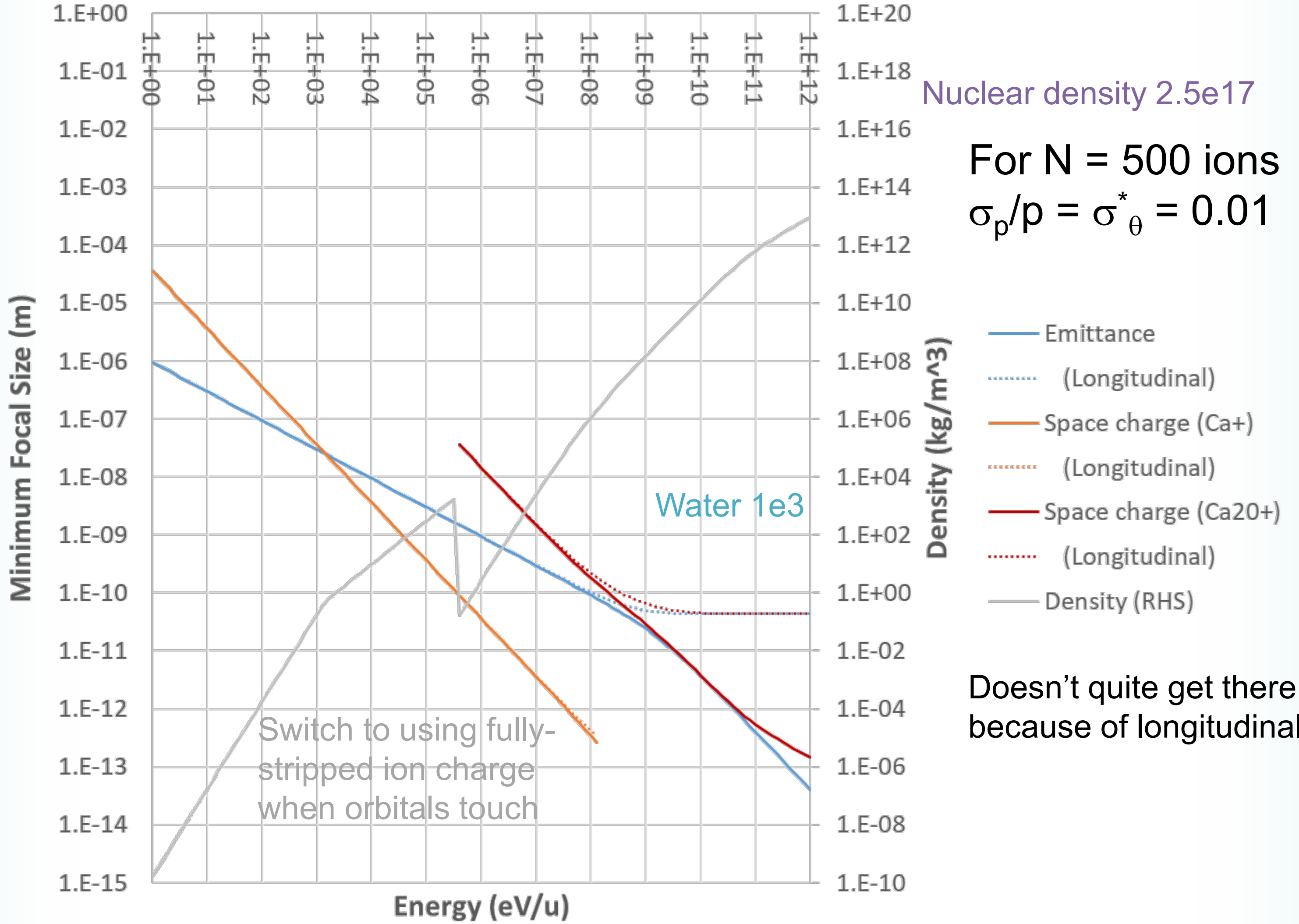
Minor Complexities

- Ca^{+} will behave as Ca^{20+} once nuclei are within each others' electron clouds (no shielding)
- Space charge forces for a non-spherical uniform charge density ellipsoid are actually coupled between the planes (but still linear)
  - Just like the 2D KV envelope space charge force

$$r_x'' + \kappa_x r_x - \frac{2Q}{r_x + r_y} - \frac{\varepsilon_x^2}{r_x^3} = 0 \qquad r_y'' + \kappa_y r_y - \frac{2Q}{r_x + r_y} - \frac{\varepsilon_y^2}{r_y^3} = 0$$

  - Integrated backwards from focal point and fitted focal ellipsoid size to match incoming σ^*_θ and σ_p/p

Energy Scaling Graph

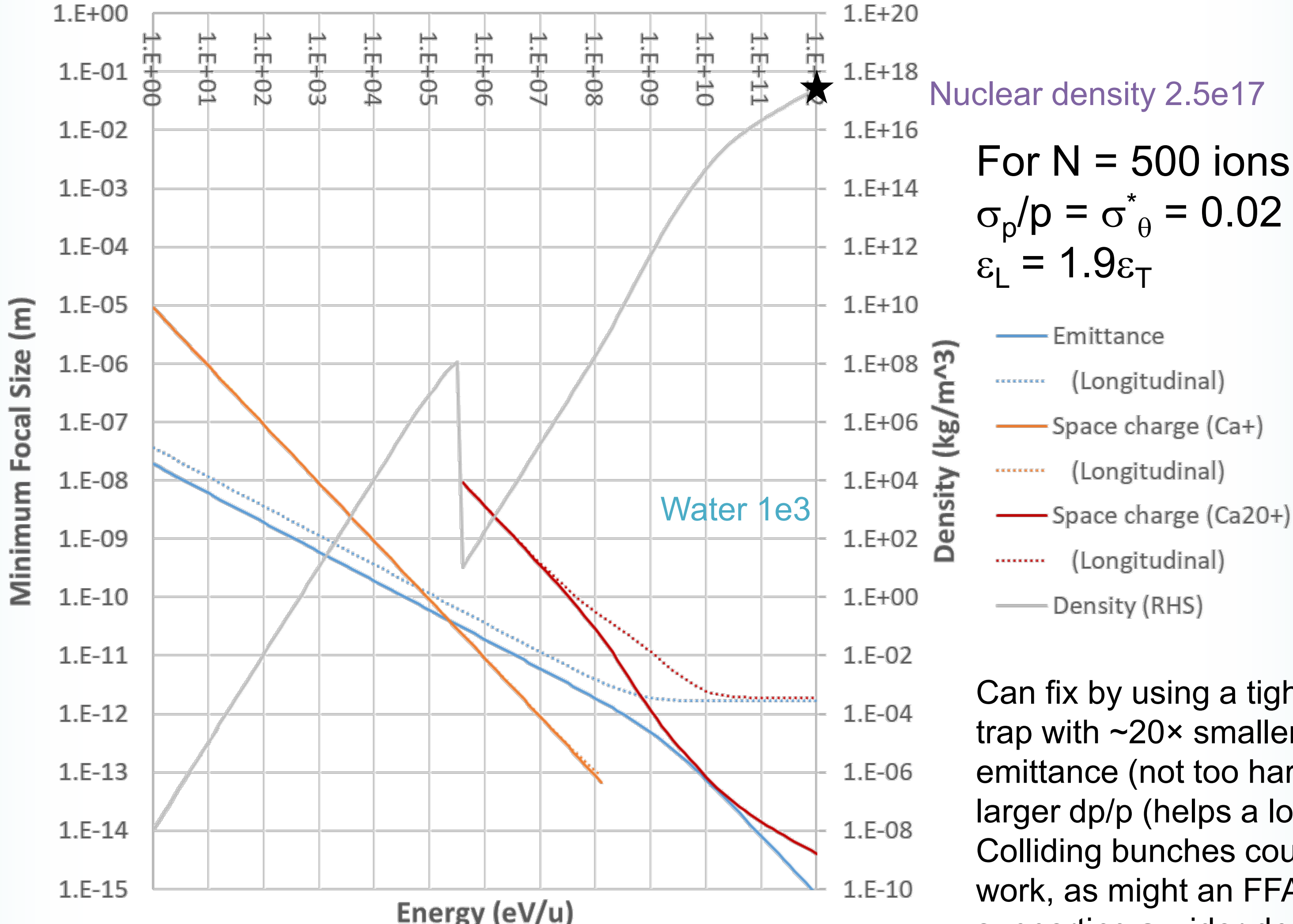


For N = 500 ions

$\sigma_p/p = \sigma^*_\theta = 0.01$

Doesn't quite get there, mainly because of longitudinal plane

Energy Scaling Graph



For N = 500 ions
$\sigma_p/p = \sigma^*_\theta = 0.02$
$\varepsilon_L = 1.9\varepsilon_T$

Can fix by using a tighter ion trap with ~20× smaller emittance (not too hard) and 2× larger dp/p (helps a lot). Colliding bunches could also work, as might an FFA channel supporting a wider dp/p.

Potential Future Applications

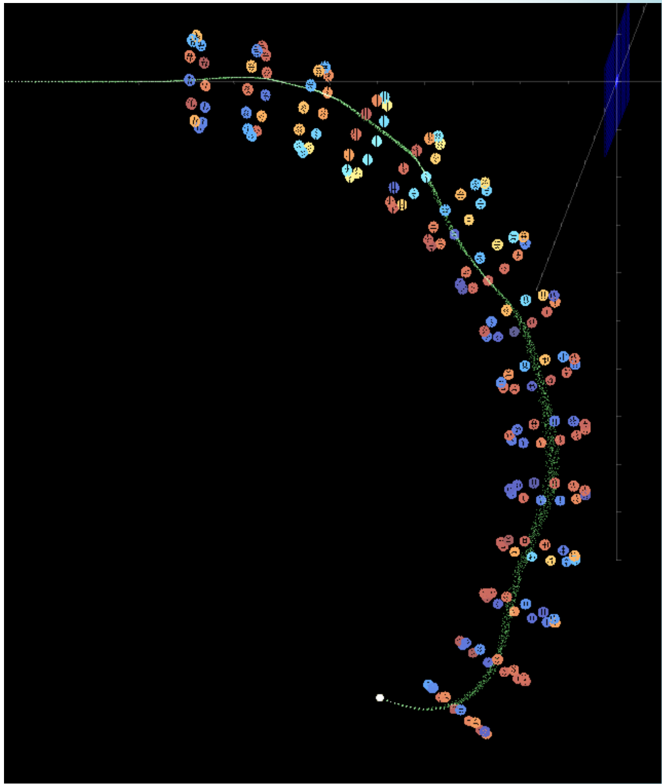
- We can already accurately position objects close to the nuclear size (e.g. LIGO mirrors)
- The low emittance (entropy) of the ion trap initial state allows nuclei to be placed in particular locations deliberately, e.g.:
  - Make white dwarf, neutron star matter
  - 3-way and multi-way particle collisions
  - Synthesise custom shapes of nuclear matter
  - Possible neutron-rich superheavy elements

What can stop us?

- Space charge, emittance and energy spread considerations as mentioned previously
- Optical aberrations! ★
  - Lenses are not linear, spherical aberration, chromaticity etc.
- Can focussing in all three planes be done? ★
- Experimental noise, jitter etc.
  - Ideas for feedback using scattered pattern from interaction point
    - Coulomb scattering has very good resolution

Focussing Beamline Simulation

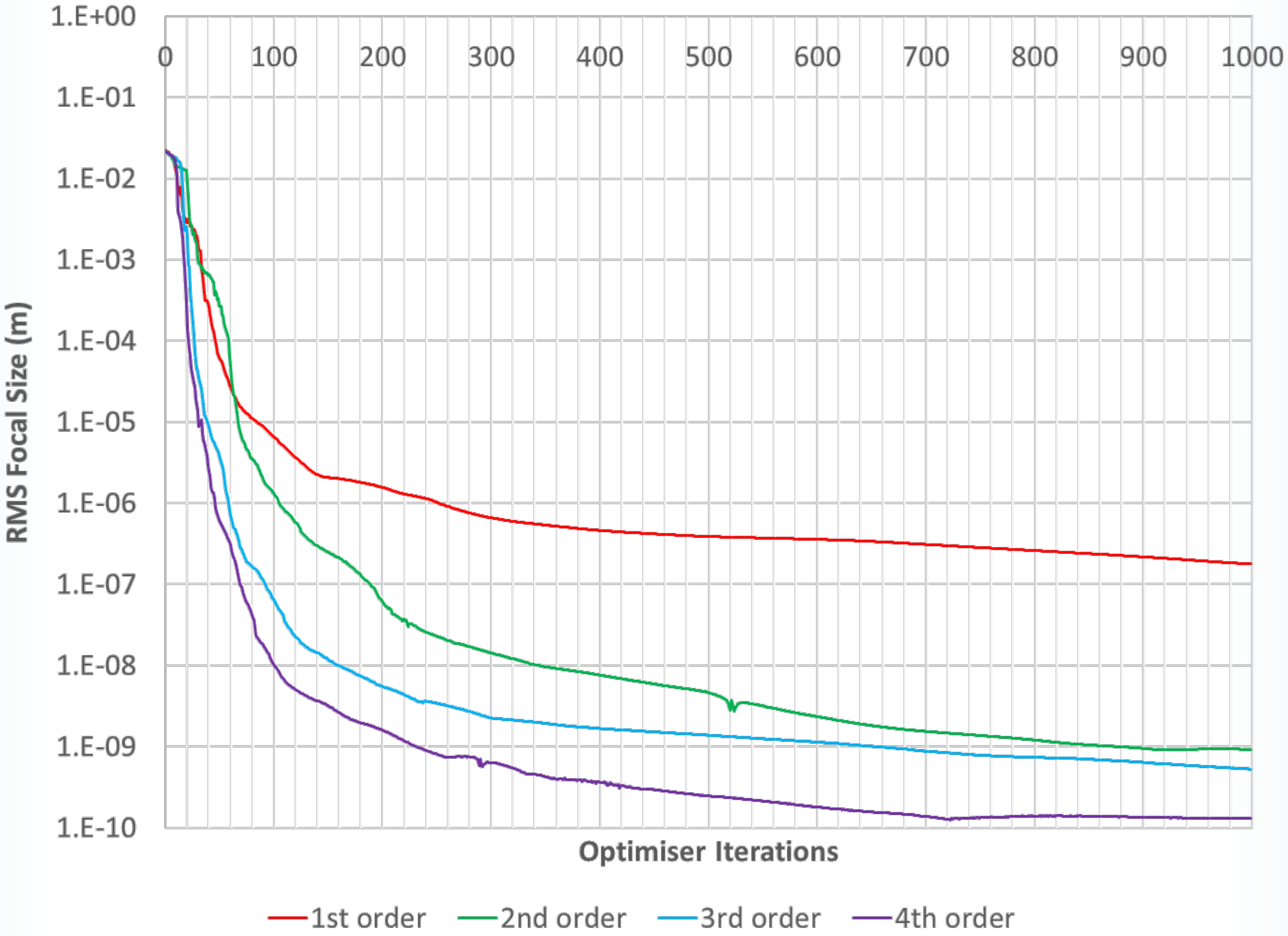
- Test of three-plane focusing and optical aberration correction (at lower energy)
  - Input N=20000, T=2mK ion trap bunch parameters
  - Given energy chirp
- Curved electrostatic beamline
  - Point-source 'electrodes'
    - Arranged in rings of 12
    - Generalised multipole lenses
- Goal: minimise 3D focal size



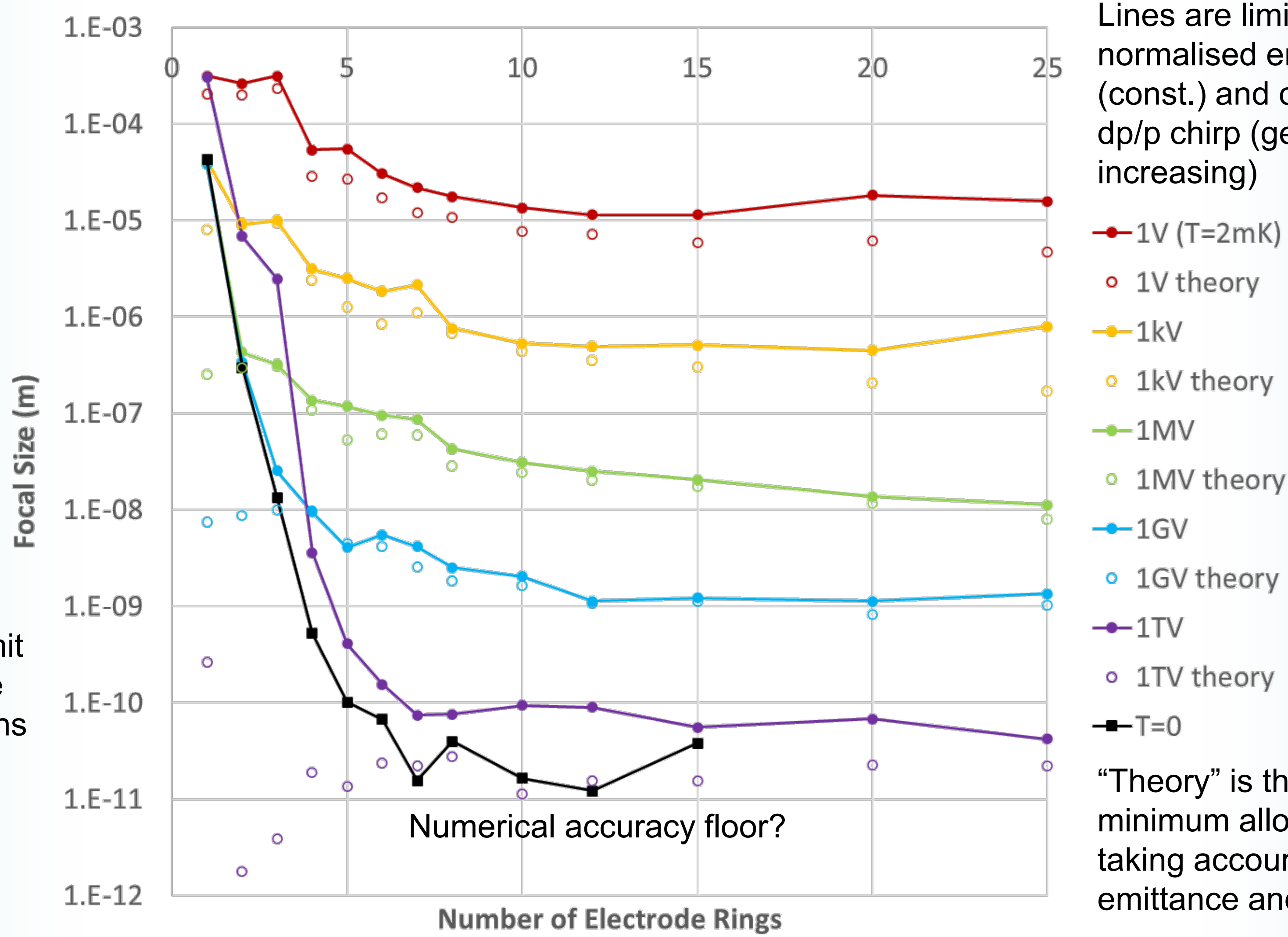
Optimisation of Electrode Voltages

- 11 electrode charges were optimised per electrode ring (12 minus one monopole)
- The bunch initial chirp was also optimised
- So 15 electrode rings → 166 parameters
- Used modified Levenburg-Marquardt method with nonlinearity correction
  - See https://arxiv.org/abs/2307.03820
    - Response matrix → SVD → try various damped inverses
    - Sample additional points to infer nonlinear behaviour

Improved L-M Optimiser



Focal Size vs. Electrodes Used

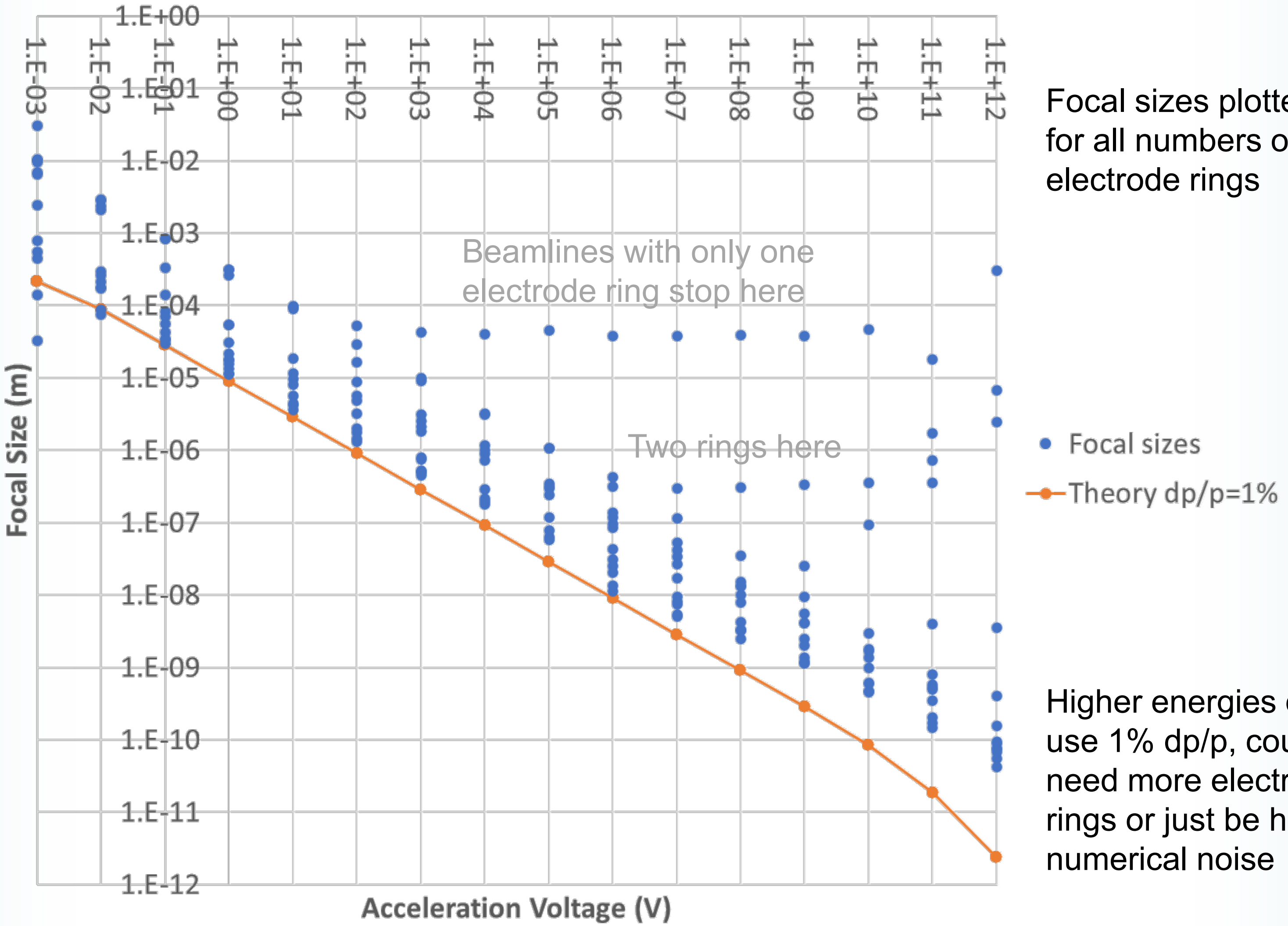


T=0 line shows limit from pure aberrations

Lines are limited by normalised emittance (const.) and chosen dp/p chirp (generally increasing)

"Theory" is the minimum allowed size taking account the emittance and chirp

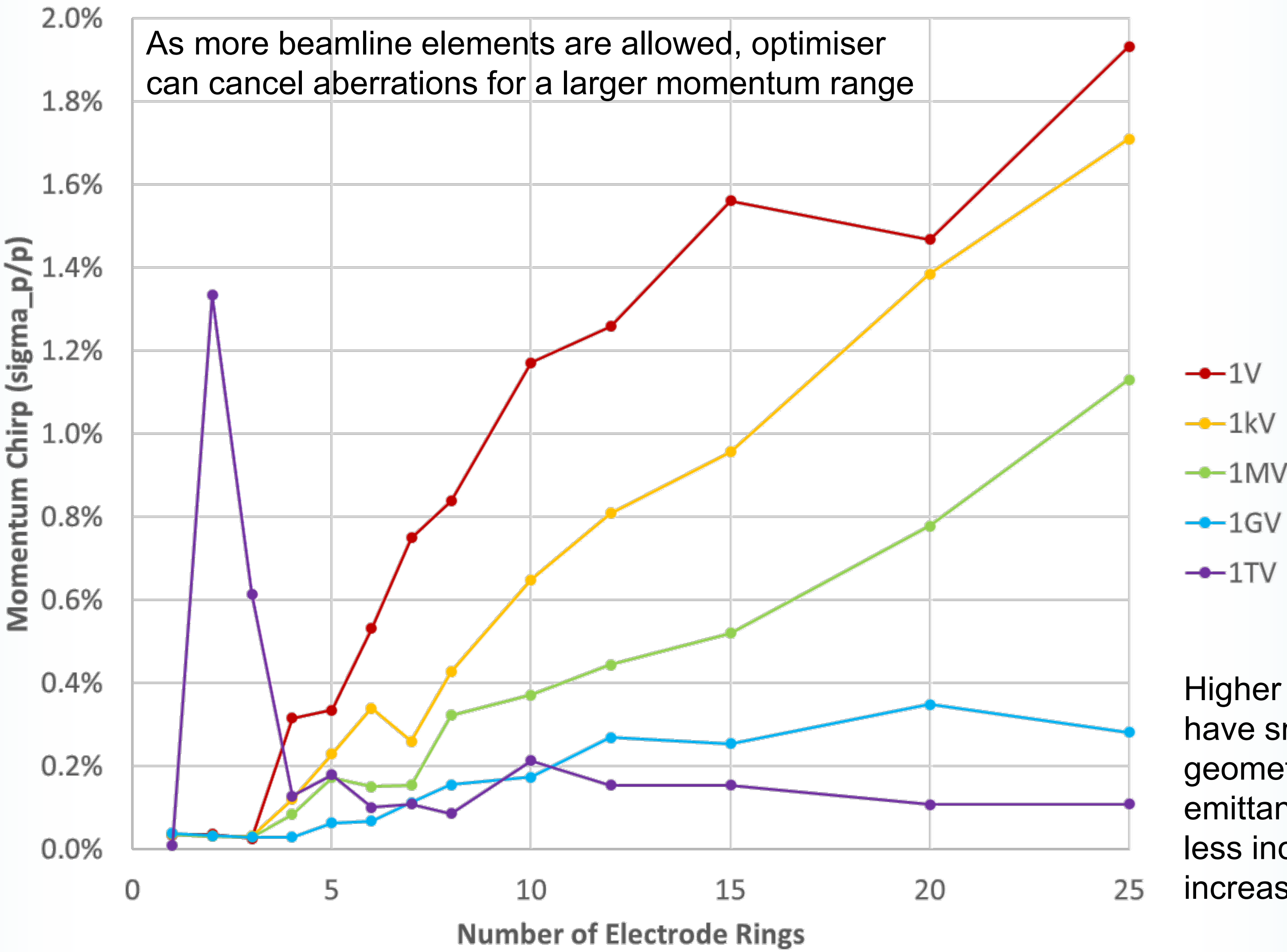
Focal Size vs. Energy



Focal sizes plotted for all numbers of electrode rings

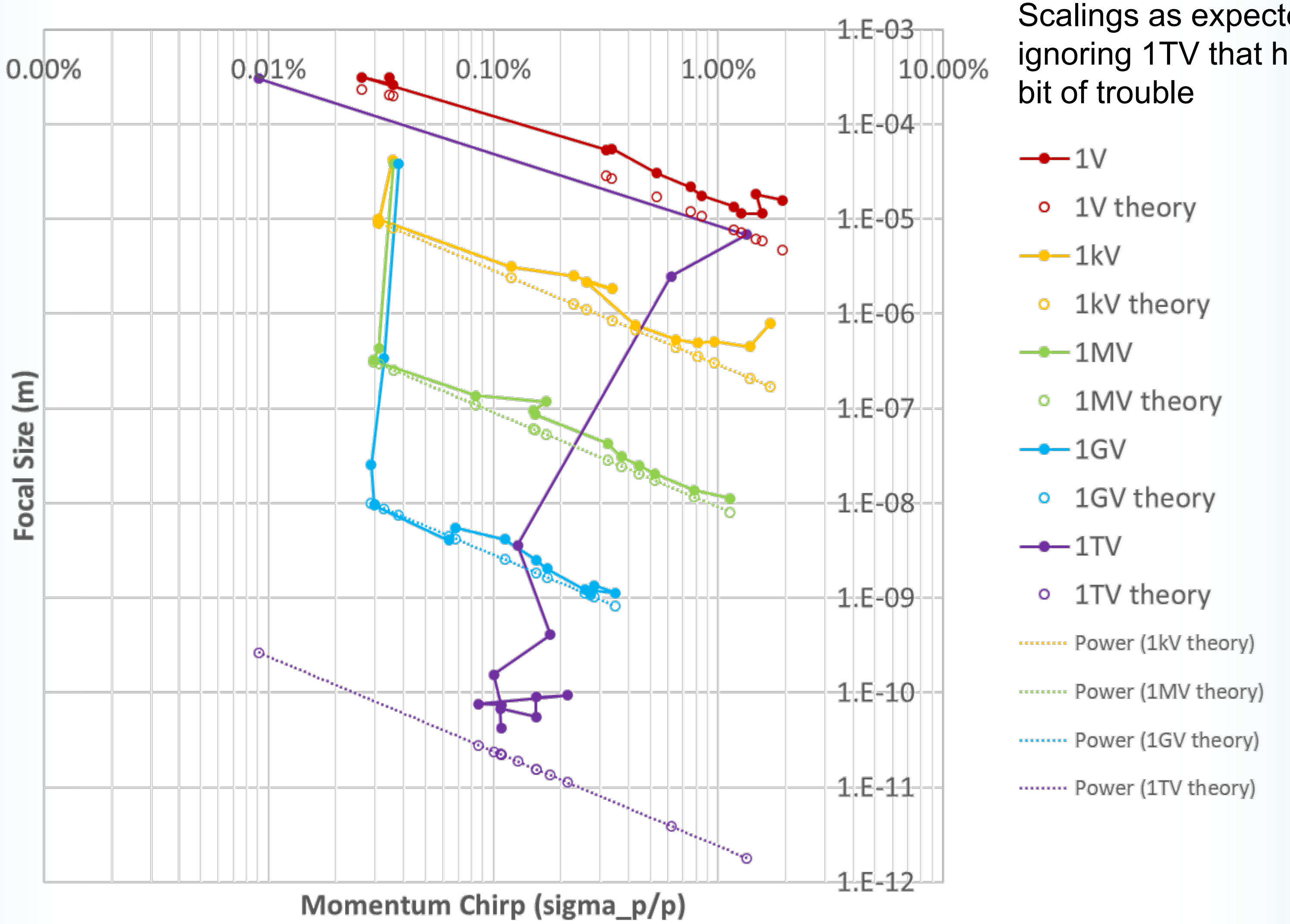
Higher energies don't use 1% dp/p, could need more electrode rings or just be hitting numerical noise

Optimal Chirp vs. Electrodes Used



Higher energies have smaller geometrical emittance already, less incentive to increase dp/p

Focal Size vs. Initial Chirp



Scalings as expected, ignoring 1TV that has a bit of trouble

Cooling at High Energy?

- Doppler cooling also works in a boosted frame
- Pros:

  PALLAS ring did this, but beam velocity only 2.8km/s

  Schramm *et al.*, Plasma Phys. Control. Fusion 44 (2002) B375–B387

  – Can create cold beams at or near final energy
    - Bypass jitter from RF and acceleration process

  – Use blue-shift to cool using harder transitions(?)

  Improves cooling rate, but increases limiting temperature
- Cons:
  – Needs a low-intra-beam-scattering (IBS) ring lattice
    - Very low phase advance per cell
    - Bunch velocity distribution near-Maxwellian

  – Too much energy or field could strip ions

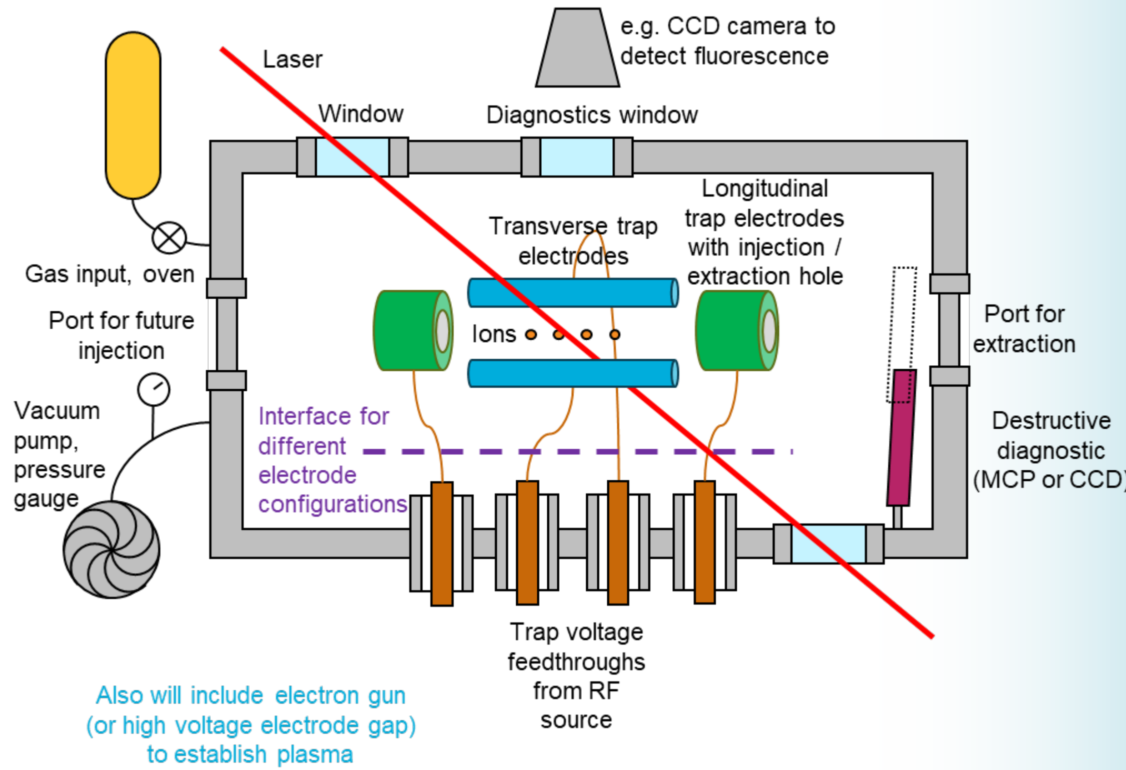
  – Ring is more expensive than an at-rest ion trap

Increasing Current Throughput

- Throughput of basic trap isn't great
  - 10^7 ions every 16ms (62.5Hz) is 100pA average
- But trap is small enough and 10ms is short enough that could make a CW cooling channel at a PALLAS-like speed of 50km/s
  - Bunches every 100ns (10MHz), spaced by 5mm
    - Average current could go as high as 16uA
  - Length of channel 16ms*50km/s = 800m
    - Could coil it up, trap can be narrow, rods few mm apart

Recent Funding at BNL

- Received Lab-Directed R&D (LDRD) funding at BNL, total $400k over 2 years, Oct ‘23-Sep ’25
- Proposed to “construct a basic foundational system”
- Probably without laser cooling to start with



Conclusion

- Ultra-low emittance bunches provide some interesting unexplored regimes
  - 10^6 times smaller emittance vs. conventional bunches
  - Extraction into an accelerator would be new
- Lower entropy initial states are going to be the long term trend as experiments improve
- Can even achieve the quantum ground state
  - Produce e.g. entangled spin states in a beam!
- Ultimately, appears capable of custom synthesis of nuclear density matter