Inverse Stability Problem in Beam Dynamics

Alexey Burov

Fermilab

HB Workshop, 10/10/2023 CERN

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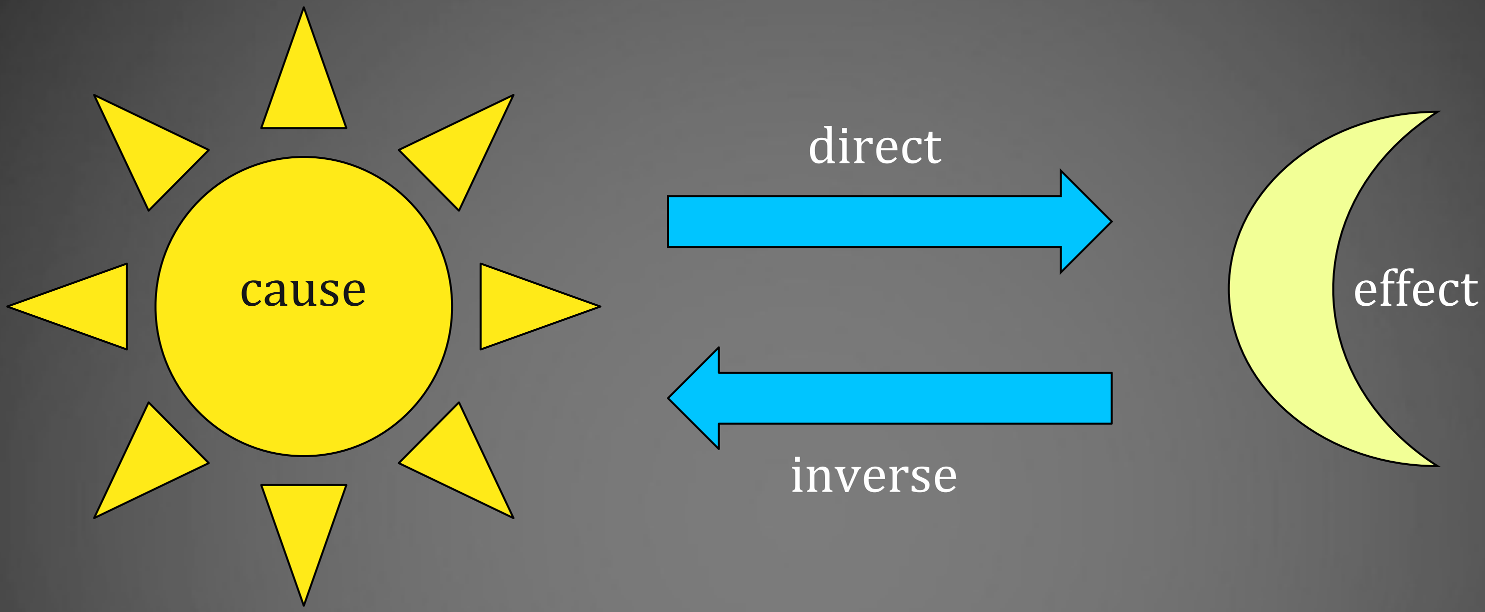
Direct and Inverse Problems



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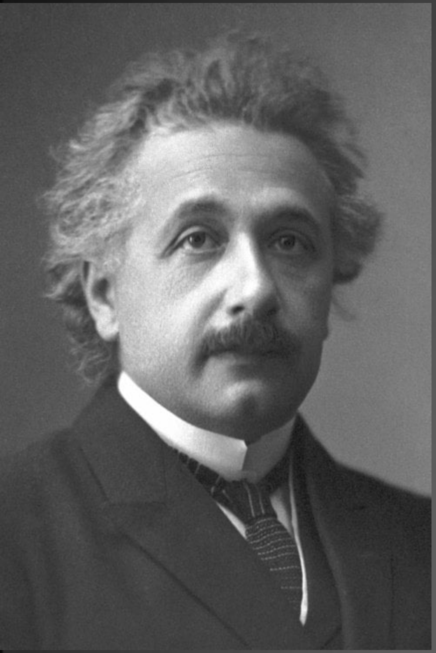
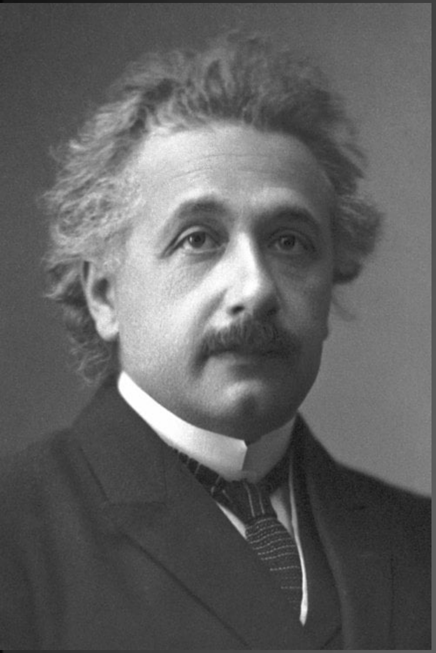


1879–1955

How a simplest unified description
for gravity and inertia
could look like?
1907–1915

Direct and Inverse Problems



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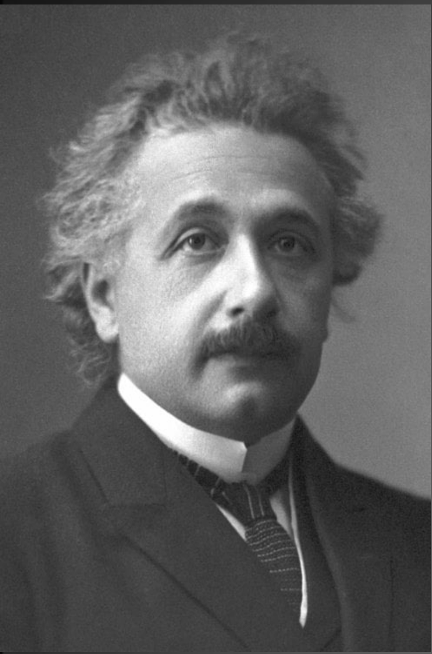

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What could be a fate of the Universe,
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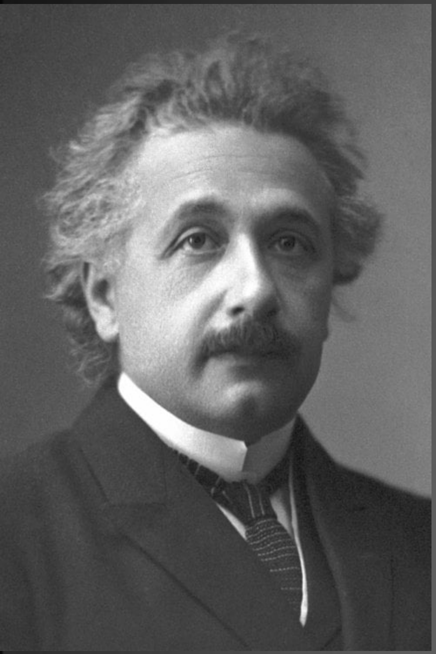

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Harmonic oscillators with an antidamper

$$\dot{a}_k + i\Delta\omega_k a_k = -ig\bar{a}$$

AB

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$$-\left[\int\int \mathrm{d}J_x \mathrm{d}J_y \frac{J_x \frac{\partial F}{\partial J_x}}{\nu - \Delta\omega(J_x, J_y) + io}\right]^{-1} = g$$

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1D, octupoles, Gaussian

alien nonlinearity

$$\Delta\omega(J_x, J_y) = kJ_y$$

→

$$\left[\int \mathrm{d}J_y \frac{F_y(J_y)}{\nu - J_y + io}\right]^{-1} = g$$

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own nonlinearity

$$\Delta\omega(J_x, J_y) = kJ_x$$

$$F_y \to -J_x \frac{\partial F_x}{\partial J_x}$$

Hereward rule

AB

1D, octupoles, Gaussian

alien nonlinearity

$$\Delta\omega(J_x, J_y) = kJ_y$$

k=1

$$\left[\int \mathrm{d}J_y \frac{F_y(J_y)}{\nu - J_y + io}\right]^{-1} = g$$

own nonlinearity

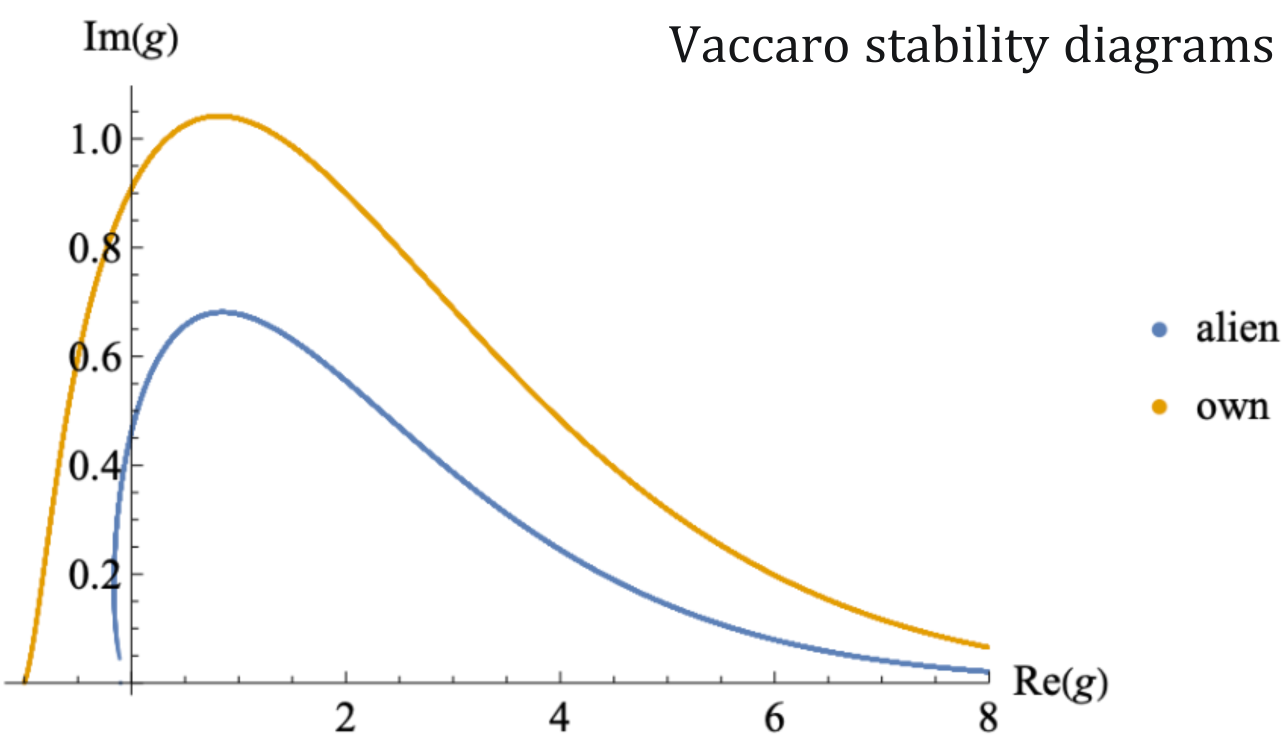
$$\Delta\omega(J_x, J_y) = kJ_x$$

$$F_y \to -J_x \frac{\partial F_x}{\partial J_x}$$

Hereward rule

$$F_{x,y} \propto e^{-J_{x,y}}$$

Vaccaro stability diagrams



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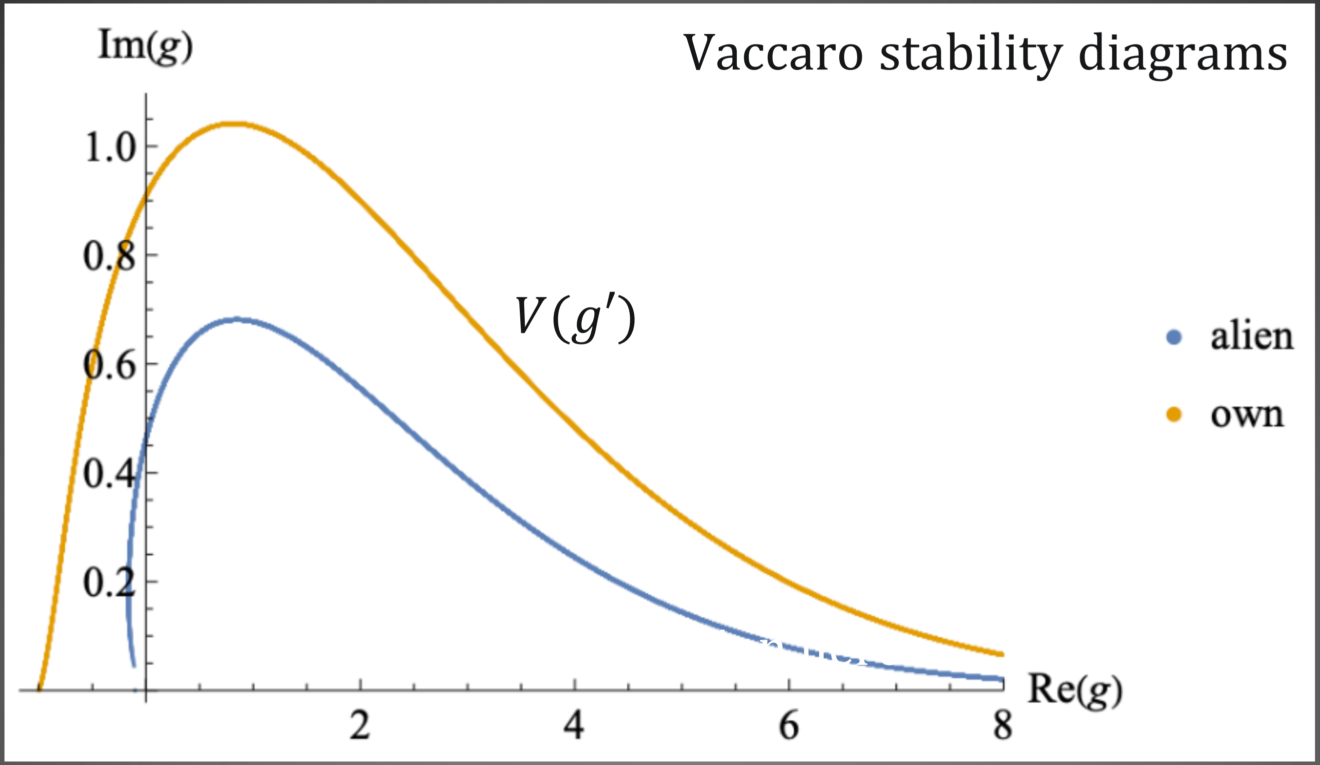
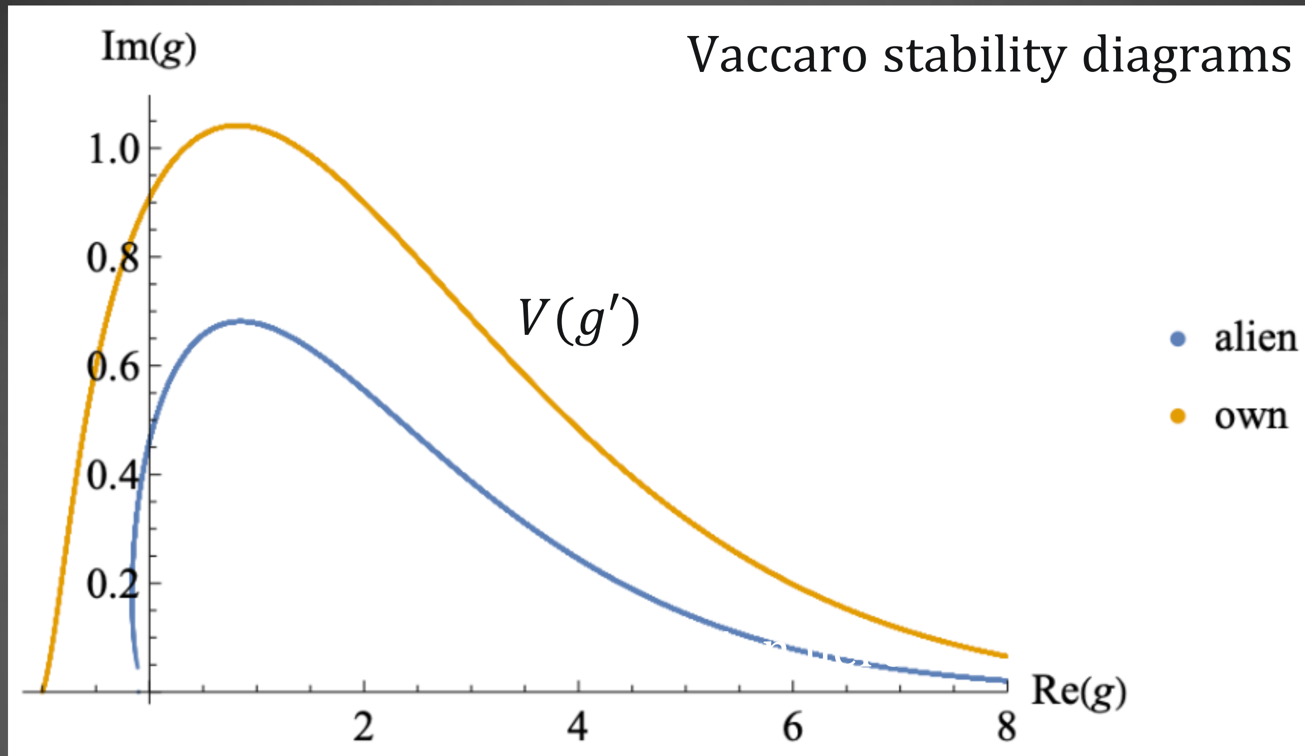
Direct and Inverse Stability Problems



Direct problem: $F(J) \rightarrow V(g')$

Inverse problem: $V(g') \rightarrow F(J)$

Direct and Inverse Stability Problems



Direct problem: $F(J) \rightarrow V(g')$

Inverse problem: $V(g') \rightarrow F(J)$; a pair of nonlinear integral equations.

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Proof-of-Principle Direct Measurement of Landau Damping Strength at the Large Hadron Collider with an Antidamper

S. A. Antipov,[1,2,*] D. Amorim,[1,3] N. Biancacci,[1] X. Buffat,[1] E. Métral,[1]
N. Mounet,[1] A. Oeftiger,[1,4] and D. Valuch[1]

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Tails: easy

$$\int \mathrm{d}J_y \frac{F_y(J_y)}{\nu - J_y + io} \simeq \frac{1}{\nu} - \pi i\, F_y(\nu)$$

$$\Re g \simeq \nu;\ \Im g \simeq \pi \nu^2 F_y(\nu)$$

$$F_y(\nu) \simeq \frac{\Im g(\nu)}{\pi \nu^2}$$

Core: fitting approach

$$A = \Delta \Re g_{\mathrm{FWHM}} / \max \Im g$$

aspect ratio for V

$$F(J) \propto \left(1 - \frac{J}{J_0}\right)^n \qquad n\, J_0 > 0$$

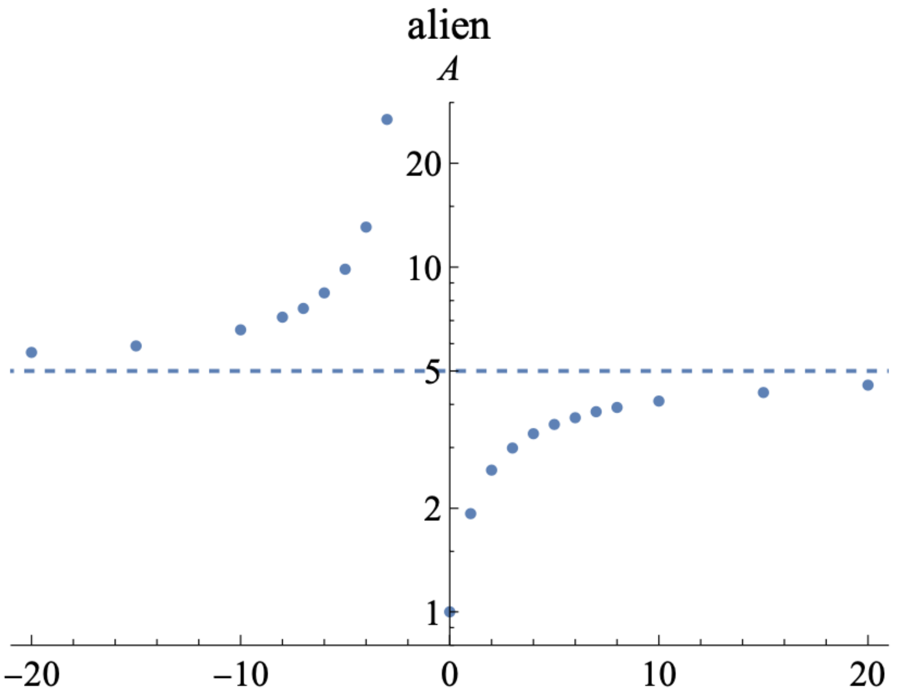

FIG. 2. Aspect ratio A of 1D stability diagram, the alien case, versus the power n of the binomial distribution function $\propto (1 - J/J_0)^n$, $nJ_0 > 0$. Note that $\lim_{n\to -2} A = \infty$. The dashed line marks the asymptote, $F(J) = J_0^{-1} e^{-J/J_0}$, $J_0 > 0$.

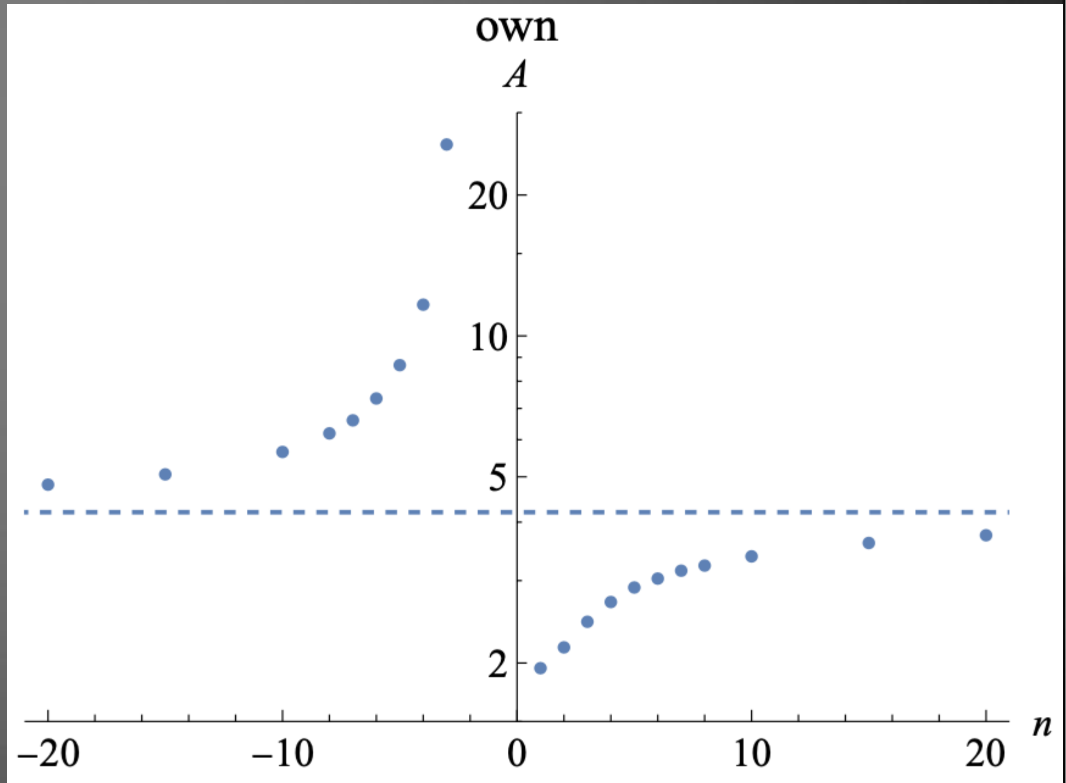

FIG. 3. The same as Fig. 2 for the own case. Here $\lim_{n\to -2} A = \infty$ as well.

Core: iterative *4-leg walk*

1. Compute integrals with your initial guess $F(J)$;
2. With that, make tables $g'(\nu)$; $g''(\nu)$;
3. Update your guess as $F(\nu) = -\pi^{-1}\Im g^{-1}(\nu)$;
4. Normalize the updated $F(J)$ and go back to 1.

Convergency Limitation

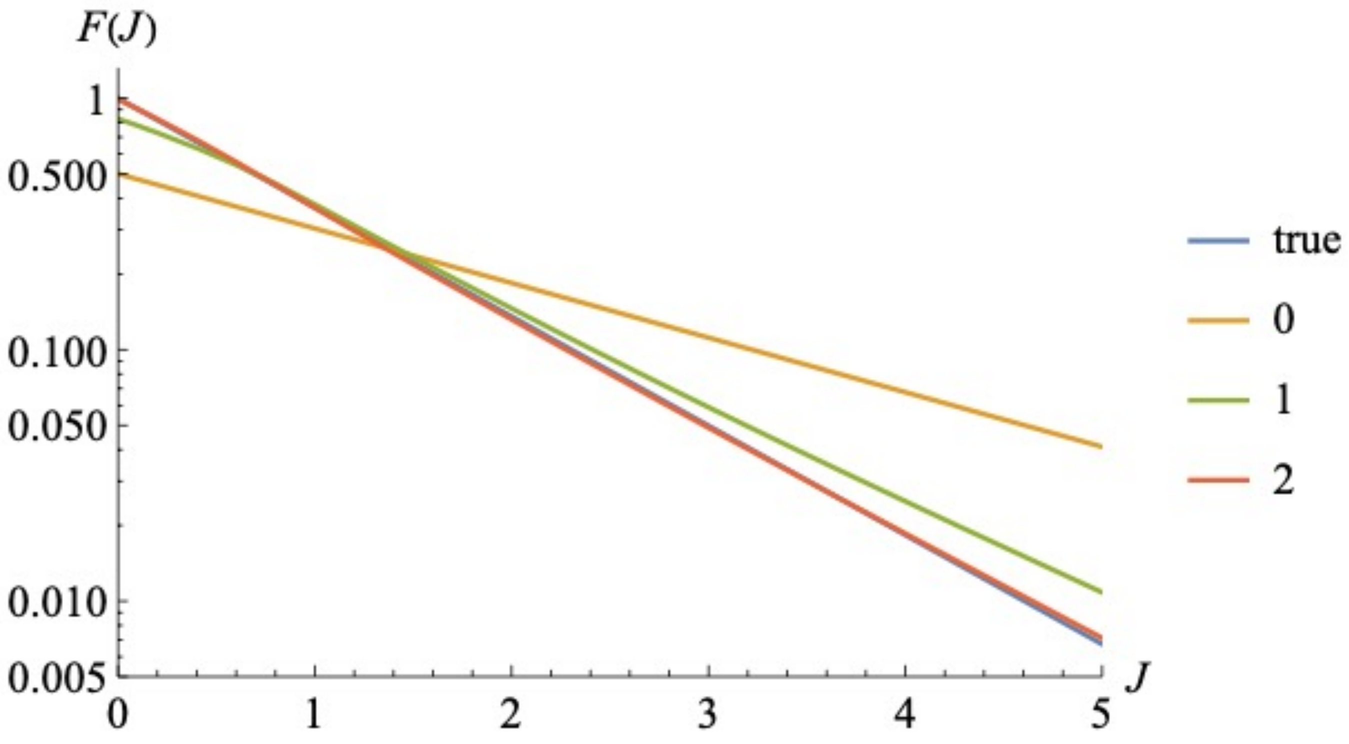

FIG. 4. An example of the iteration convergence for the alien case, $\nu_{\min} = 0.7$. Here "true" means the distribution responsible for the "measured diagram"; "0" means the initial guess of the distribution, while "1" and "2" stand for the output distributions after the first and second four-leg moves of the algorithm. The latter is clearly very fast, but it becomes unstable at small actions, $J \lesssim 0.5$, for a slightly smaller border $\nu_{\min}$.

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2D case $\Delta\omega(J_x, J_y) = k_x J_x - k_y J_y$

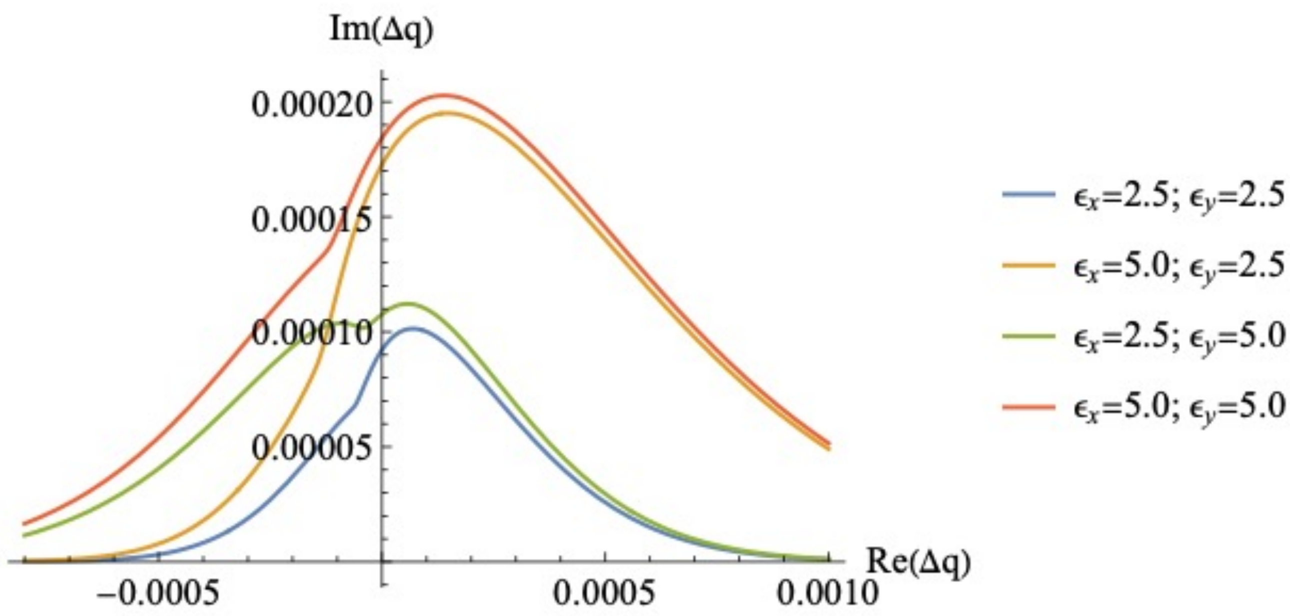

FIG. 7. Vaccaro diagrams calculated for a Gaussian bunch at the LHC top energy for 550A of the octupole current, yielding $k_x = 1.0 \cdot 10^{-4}$, $k_y = 0.7 \cdot 10^{-4}$ for the normalized rms emittances 2.5mm·mrad; for more details see Ref. [5]. Gaussian normalized rms emittances for each curve are shown.

Positive tune shifts mostly correspond to *x*, negative — to *y*.
The problem is effectively factorized, reducing to 1D case.

Chromaticity effects

If $|g| \ll \omega_s$ then the gain is distributed between the headtail modes:

$$g \rightarrow g_l = g\, K_l(\zeta) \qquad \text{with } \zeta = \text{rms HT phase}$$

$$K_l(\zeta) = \exp(-\zeta^2)\mathrm{I}_l(\zeta^2)$$ for the longitudinally Gaussian case

$$K_l = \int_0^\infty \mathrm{J}_l^2(\zeta r) f(r) r \mathrm{d}r$$ in general

$$\sum_{l=-\infty}^{\infty} K_l = 1$$

If $|g| \gg \omega_s, |\zeta|\omega_s$, the single rigid–bunch mode is formed, taking the entire gain.

Many thanks!