New understanding of longitudinal (bunched) beam instabilities and comparison with measurements

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Acknowledgments:
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Outline

1. Loss of Landau Damping

I. Karpov, T. Argyropoulos, E. Shaposhnikova, Thresholds for loss of Landau damping in longitudinal plane, 2021

2. Single-bunch instabilities

I. Karpov, Longitudinal mode-coupling instabilities of proton bunches in the CERN Super Proton Synchrotron, 2023

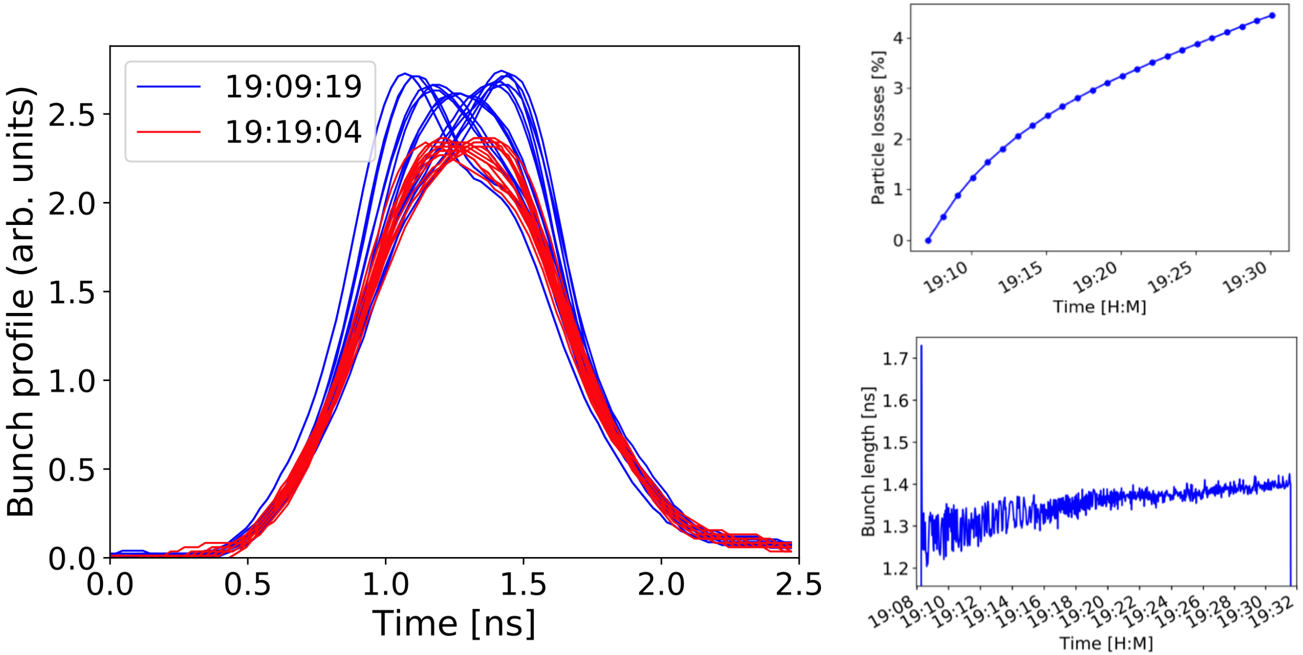
3. Multi-bunch instabilities

I. Karpov, E. Shaposhnikova, Generalized threshold of longitudinal multi-bunch instability in synchrotrons, 2023

Loss of Landau damping

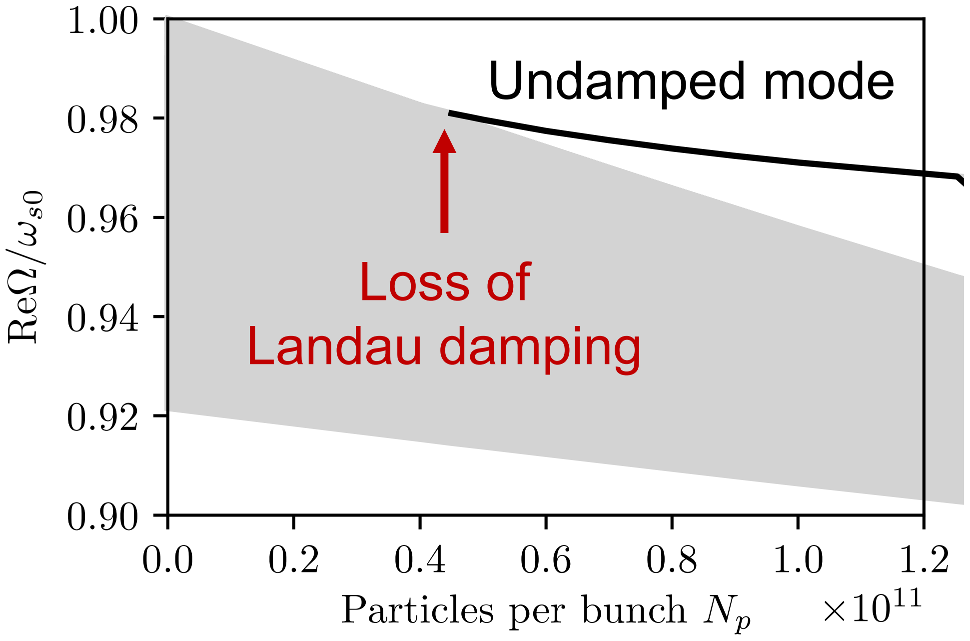
Loss of Landau damping (LLD)

Long-lasting oscillations were observed in SPS, RHIC, Tevatron, LHC, …



H. Timko et al, Beam instabilities after injection to the LHC, 2018

Longitudinal particle oscillations can be described as van Kampen modes*



Dominant inductive impedance above transition

**Y. H. Chin, K. Satoh, and K. Yokoya, Instability of a bunched beam with synchrotron frequency spread, 1983, and A. Burov, Van Kampen modes for bunch longitudinal motion, 2010*

Lebedev equation*

A system of equations for line-density harmonics $\tilde{\lambda}_k$ for coherent mode Ω

Beam and RF parameters

Impedance at $k\omega_0 + \Omega$

$$\sum_{k=-\infty}^{\infty} \left[\delta_{k'k} + \boxed{\frac{qN_p\omega_{\text{rf}}}{V_0}} \boxed{G_{k'k}(\Omega)} \frac{\boxed{Z_k(\Omega)}}{k} \right] \tilde{\lambda}_k(\Omega) \equiv \sum_{k=-\infty}^{\infty} \mathcal{M}_{k'k}(\Omega)\tilde{\lambda}_k(\Omega) = 0$$

Beam transfer matrix

$k = \omega/\omega_0$
N_p – number of particles
q – charge
ω_0 – revolution frequency
ω_{rf} – rf frequency
V_0 – rf voltage

→ The mode Ω is a solution if the determinant is zero, $\det \mathcal{M} = 0$

**A. N. Lebedev, Coherent synchrotron oscillations in the presence of a space charge, 1968*

Approximate analytic solution

$$\det \mathcal{M} = \det[I + \varepsilon X(\varepsilon)] = \det(\exp\{\ln[I + \varepsilon X(\varepsilon)]\}) = \exp(\mathrm{tr}\{\ln[I + \varepsilon X(\varepsilon)]\})$$

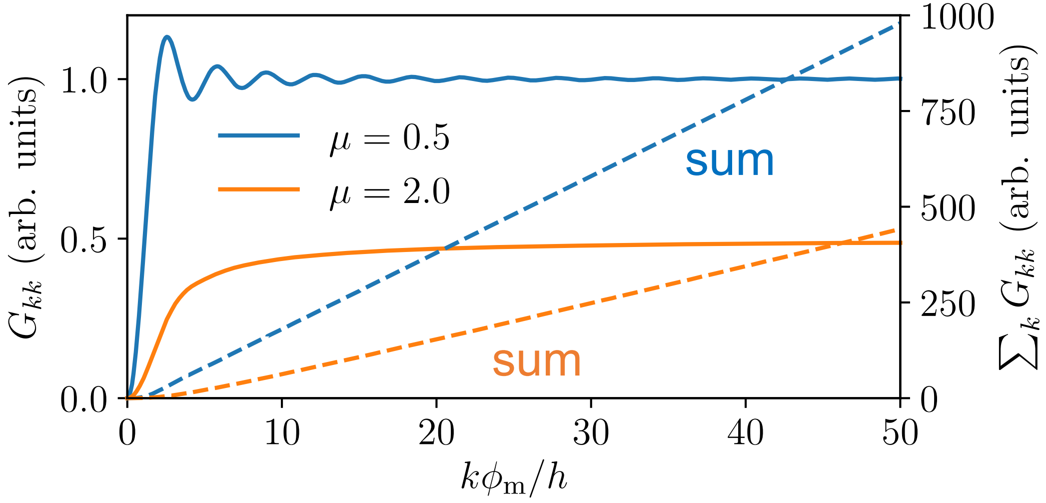
$$= 1 + \varepsilon\, \mathrm{tr}[X(0)] + \mathcal{O}(\varepsilon^2)$$

$\det(\exp A) = \exp(\mathrm{tr} A)$

The LLD threshold for dipole mode: $N_{\mathrm{LLD}} \approx \frac{V_0}{q\omega_{\mathrm{rf}}}\left[\sum_{k=-\infty}^{\infty} G_{kk}(\Omega)\frac{Z_k(\Omega)}{k}\right]^{-1}$

Assuming:

- Reactive impedance $Z_k/k = i\mathrm{Im}Z/k =$ const.
- Beam above transition in single rf: $\Omega = \omega_s(0)$
- Short bunch approximation $\phi_\mathrm{m} = \omega_\mathrm{rf}\,\tau/2 \ll \pi$
- Binomial distribution $\lambda(\phi) \propto [1 - \phi^2/\phi_\mathrm{m}^2]^{\mu+1/2}$



→ Elements G_{kk} saturate for $k \to \infty$

→ LLD threshold is zero for commonly used inductive impedance $\mathrm{Im}Z/k =$ const

LLD threshold

One needs to introduce a cutoff frequency $f_c = k_c f_0$ and then

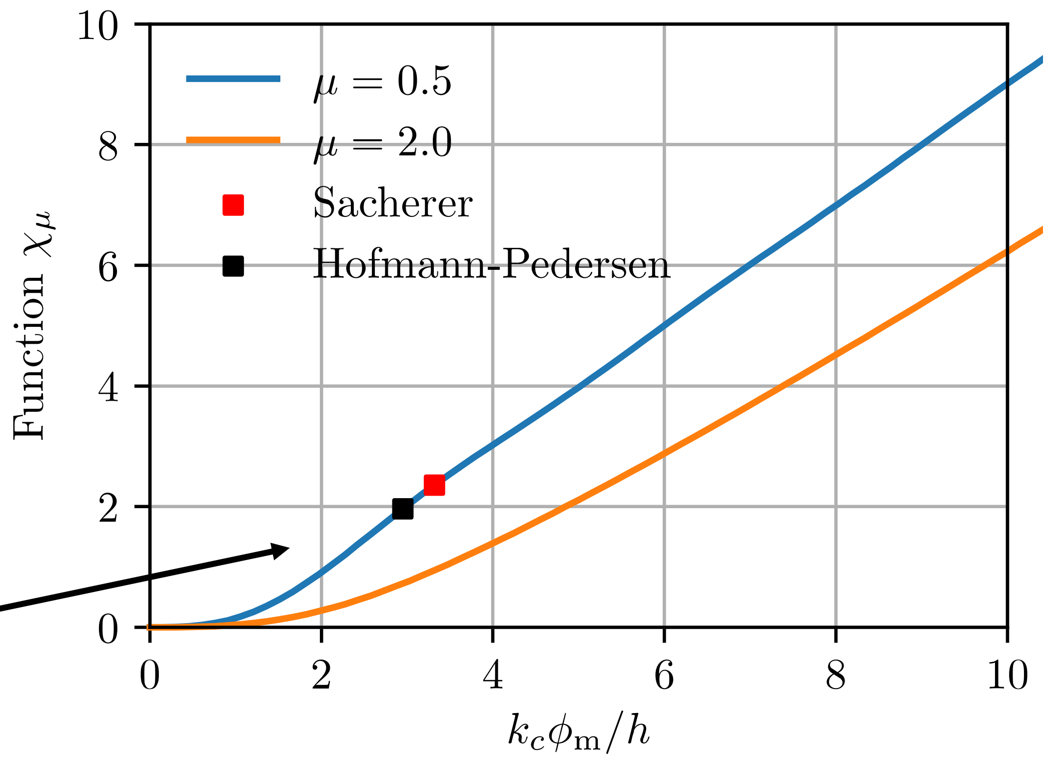
$$\boxed{N_{\mathrm{LLD}} \approx \frac{\pi}{32qh\omega_{\mathrm{rf}}\mu(\mu+1)} \frac{V_0\phi_{\mathrm{m}}^5}{\chi_\mu(k_{\mathrm{c}}\phi_{\mathrm{m}}/h)\mathrm{Im}Z/k}}$$

For $f_c \to \infty$ $N_{\mathrm{LLD}} \approx \frac{1}{64qh\mu(\mu+1)} \frac{V_0\phi_{\mathrm{m}}^4}{f_{\mathrm{c}}\mathrm{Im}Z/k}$

so that $N_{\mathrm{LLD}} \propto 1/f_c$ and $\phi_{\mathrm{m}}^5 \to \phi_{\mathrm{m}}^4$

→ N_{LLD} based on Sacherer* and Hofmann-Pedersen** formalisms ($\mu = 0.5$) is reproduced for $f_c \approx 1/\tau$ $(k_c\phi_{\mathrm{m}} \approx \pi)$

Function $\chi_\mu(y) = y\left[1 - {}_2F_3\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},2,\mu;-y^2\right)\right]$



*F.J. Sacherer, Methods for computing bunched-beam instabilities, 1972

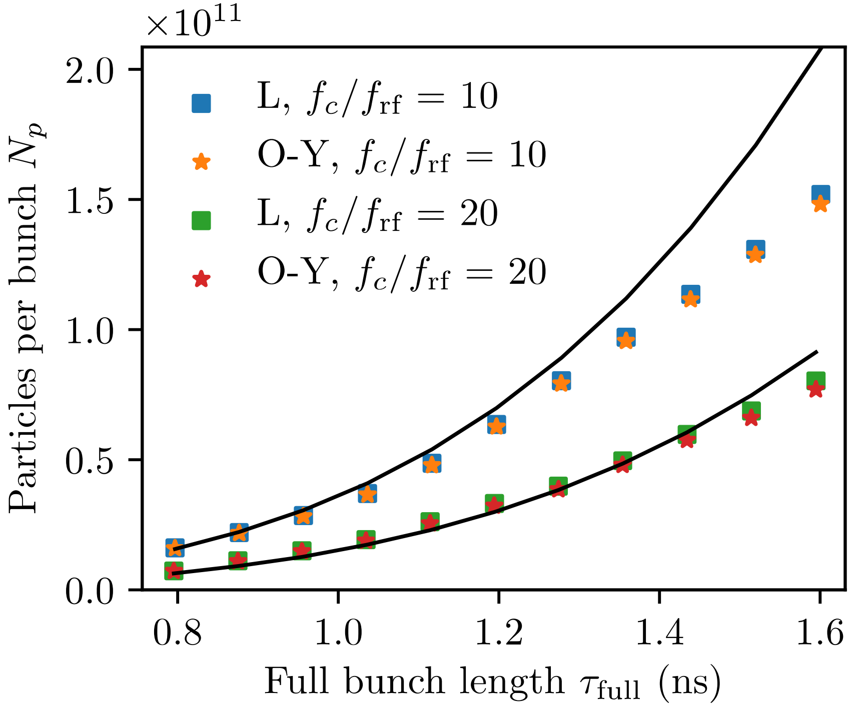
**A. Hofmann and F. Pedersen, Bunches with local elliptic energy distributions, 1979

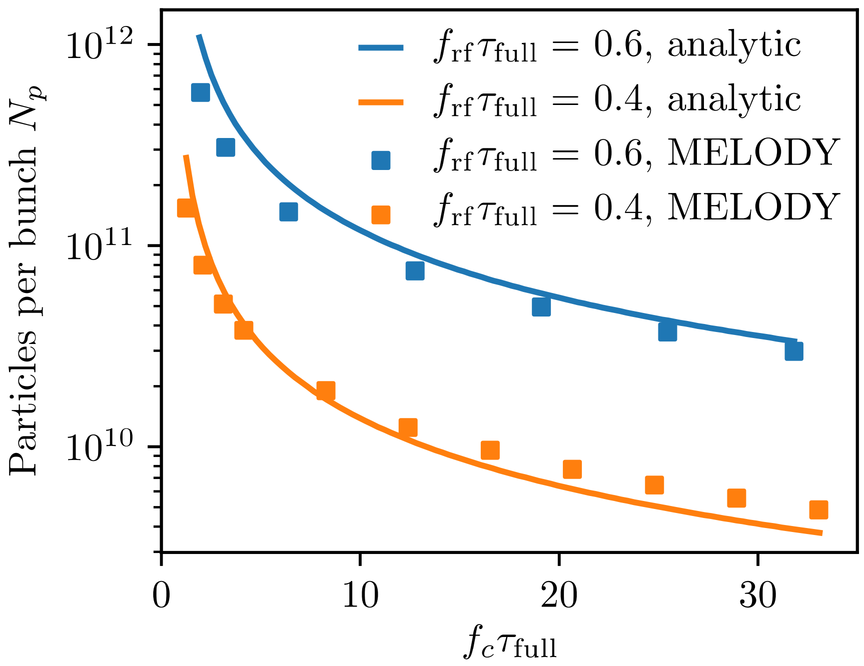
LLD threshold: numerical approaches

(1) The Oide-Yokoya discretization method (O-Y)*: Originally applied for analysis of single-bunch instabilities and later for LLD studies**

(2) Direct solution of the Lebedev equation (L): recently implemented in code MELODY***

Example for LHC:
450 GeV, $\mu = 2$,
truncated inductive
impedance with
$\mathrm{Im}Z/k = 0.07$ Ohm





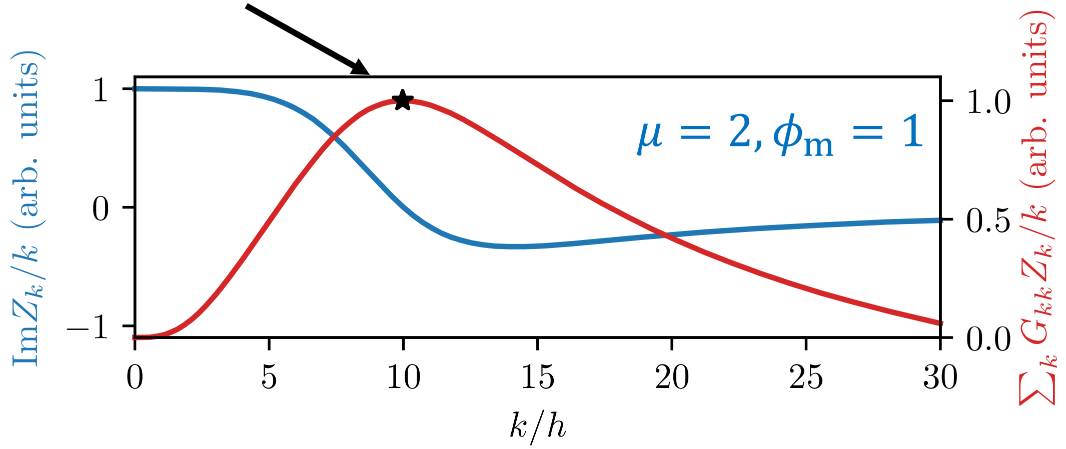
**K. Oide and K. Yokoya, Longitudinal single bunch instability in electron storage rings, 1990*
***A. Burov, Van Kampen modes for bunch longitudinal motion, 2010*
****IK, Matrix Equations for LOngitudinal beam DYnamics*

LLD for effective impedance

Since $N_{\mathrm{LLD}} \approx \frac{V_0}{q\omega_{\mathrm{rf}}}\left[\sum_{k=-\infty}^{\infty} G_{kk}(\Omega)\frac{Z_k(\Omega)}{k}\right]^{-1}$

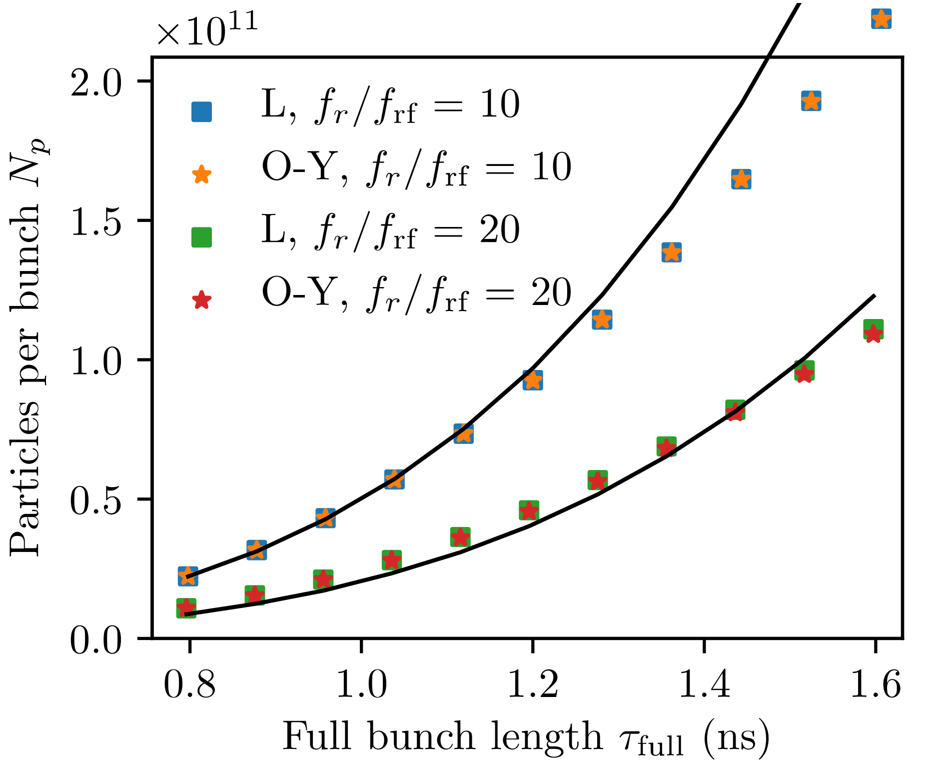
naturally $(\mathrm{Im}Z/k)_{\mathrm{eff}} = \frac{\sum_{k=1}^{k_{\mathrm{eff}}} G_{kk}\mathrm{Im}(Z_k/k)}{\sum_{k=1}^{k_{\mathrm{eff}}} G_{kk}}$

where k_{eff} maximizes the nominator*



Broadband resonator with $Q = 1,\ f_r = 10f_{\mathrm{rf}}$

All work with $k_c \rightarrow k_{\mathrm{eff}}$ & $\mathrm{Im}Z/k \rightarrow (\mathrm{Im}Z/k)_{\mathrm{eff}}$



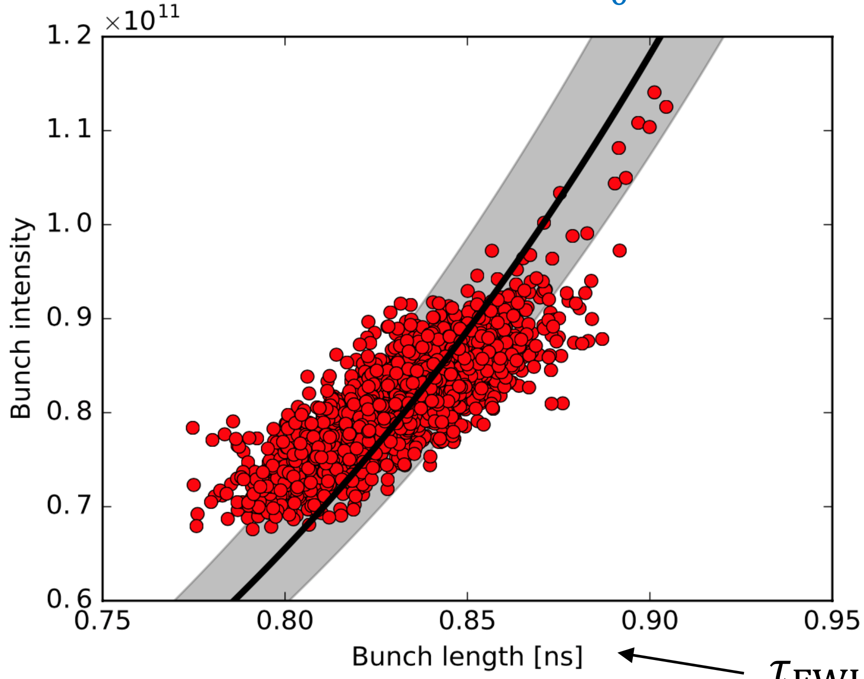
LHC, 450 GeV, $\mu = 2$, broadband impedance with $R = 0.07f_r/f_0$ Ohm and $Q = 1$

*S. Nese, *Effective impedance for the threshold of loss of Landau damping, 2021*

Beam measurements of LLD

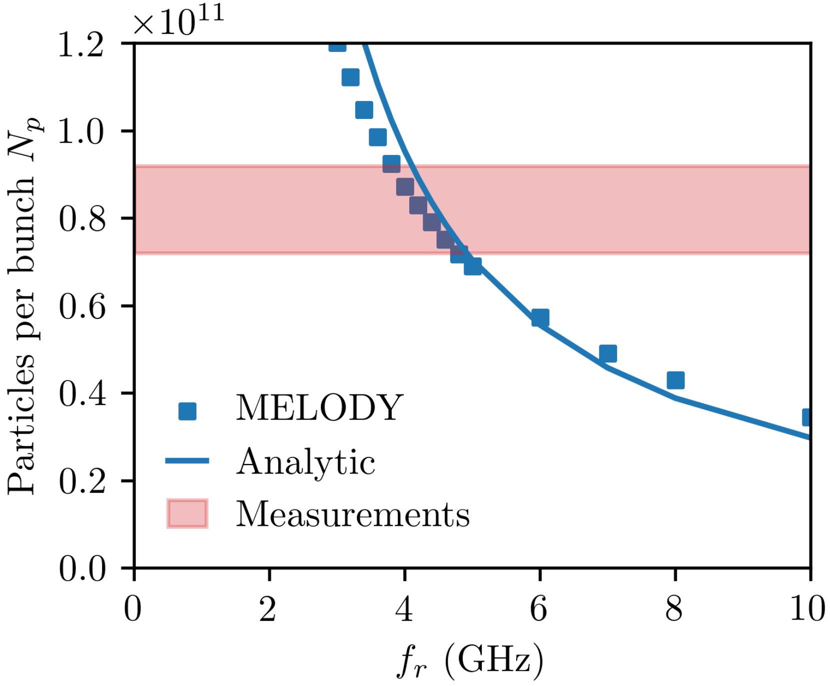
LLD was the first and only intensity effect observed in the LHC in the longitudinal plane*

Measured parameters of bunches with LLD in LHC at 6.5 TeV with $V_0 = 10$ MV*



$\tau_{\mathrm{FWHM}}\sqrt{2/\ln 2}$

LLD threshold for LHC at 6.5 TeV with $V_0 = 10$ MV, $\mu = 2$, $\mathrm{Im}Z/k = 0.076$ Ohm



→ Calculations are consistent with observations for $f_r \approx 5$ GHz (cutoff of LHC beam pipe)

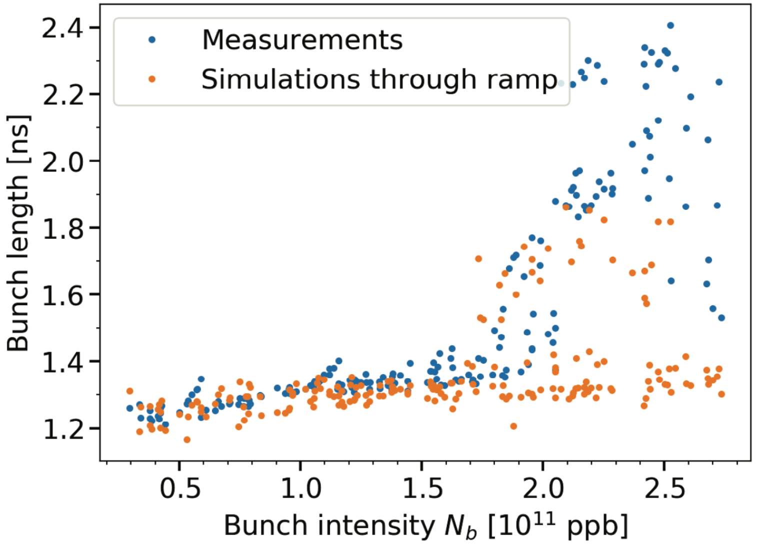
**E. Shaposhnikova et al, Loss of Landau damping in the LHC, 2011*
J.F. Esteban Müller, Longitudinal intensity effects in the CERN Large Hadron Collider, PhD, 2016

Single-bunch instabilities

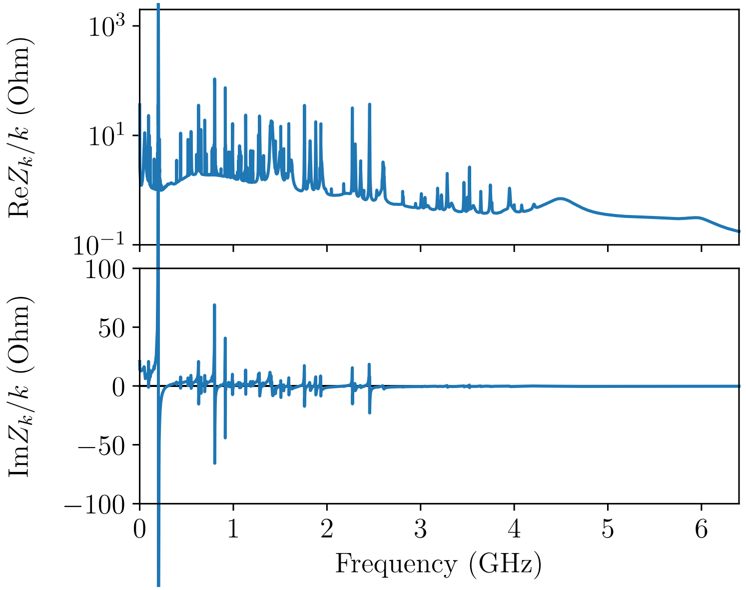
Instability of proton bunch in SPS

Uncontrolled emittance blowup during the acceleration of single bunches was observed

Bunch parameters after acceleration from 26 to 450 GeV*



SPS impedance model (2018)***

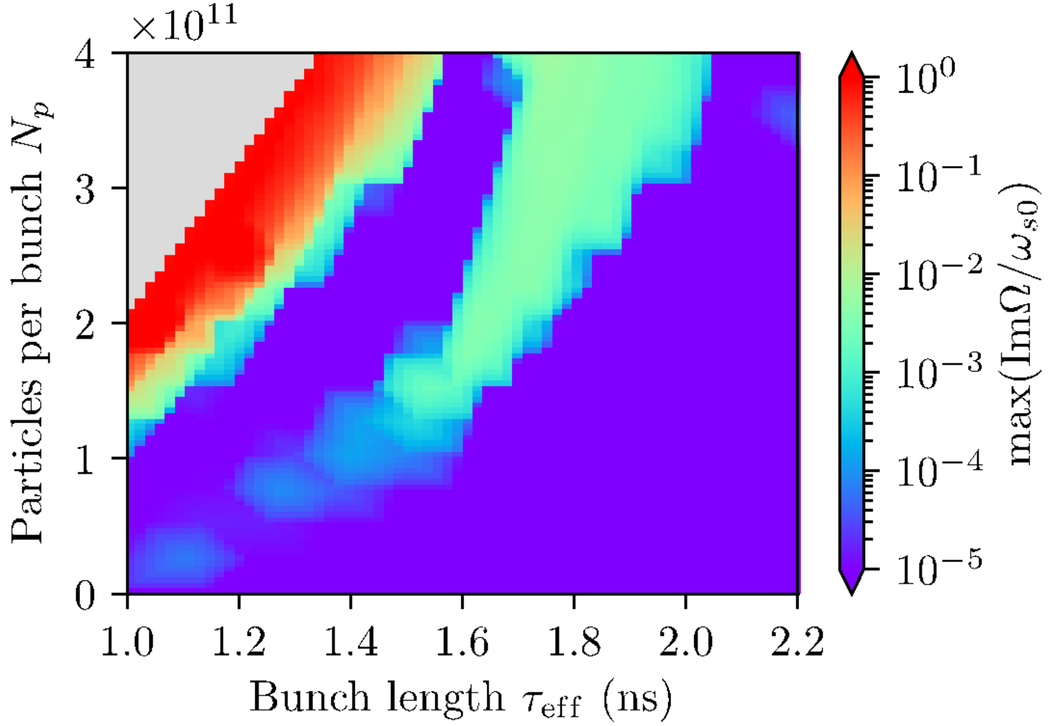


The simulation results (with code BLonD**) for the complicated impedance model were consistent with the measured instability threshold*, however, the instability mechanism was not known

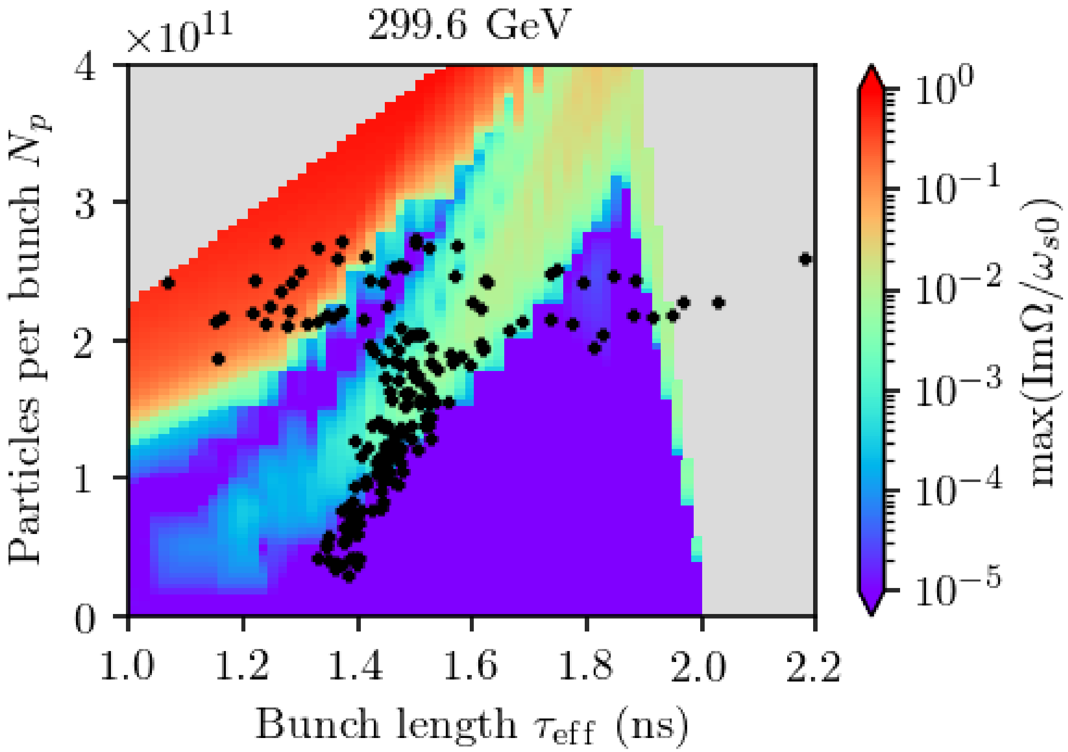
*A. Lasheen, Beam measurements of the longitudinal impedance of the CERN Super Proton Synchrotron, PhD, 2017
J. Repond, Possible mitigations of longitudinal intensity limitations for HL-LHC beam in the CERN SPS, PhD, 2019
**H. Timko et al, Beam Longitudinal Dynamics Simulation Suite BLonD, 2022
***CERN SPS Longitudinal Impedance Model, https:// gitlab.cern.ch/longitudinal-impedance/SPS

Stability maps during acceleration

Calculations at flattop



Calculations during ramp (✚ - measurements)

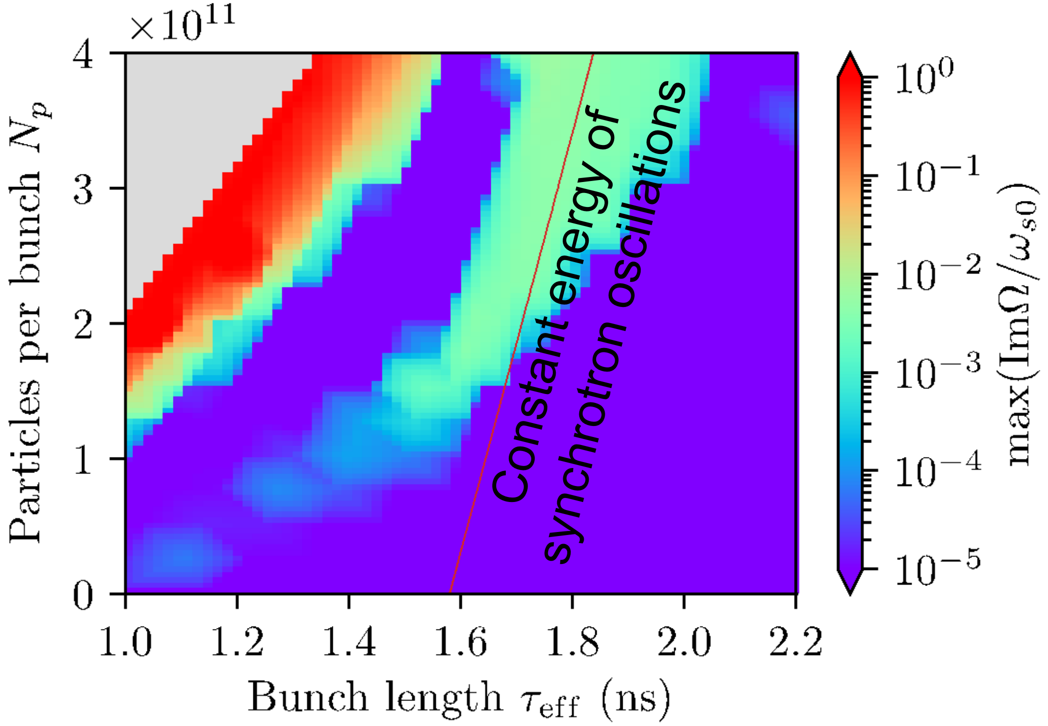


The island found in simulations at 450 GeV* is also present earlier in the acceleration cycle**
→ Measured parameters of unstable bunches (✚) are crossing the island

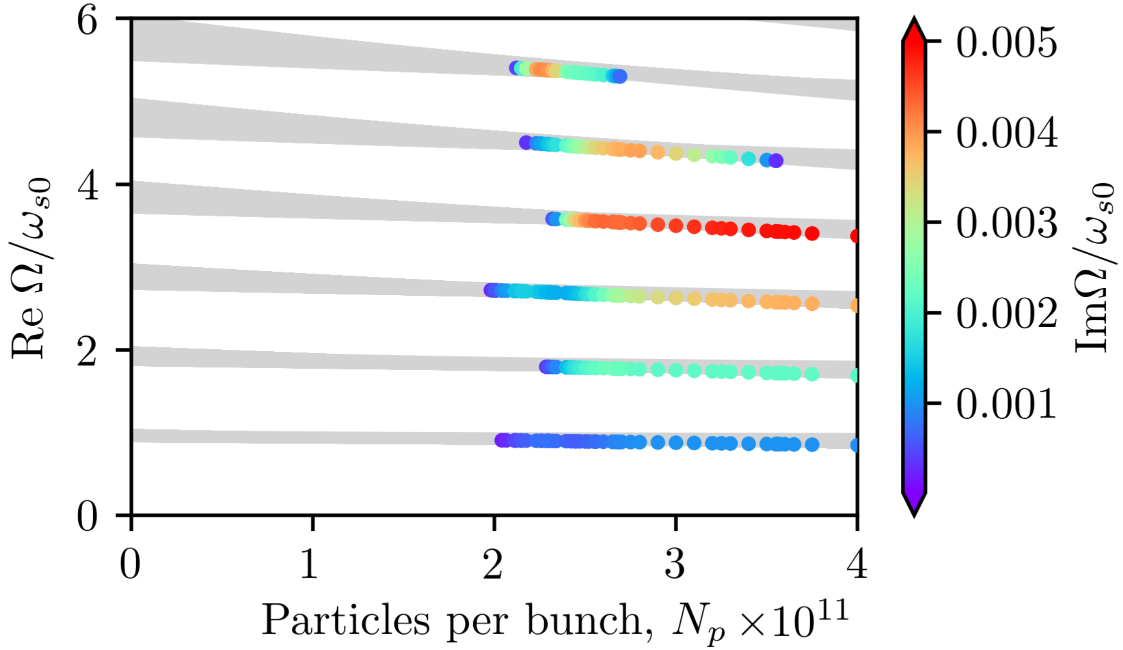
***M.Gadioux, Evaluation of longitudinal single-bunch stability in the SPS and bunch optimization for AWAKE, 2020*

Unstable island

Calculations at flattop



van Kampen modes



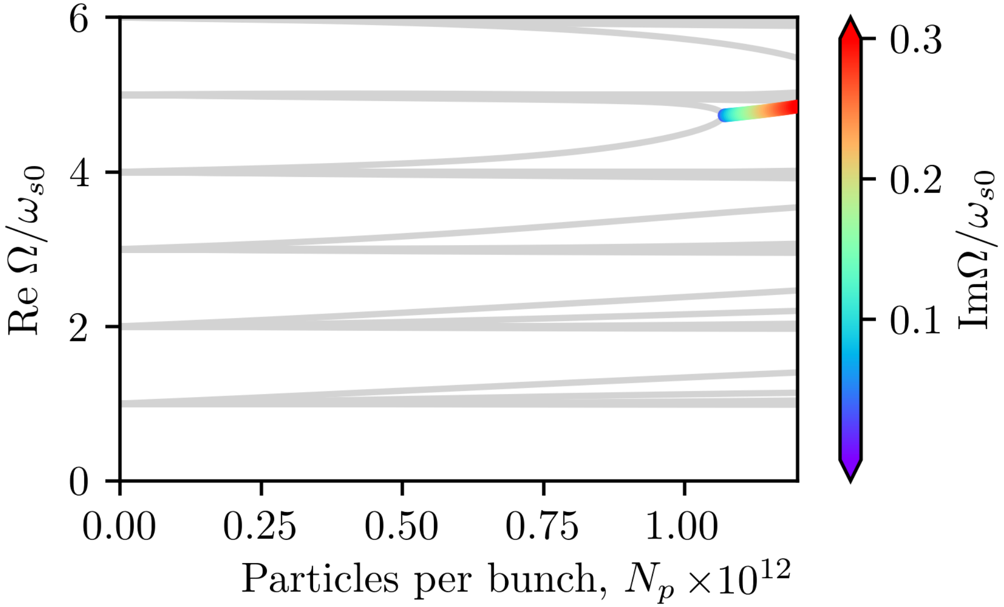
→ Radial mode-coupling instability** since there is no overlap of modes from different azimuthal bands

→ Coupling is present in many azimuthal modes simultaneously (microwave regime)

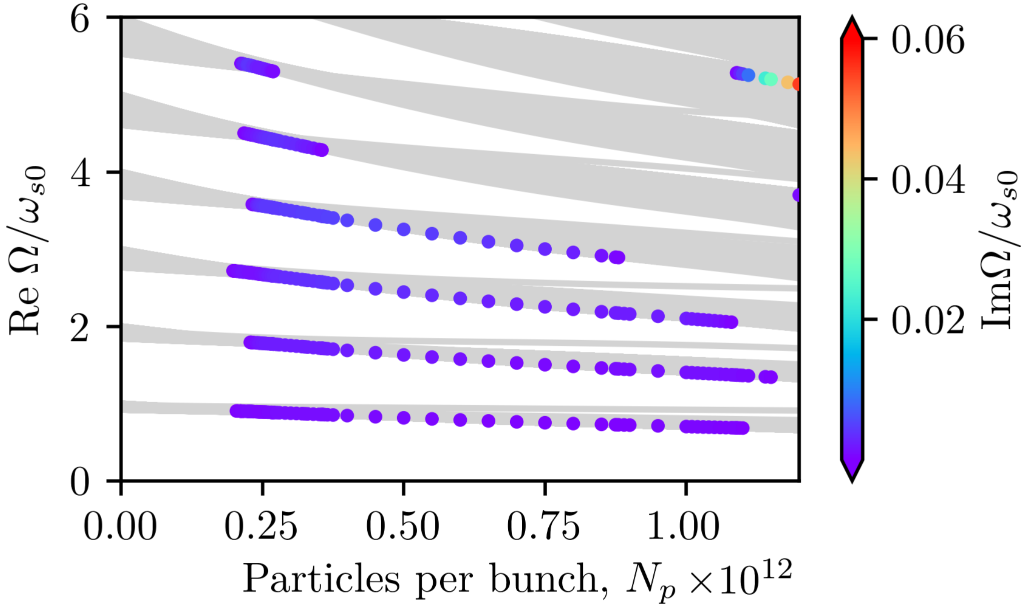
**K. Oide and K. Yokoya, Longitudinal single bunch instability in electron storage rings, 1990*

Role of rf nonlinearity

van Kampen mode for linear rf without potential-well distortion (PWD)



van Kampen modes



If PWD and rf nonlinearity are neglected, the instability threshold is 5 times higher (azimuthal mode-coupling instability*) than for radial mode-coupling instability

In a self-consistent approach, a strong radial mode-coupling instability emerges at this intensity
→ rf nonlinearity can significantly reduce the threshold

**F. J. Sacherer, Bunch lengthening and microwave instability, 1977*

Multi-bunch instabilities

Instability due to narrowband impedance

Coupled-bunch mode l of M equidistant bunches can be driven by impedance with $k_{\mathrm{nb}} = \lfloor f_{r,\mathrm{nb}}/f_0 \rfloor = pM + l$

The threshold can be obtained from the Lebedev equation. If the resonator bandwidth $\Delta\omega \ll M\omega_0$ and k_{nb} is far from $M/2$ harmonics*

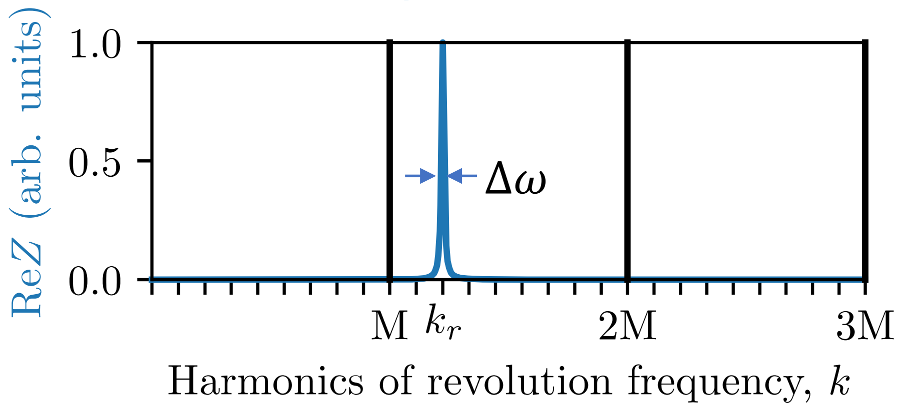
The coupled-bunch instability (CBI) threshold for the binomial distribution is the lowest for $m = 1$**

$$N_{\mathrm{CBI}} \approx \frac{V_0 \phi_{\mathrm{m}}^4 k_{\mathrm{nb}}}{16 q \omega_{\mathrm{rf}} M R_{\mathrm{nb}}} \min_{y \in [0,1]} \left[\frac{(1-y^2)^{1-\mu}}{\mu(\mu+1)} J_1^{-2} \left(\frac{y k_{\mathrm{nb}} \phi_{\mathrm{m}}}{h} \right) \right]$$

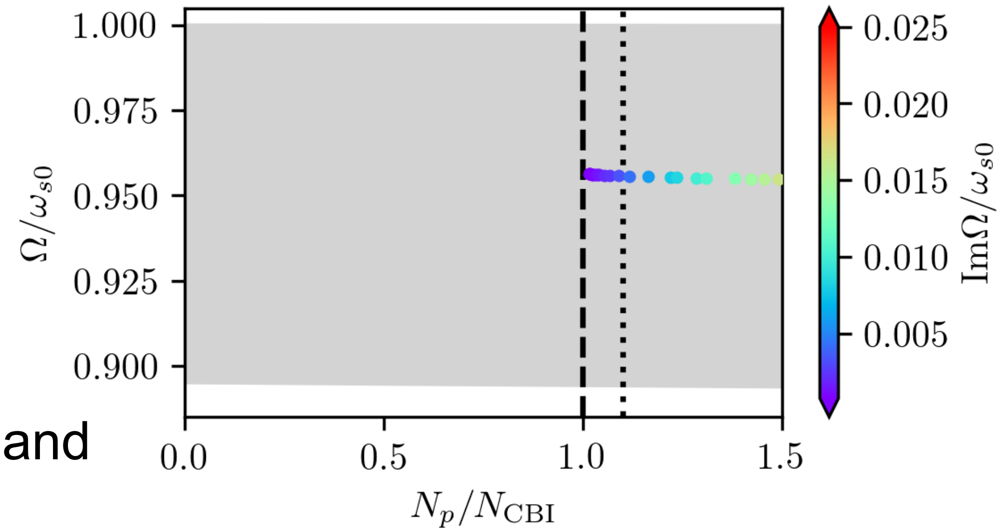
Bessel function

→ Unstable mode Ω_{CBI} is inside the incoherent frequency band

Resonator impedance with $Q = 100$



Example of unstable dipole mode



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*V. I. Balbekov and S. V. Ivanov, Longitudinal beam instability threshold beam in proton synchrotrons, 1986

**IK and E. Shaposhnikova, “Longitudinal coupled-bunch instability evaluation for FCC-hh, 2019

Generalized threshold

Typically, broadband (bb) and narrowband (nb) impedance sources are treated separately, except in a few examples of CBI growth rate calculations*

Including them in the Lebedev equation simultaneously

$$N_g(\Omega_g) \approx \frac{V_0}{q\omega_{\mathrm{rf}}}\left[\sum_{k=-\infty}^{\infty} G_{kk}(\Omega_g)\frac{Z_k^{\mathrm{bb}}(\Omega_g)+Z_k^{\mathrm{nb}}(\Omega_g)}{k}\right]^{-1}$$

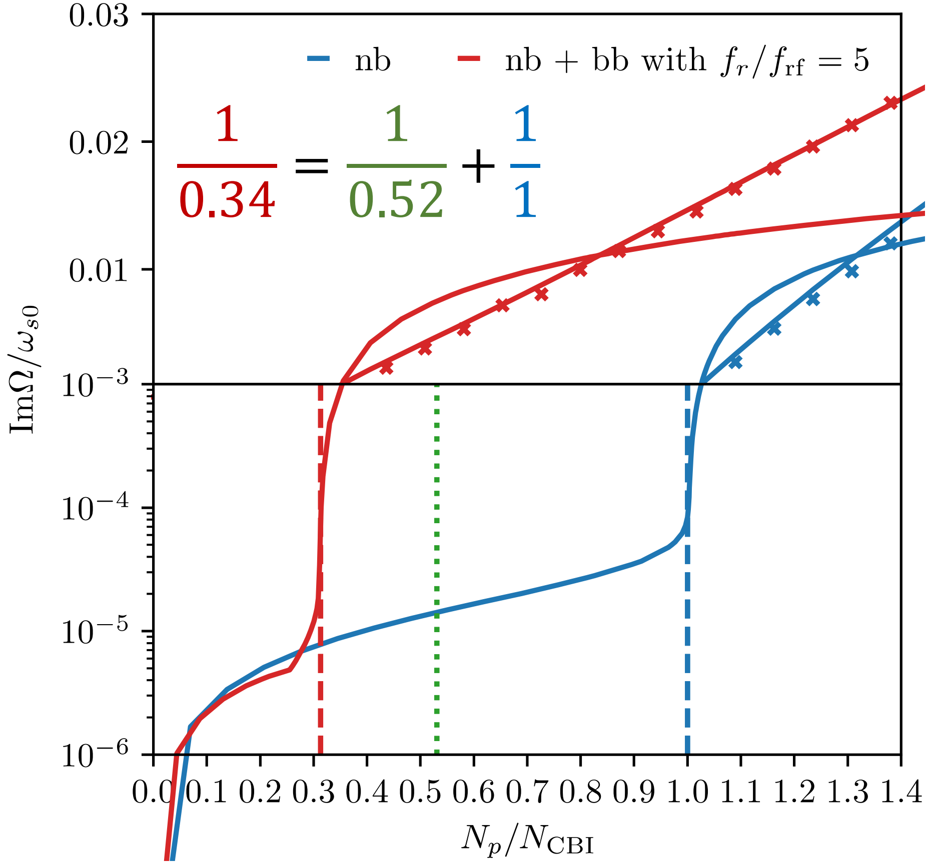
$\Omega_g \neq \Omega_{\mathrm{LLD}}$ and $\Omega_g \neq \Omega_{\mathrm{CBI}}$

→ Approximate threshold (first estimate)

$$\frac{1}{N_g} \approx \frac{1}{N_{\mathrm{LLD}}} + \frac{1}{N_{\mathrm{CBI}}}$$

→ Instability develops below the LLD threshold

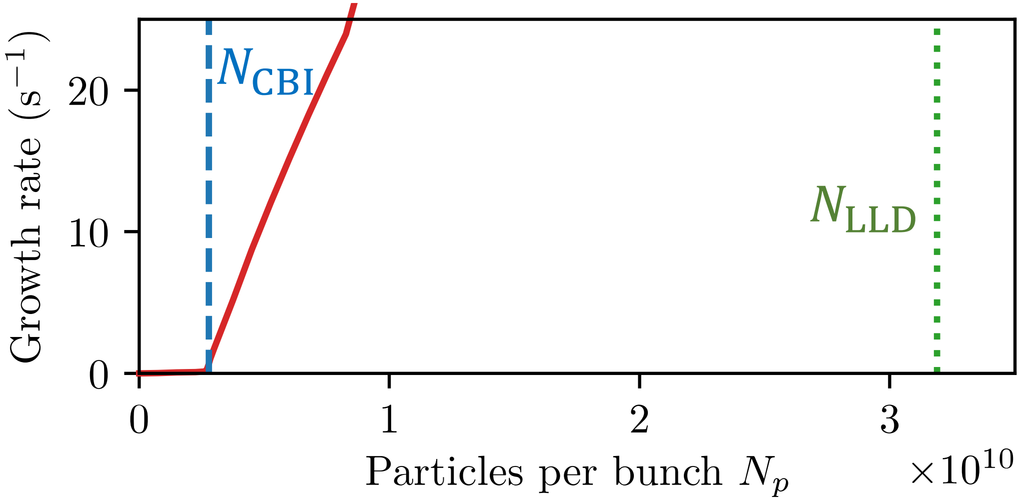
Growth rates of most unstable modes for 9 bunches (MELODY - lines, BLonD - crosses)



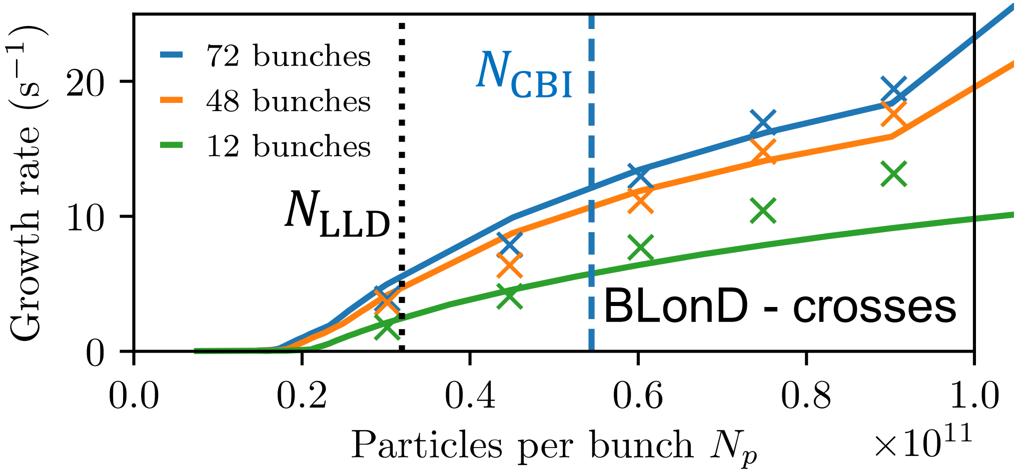
*M. Blaskiewicz, Longitudinal stability calculations, 2009, and recently in A. Burov, Longitudinal modes of bunched beams with weak space charge, 2021

Multi-bunch instabilities in the SPS

Growth rates of most unstable modes for full ring (5 ns bunch spacing)



Growth rates of most unstable modes for LHC-type trains (25 ns bunch spacing)



Instability of fixed-target beams (5 ns spacing) is driven by Higher Order Mode (HOM) of 200 MHz rf system at 914 MHz*

→ LLD has no impact since N_{CBI} is very low

→ Instability of bunch trains is enhanced by LLD (weak dependence on number of bunches)
→ Stability is improved with an additional 800 MHz rf system and controlled emittance blowup (LLD threshold is increased)**

**E. Shaposhnikova, Analysis of coupled bunch instability spectra, 1999*
***LHC Injectors Upgrade, Technical Design Report, Vol. I: Protons, 2014*

Expectations for HL-LHC

Coupled-bunch instabilities (CBI) driven by higher-order modes (HOM) have not been observed in the LHC so far

Bunch intensity for HL-LHC is doubled compared to LHC, and crab cavities with strongly damped HOMs will be installed

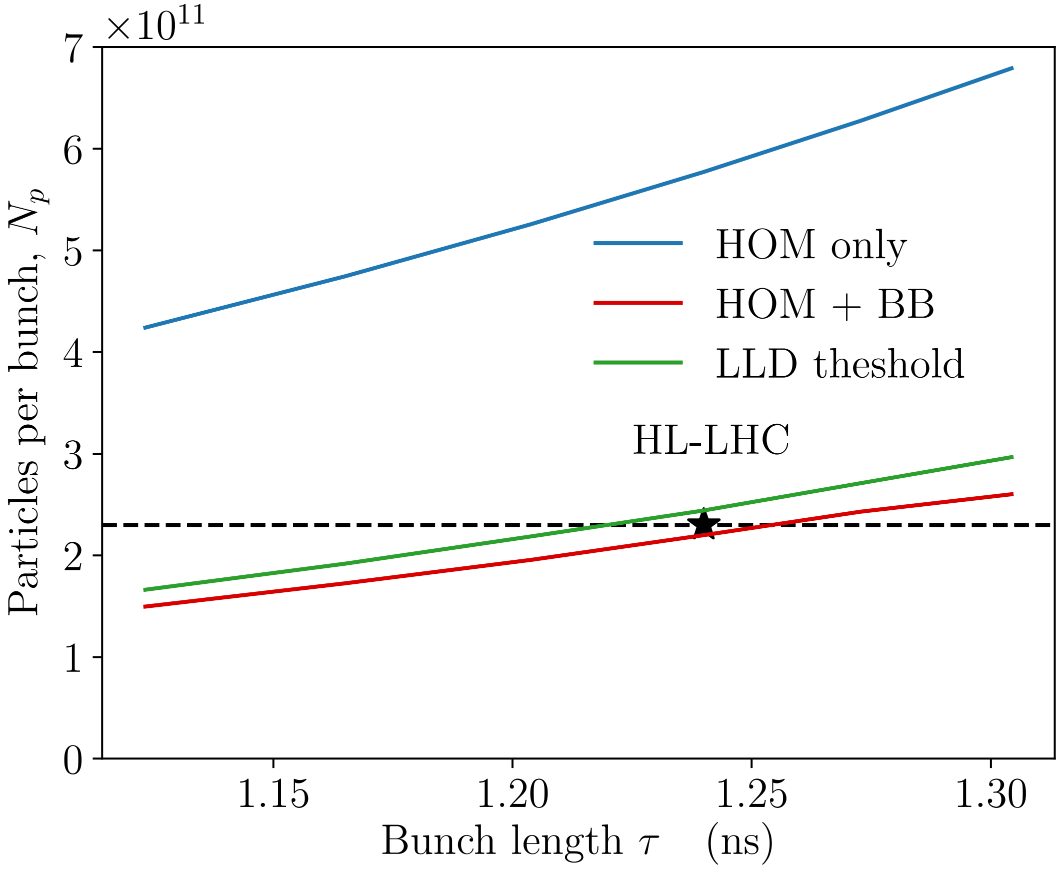
→ In the presence of BB impedance, the instability threshold is reduced below the LLD threshold

→ Precise BB impedance model (f_c) is necessary to predict stability margins

Instability thresholds at $E = 450$ GeV for V_0 = 8 MV:

nb - $R_{\mathrm{nb}} = 4 \times 71$ kOhm, f_r = 582 MHz

bb - $(\mathrm{Im}Z/k)_{\mathrm{eff}} \approx 0.075$ Ohm, f_r = 5 GHz



Summary

Threshold for loss of Landau Damping (LLD) for binomial distribution:

- is inversely proportional to cutoff frequency (vanishes for ImZ/k = const)
- has weaker dependence on the bunch length (4th instead of 5th power)
- can be evaluated for arbitrary impedance using effective-impedance parameters

Single bunch instability threshold:

- is mainly determined by the radial mode-coupling mechanism
- can be reduced by rf nonlinearity

Multi-bunch instability threshold:

- is defined by both broadband and narrowband impedance contributions
- can be below the LLD threshold

These findings are supported by numerical calculations and beam measurements

Thank you for your attention!