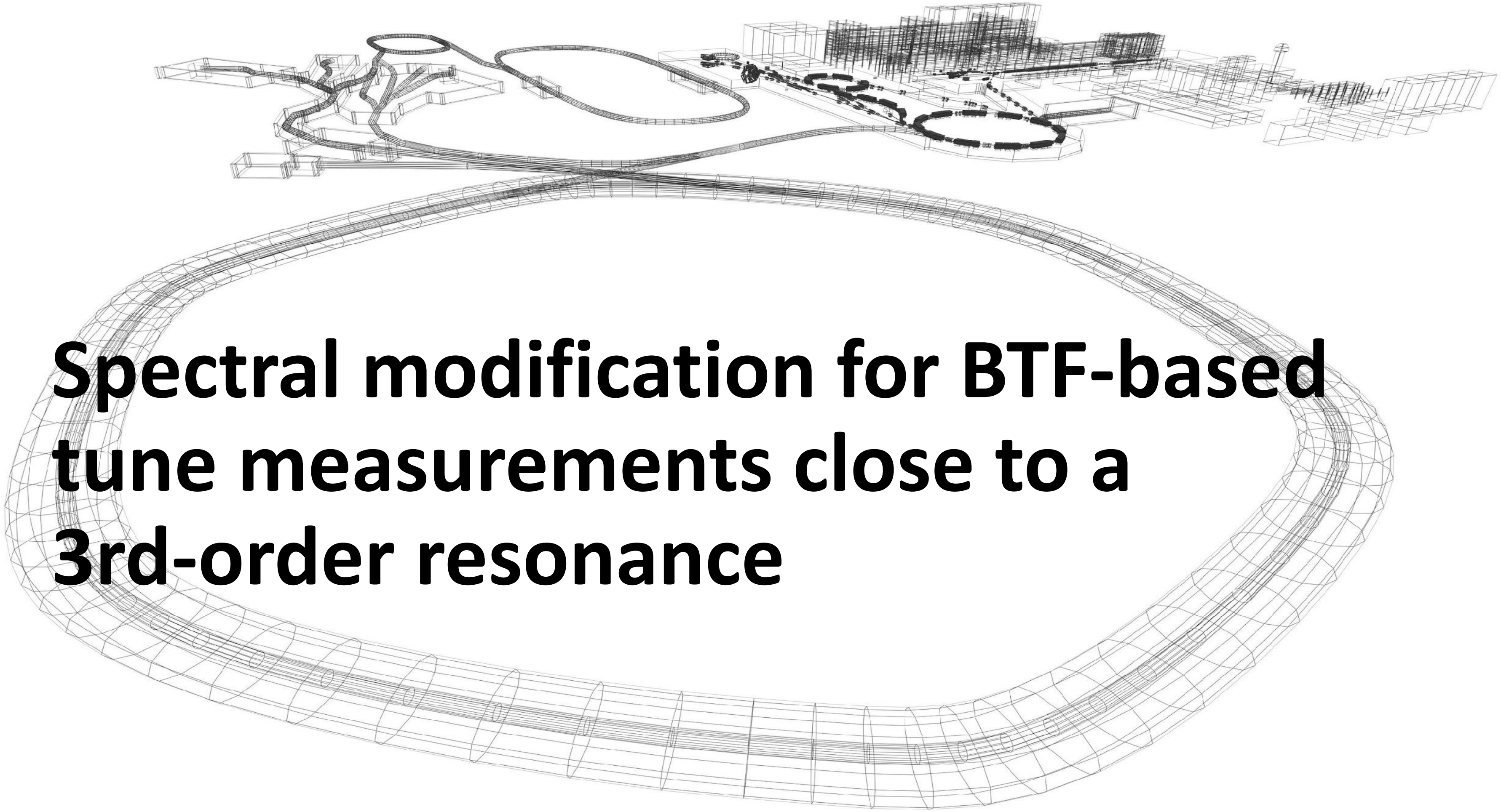
Spectral modification for BTF-based tune measurements close to a 3rd-order resonance



E.C. Cortés García, E. Benedetto, E. Feldmeier, T. Haberer, M. Hun, P. Niedermayer, R. Singh, R. Taylor

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   - Dynamics near the third integer resonance
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   - Single particle dynamics
   - Multiparticle dynamics
5. Summary

Motivation and introduction

- Understand the dynamics near the third order resonance to excite the particles the most efficient way possible
- Application to resonant extraction

Beam Transfer Function measurement

- Observe beam reaction to different excitation frequencies and deduce the dynamics
- Established theoretical framework
- Experimentally available

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Theory

Hamiltonian dynamics

- Machine with no multipole components

$$Q_x = n + \frac{1}{3} + \Delta q_x, n \in \mathbb{N}_0$$

- Linear Hamiltonian

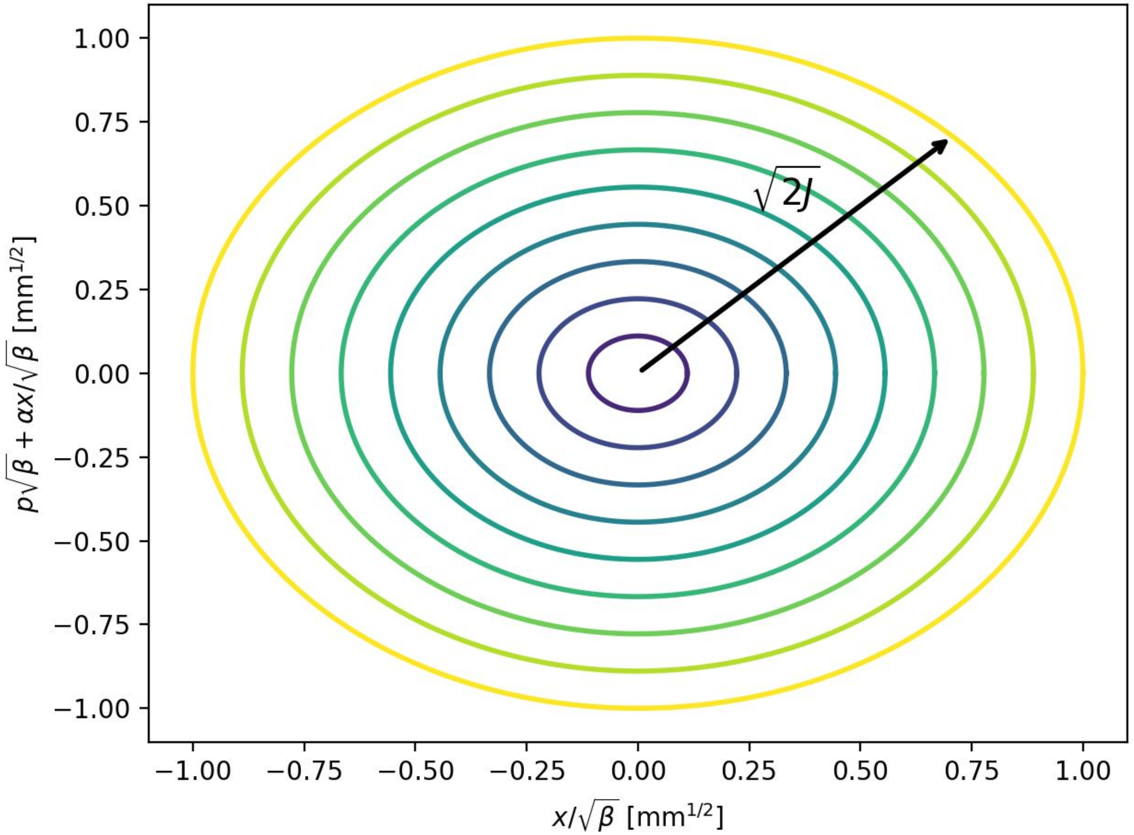
$$H_0 = \frac{\mu_x}{2}\left(X^2 + P^2\right) = \mu_x J, \qquad \mu_x = 2\pi Q_x$$

$$X = x/\sqrt{\beta_x}, \quad P = p_x\sqrt{\beta_x} + \alpha_x X$$

- One-turn phase-advance

$$\frac{1}{2\pi}\frac{\partial H}{\partial J} = Q_{x,0}$$

Equipotential lines in normalized phase-space in a linear machine



Theory

Kobayashi Hamiltonian

- Tune near a third integer resonance

$$Q_x = n + \frac{1}{3} + \Delta q_x, n \in \mathbb{N}_0$$

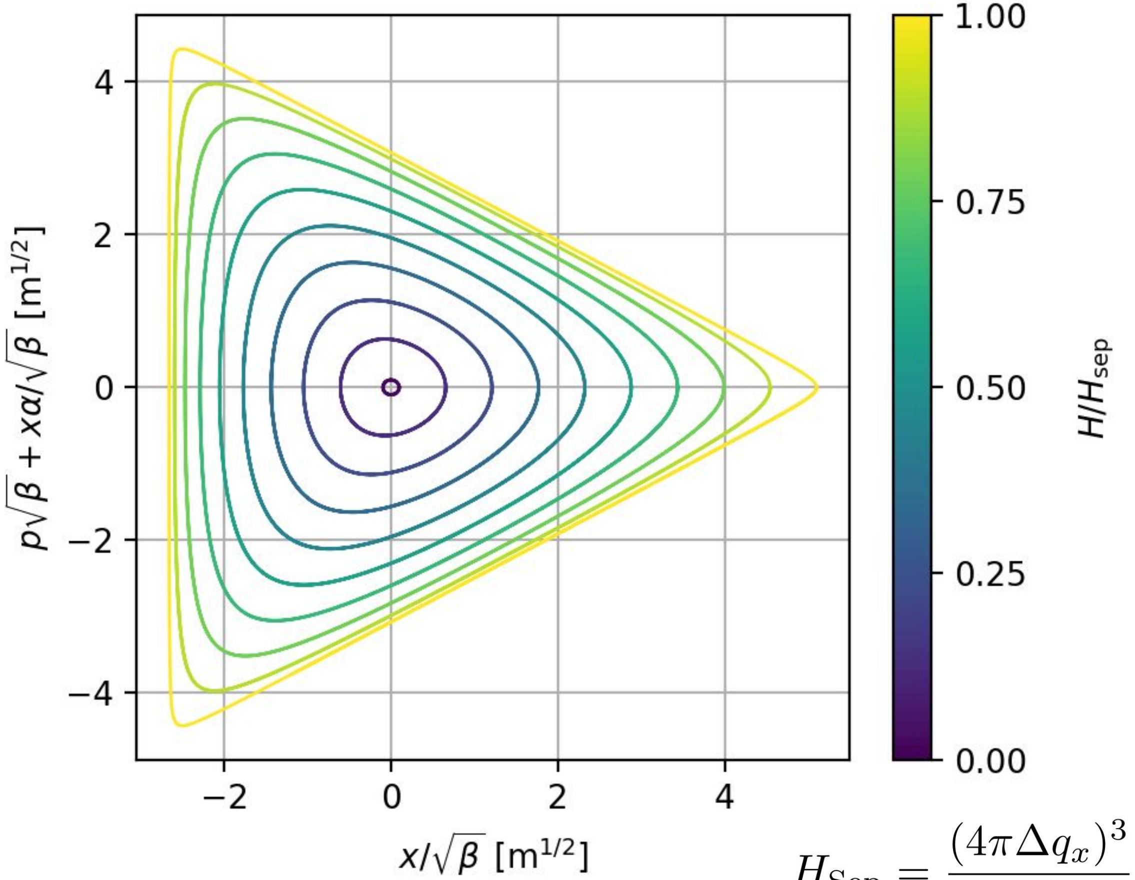
- Resonance driven by a sextupole component S
- The dynamics can be effectively described by [1]

Linear theory — Non-linear term

$$H = 3\pi\Delta q_x(X^2 + P^2) + \frac{S}{4}(3XP^2 - X^3)$$

$$X = x/\sqrt{\beta_x}, \quad P = p_x\sqrt{\beta_x} + \alpha_x X$$

Equipotential lines in normalized phase-space described by the Kobayashi Hamiltonian



$$H_{\text{Sep}} = \frac{(4\pi\Delta q_x)^3}{S^2}$$

[1] Y. Kobayashi and H. Takahashi, Improvement of the emittance in the resonant ejection, in *Proc. VIth Int. Conf. High Energy Accelerators* (Massachusetts, 1967) pp. 347–351.

Theory

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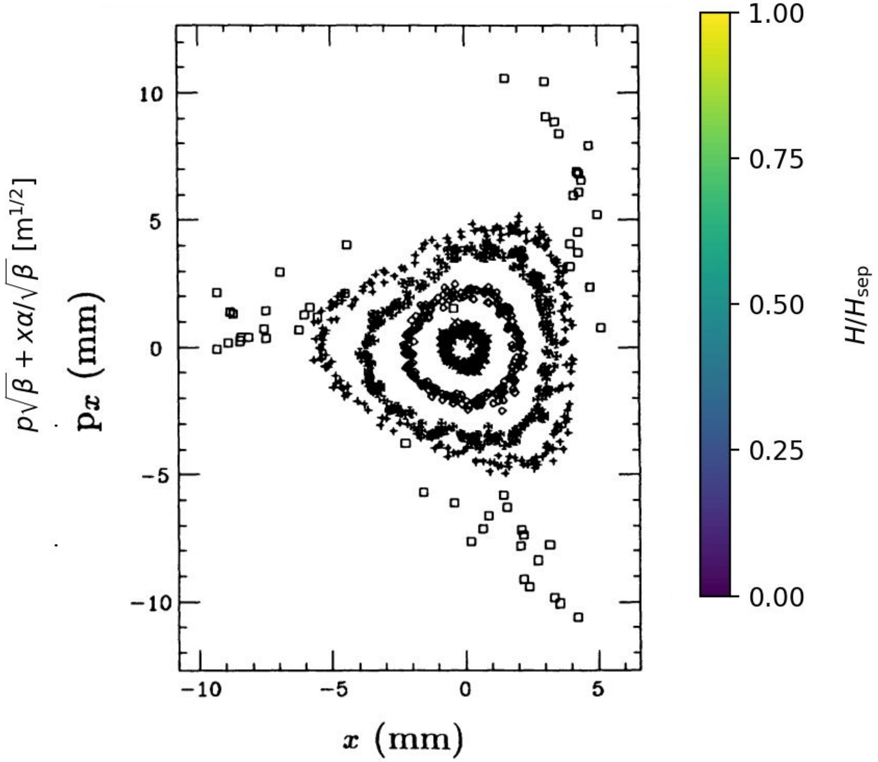
Measurement of the phase-space near the third integer resonance at the IUCF cooler ring (1992) [2].



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[2] D.D. Caussyn, et. al., Experimental studies of nonlinear beam dynamics. Phys. Rev. A (1992).

Theory

Non-linear detuning

$$H = 3\pi\Delta q_x(X^2 + P^2) + \frac{S}{4}(3XP^2 - X^3)$$

- From normalized coordinates to action-angle variables

$$H = 6\pi\Delta q_x J + \frac{S}{\sqrt{2}}J^{3/2}\sin 3\phi$$

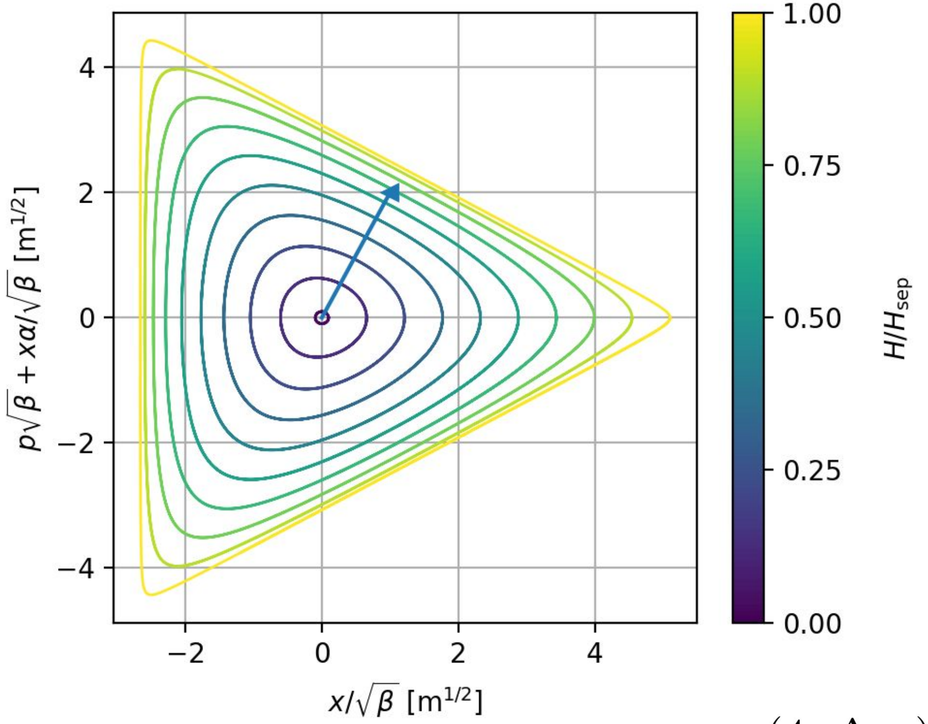
- Particle's tune (One-turn phase-advance)

Linear theory contribution — Non-linear detuning

$$\frac{1}{6\pi}\frac{\partial H}{\partial J} + \frac{n}{3} = q_{x,0} + q_{x,1}$$

$$q_{x,1} = \frac{3S}{\sqrt{2^5}\pi}J^{1/2}\sin 3\phi$$

Equipotential lines in normalized phase-space described by the Kobayashi Hamiltonian



$$H_{\text{Sep}} = \frac{(4\pi\Delta q_x)^3}{S^2}$$

Non-linear detuning

- From normalized coordinates to angle-action variables

$$H = 6\pi\Delta q_x J + \frac{S}{\sqrt{2}} J^{3/2} \sin 3\phi$$

$$H = J\delta - \frac{(2J)^{3/2}F}{48\pi}\cos[3(\phi+\xi)] \quad [2]$$

- Particle's one-turn phase-advance

Linear theory contribution — Non-linear detuning

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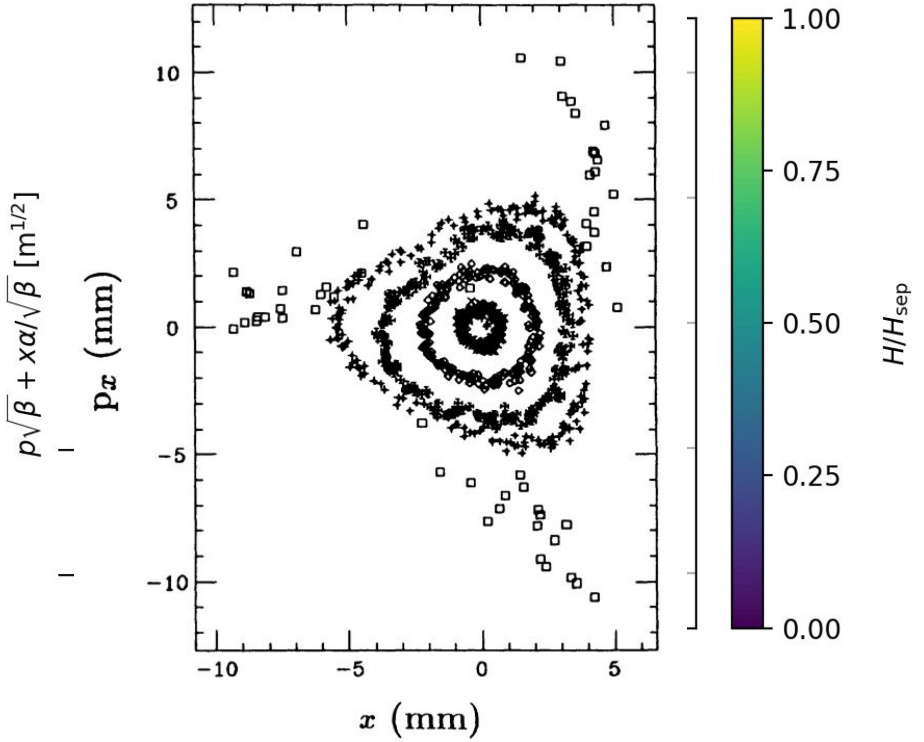

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Theory

Non-linear detuning

- Particle's one-turn phase-advance

Linear theory contribution — Non-linear detuning

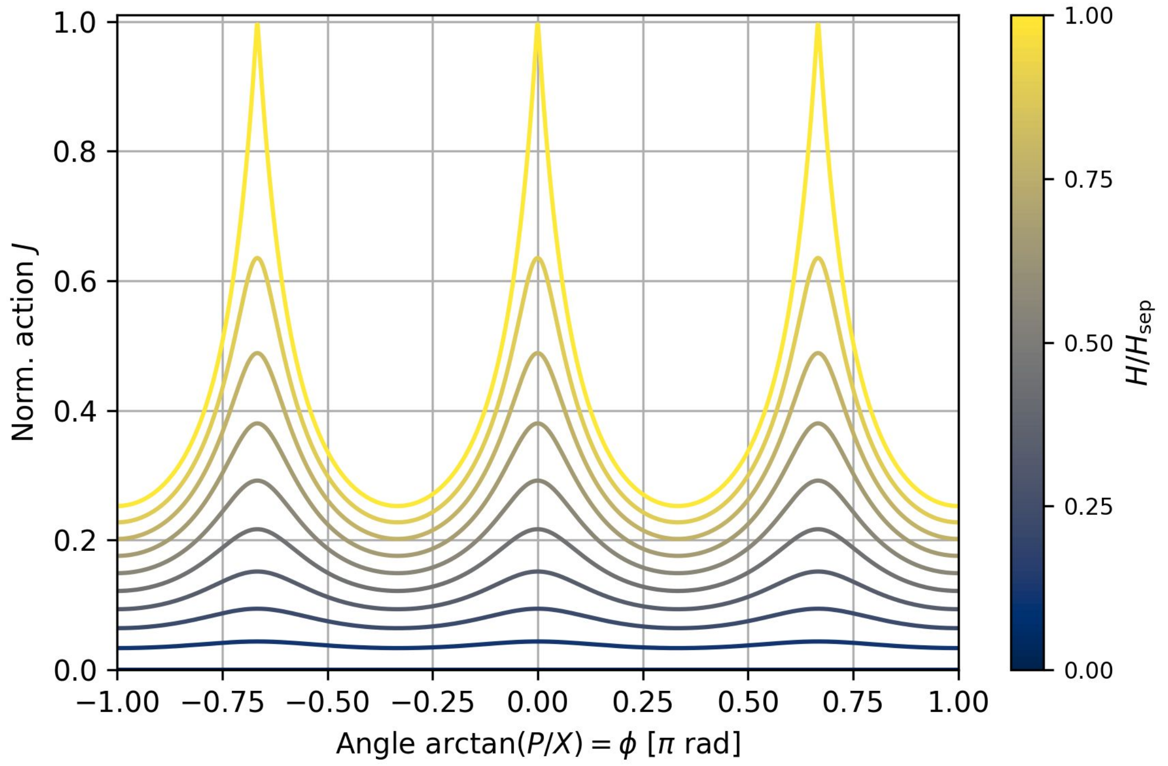
$$\frac{1}{6\pi}\frac{\partial H}{\partial J}+\frac{n}{3}=q_{x,0}+q_{x,1}$$

$$q_{x,1}=\frac{3S}{\sqrt{2^5}\pi}J^{1/2}\sin 3\phi$$

- Near the resonance there is a **phase-amplitude** modulation
- The average detuning over many turns gives a non-vanishing contribution
- The average detuning deviates in the direction of the nearest resonance

- Kobayashi Hamiltonian in action-angle variables

$$H=6\pi\Delta q_x J+\frac{S}{\sqrt{2}}J^{3/2}\sin 3\phi$$



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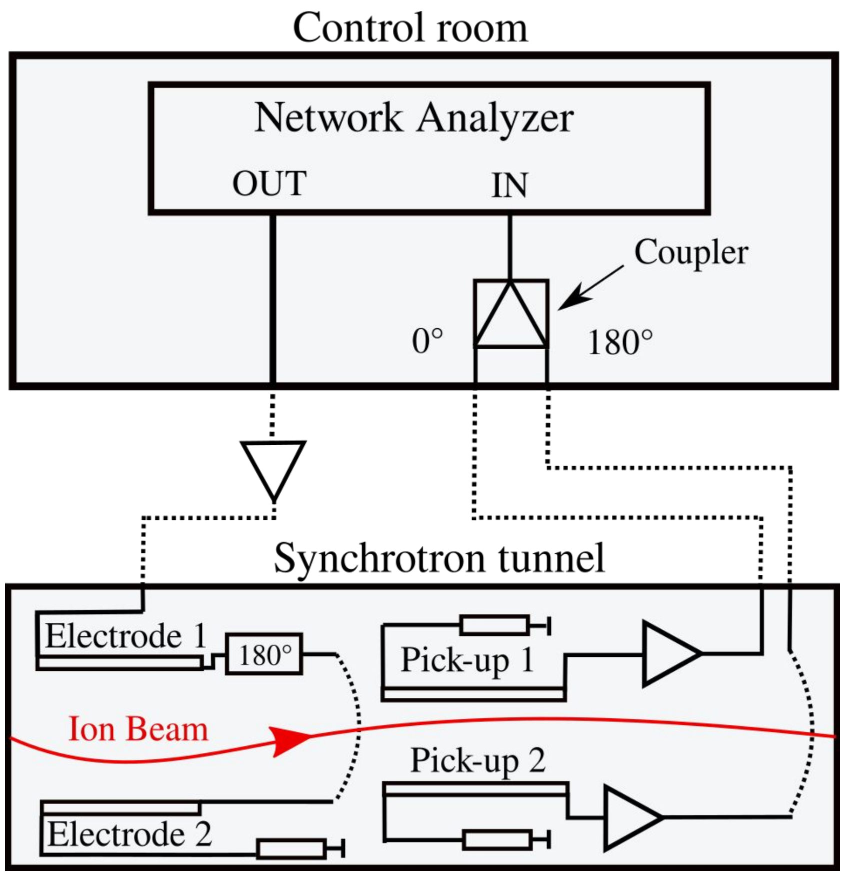
Measurements

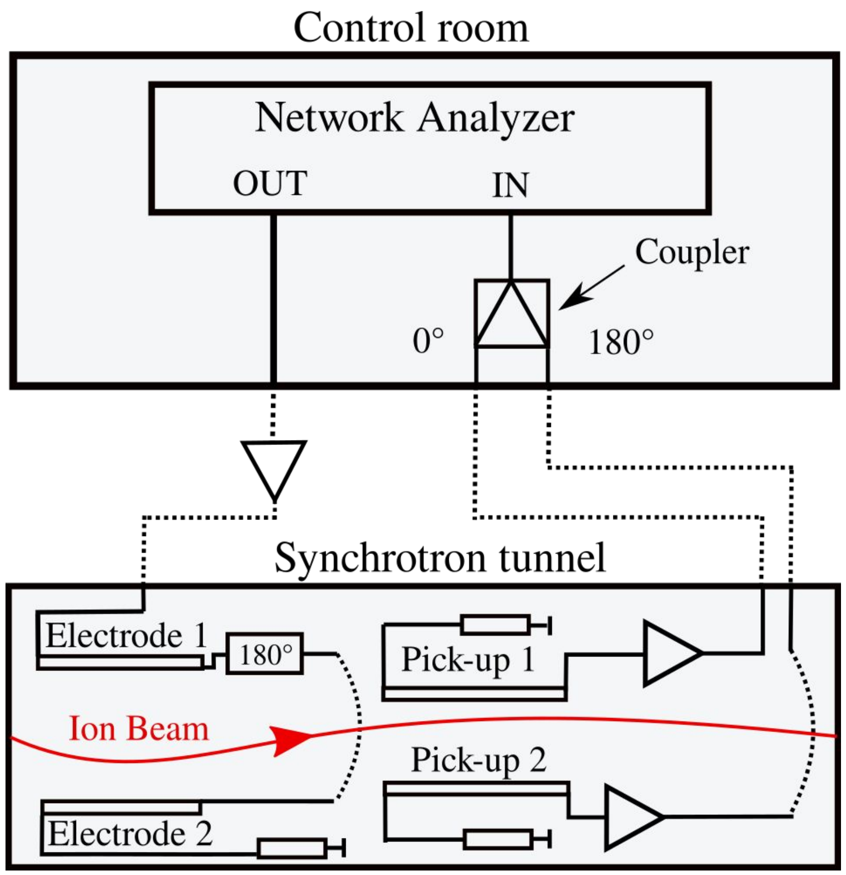
Beam Transfer Function measurement



1. Excite the beam with a single frequency (sinusoidal) signal
   - Generates a beam centroid oscillation with an amplitude of < 500 microns
   - Beam pipe radius is 8 cm
2. Observe centroid response signal
3. Extract the frequency component of the excitation signal
4. Go to next frequency and start from Step 1.

Measurements

Beam Transfer Function measurement



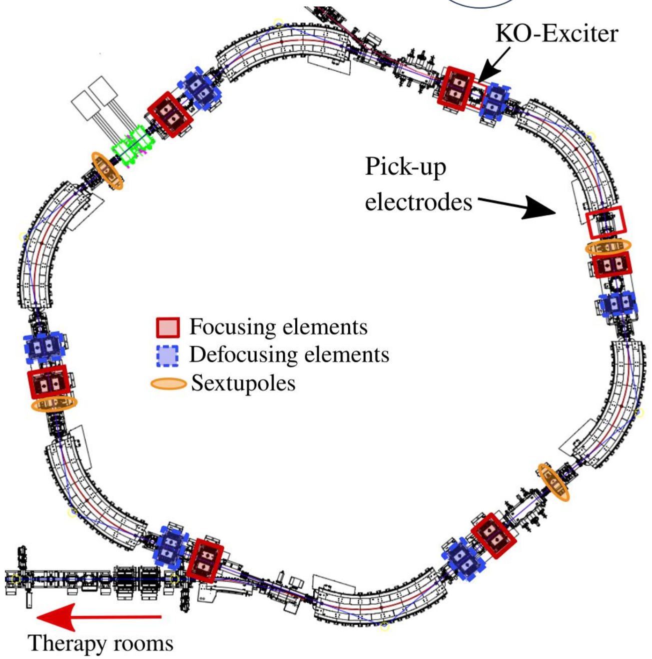
1. Excite the beam with a single frequency (sinusoidal) signal
2. Observe centroid response signal
3. Extract the frequency component of the excitation signal
4. Go to next frequency and start from Step 1.

- Investigation of **coasting beams**
- Low intensity (10^8 - 10^9 particles)
- Momentum spread ~ 10^{-3}
- Measurement campaigns at Heidelberg with Carbon-ions
- Measurement campaigns at GSI with Argon- and Uranium-ions

Measurements

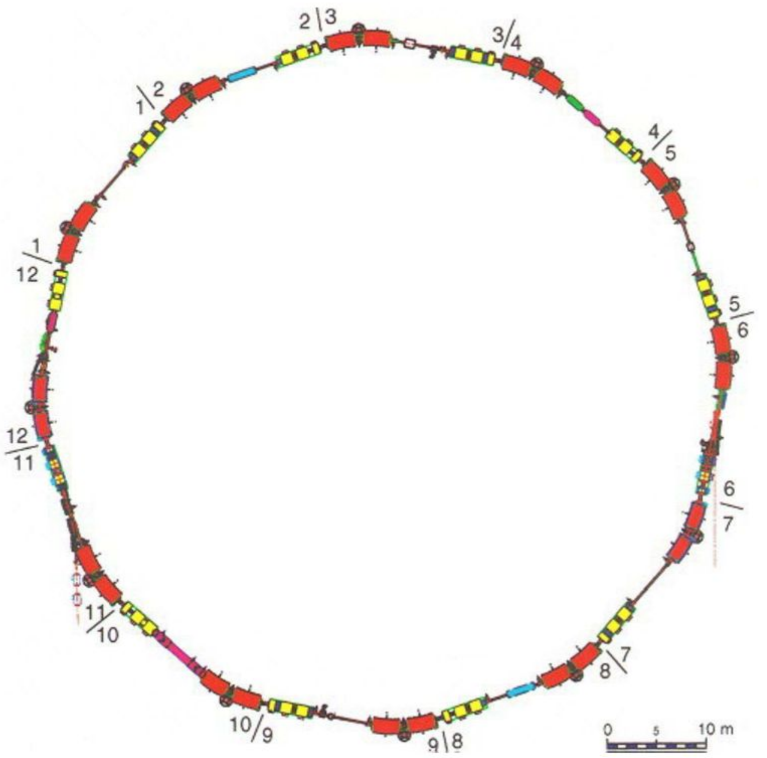
Heidelberg Ion-Beam Therapy Center synchrotron

GSI Heavy Ion Synchrotron



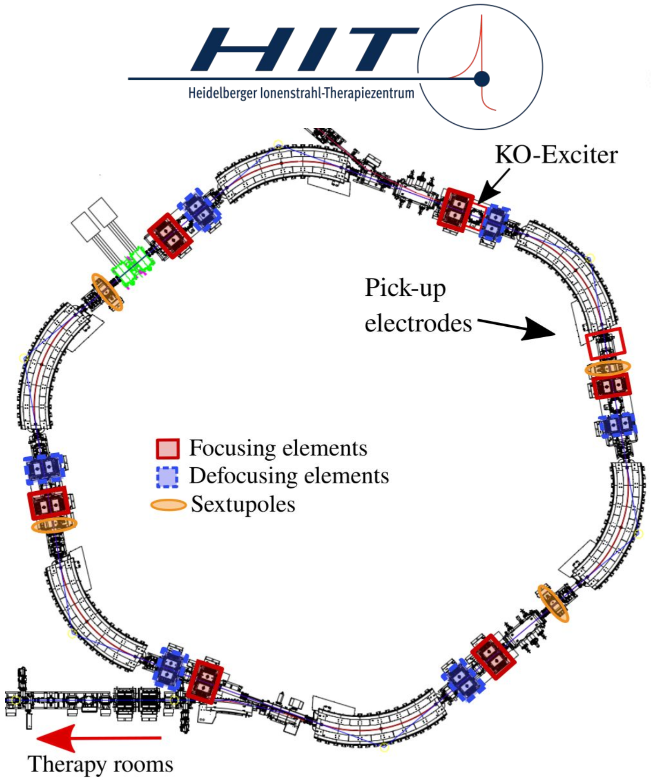
Parameter	HIT	GSI
Circumference	64.986 m	216.720 m
Tunes (Q_x, Q_y)	(1.67,1.74)	(4.29, 3.27)
Chromaticity (ξ_x, ξ_y)	(-1.7, -1.6)	(-5.5, -5.0)
Harmonic n	2	4
Ion types	p^+, He^{2+}, C^{6+}, O^{8+}	p^+ to U^{73+} Ar^{10+}

- Compact synchrotron designed for therapy
- Very flexible synchrotron for the acceleration of diverse types of ions



Measurements

Heidelberg Ion-Beam Therapy Center synchrotron



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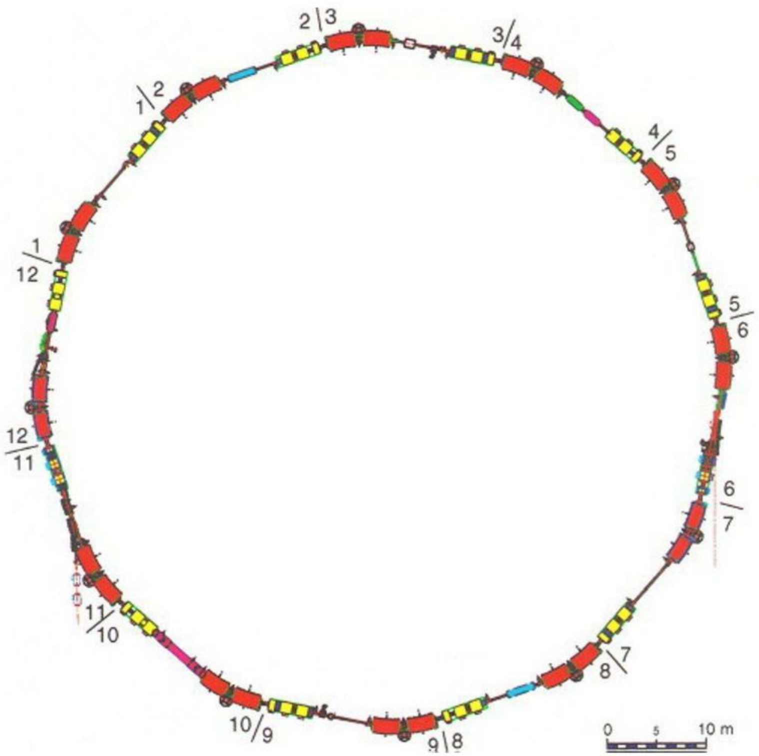


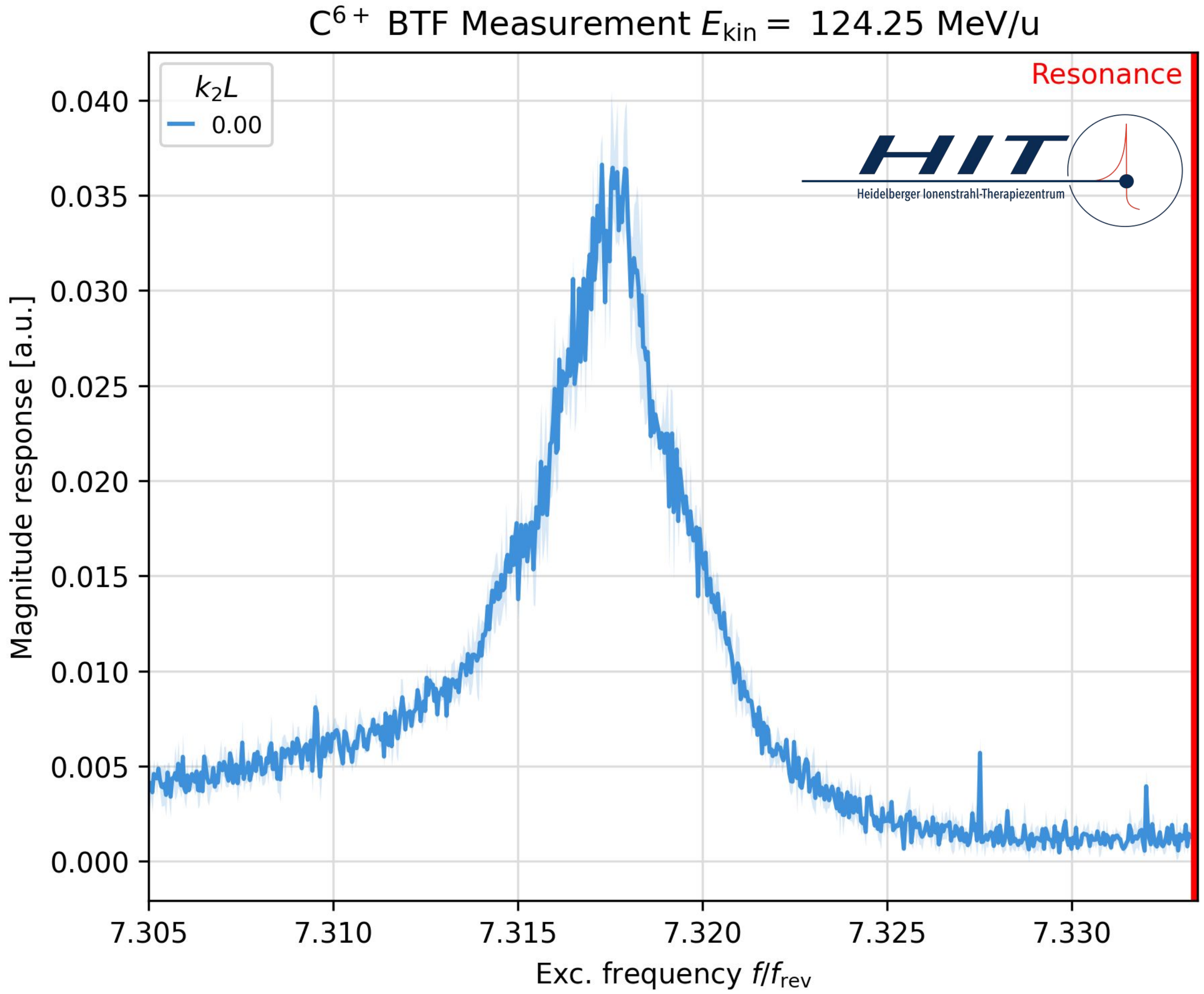
Measurements

Scans over sextupole strength

- Excitation strength set to -10dBm (~200 nrad kick)
- 701 points
- 3 shots
- 10 s measurement time per shot
- Investigation of lower 8th betatron band

$$f/f_{\mathrm{rev}} = n - q_x$$

- Single peak (linear case)



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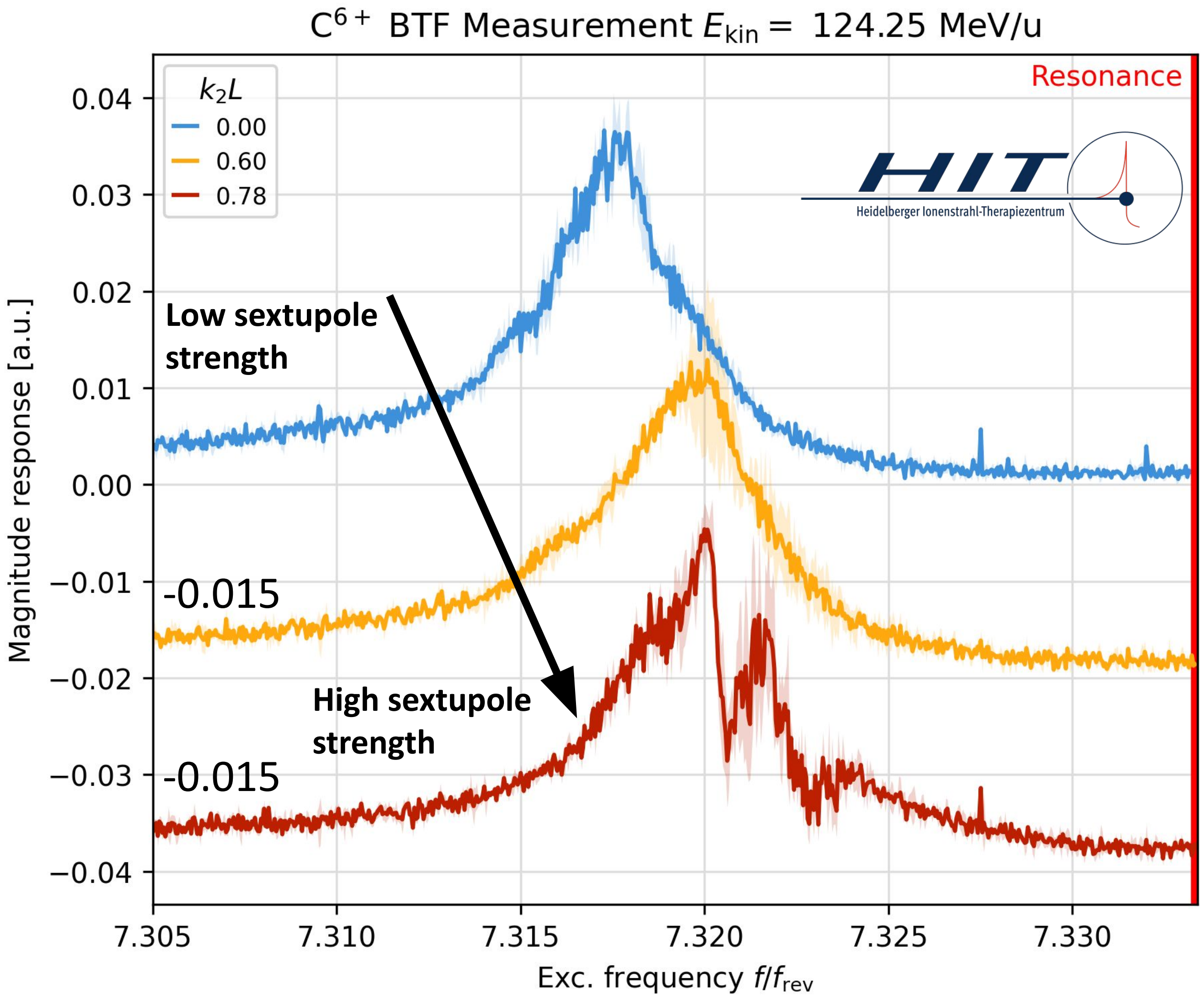
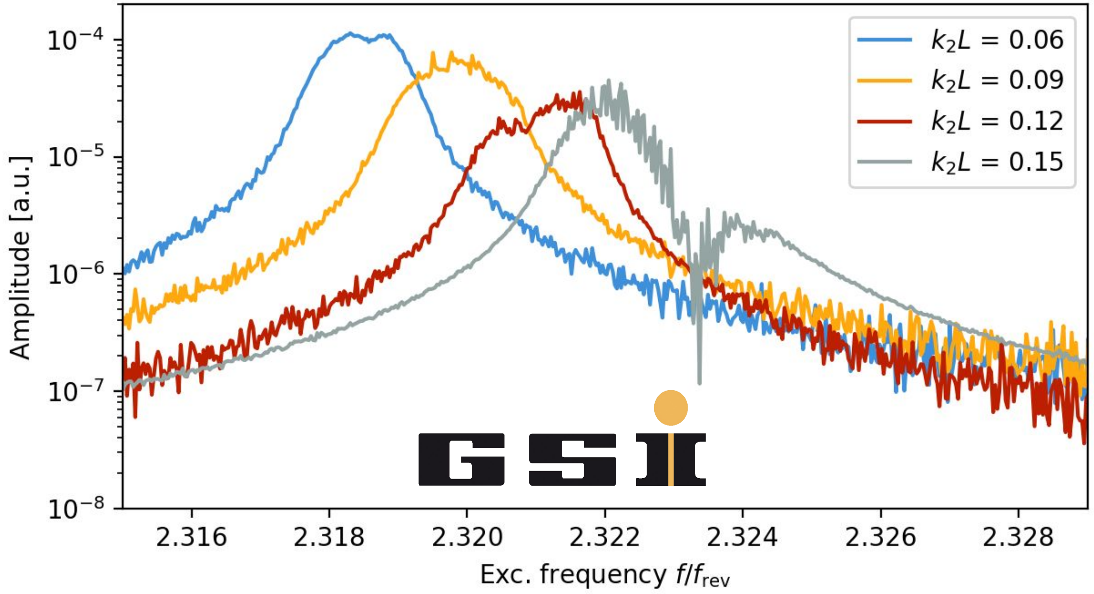
- Splitting is observed

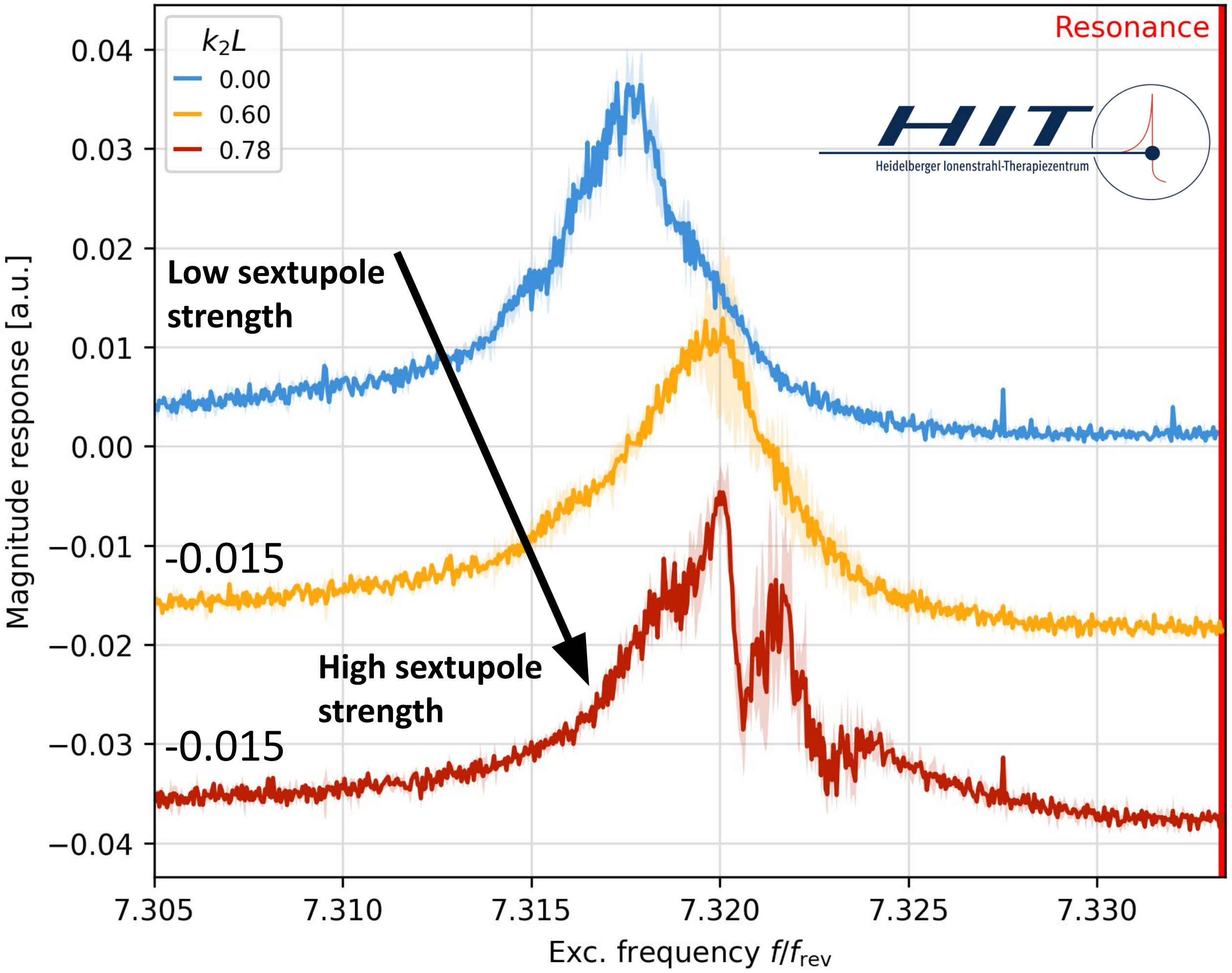


Measurements

Scans over sextupole strength

- Sextupole component distorts the signal
- Splitting is observed
- Qualitative behaviour confirmed with GSI measurements
- Initial conditions play a decisive role





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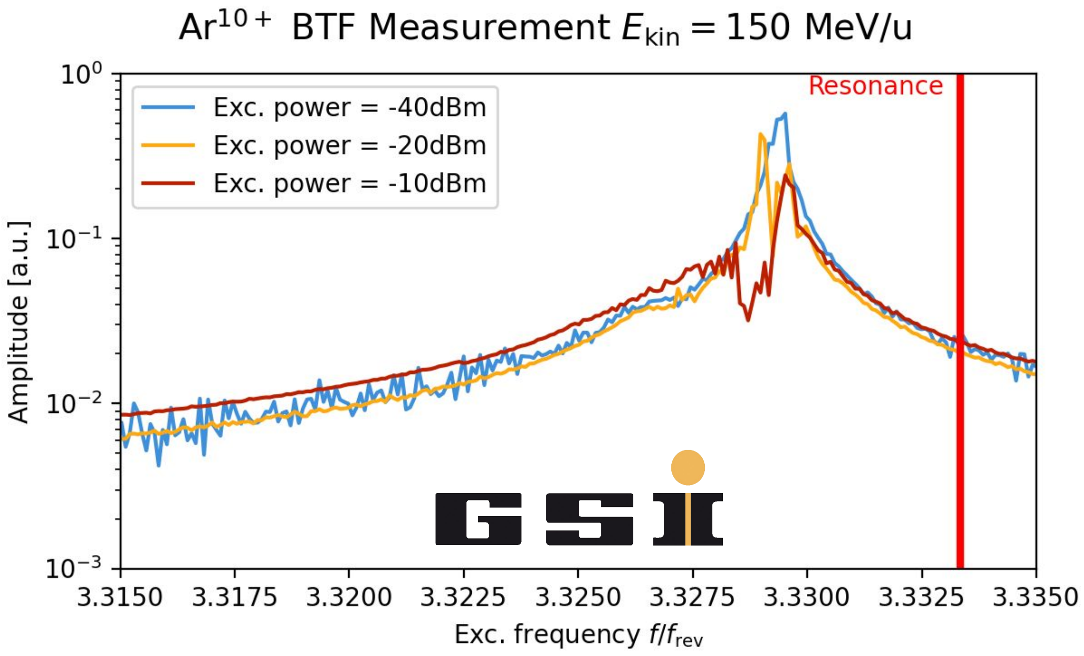

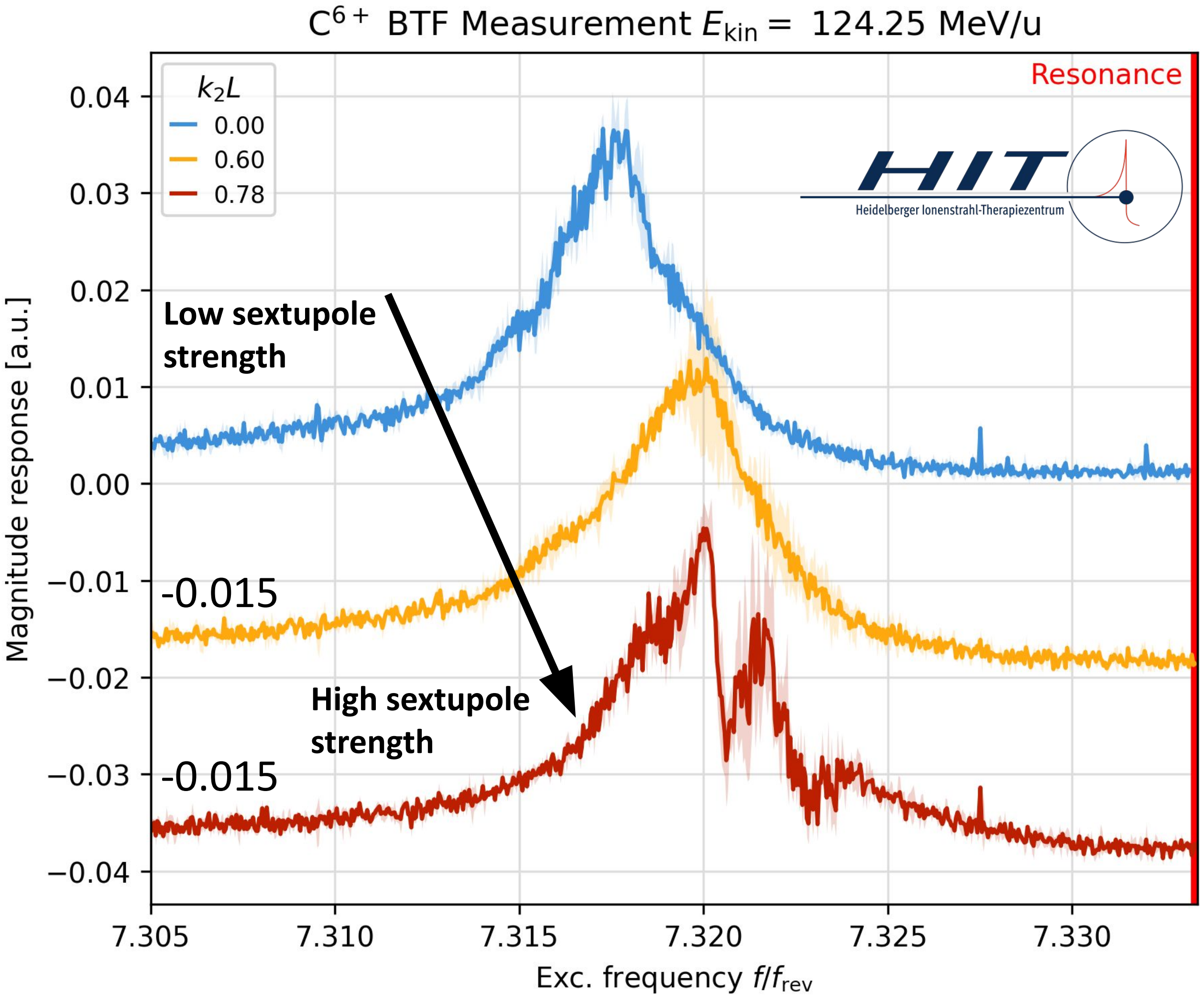

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Simulation

Simulation results for the Heidelberg machine

- MAD-X
- Maptrack
- **X-Suite**

Typical parameters for simulation HPC (XSuite)

Parameter	Simulation	Experiment
N parts.	10^3 to 10^6	10^8
Turns per exc. freq.	512 to 30720	31 000
Excitation steps	201 to 251	701
Exc. freq. range f/f_{rev}	[7.305, 7+1/3]	[7.305, 7+1/3]
Time	$\leq$ 2:30 hours	10 s
Samples per turn	$\leq$ 20	continuous

Simulation

Simulation results for the Heidelberg machine

- For phenomenology studies the parameters have to be scaled-down
- All type of scans can be performed
- Current parameters:

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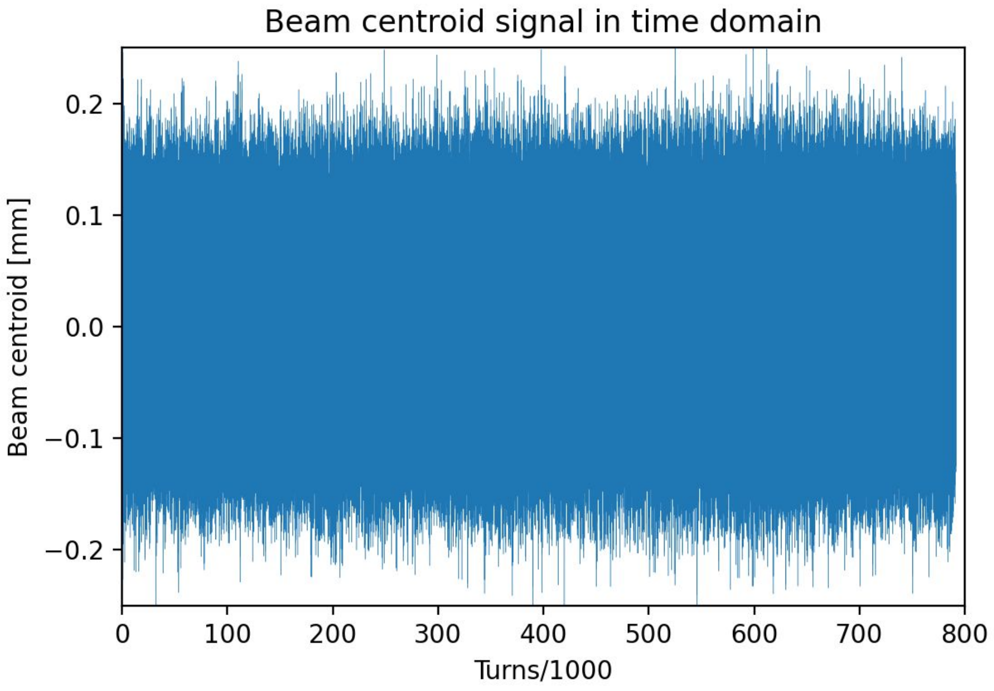
Simulation

Simulation results for the Heidelberg machine

Typical signal from simulation



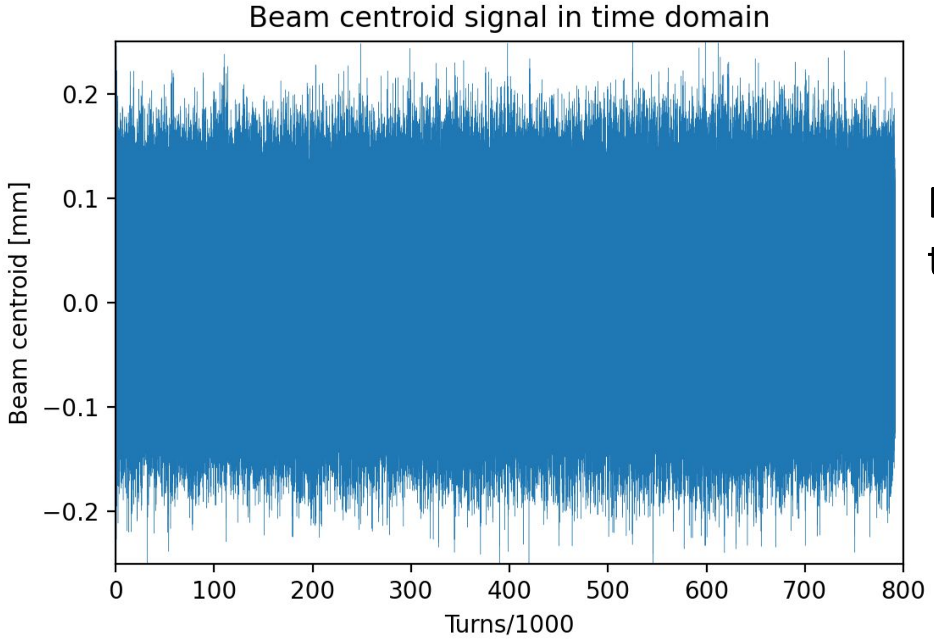
Data analysis:

- Implementation of centroid monitor and beam size in XSuite (P. Niedermayer. https://github.com/xsuite/xtrack/pull/378)
- (Coasting) Beam sliced in longitudinal n-bins (user-defined)
- Arbitrary increase in resolution for the coasting beam case

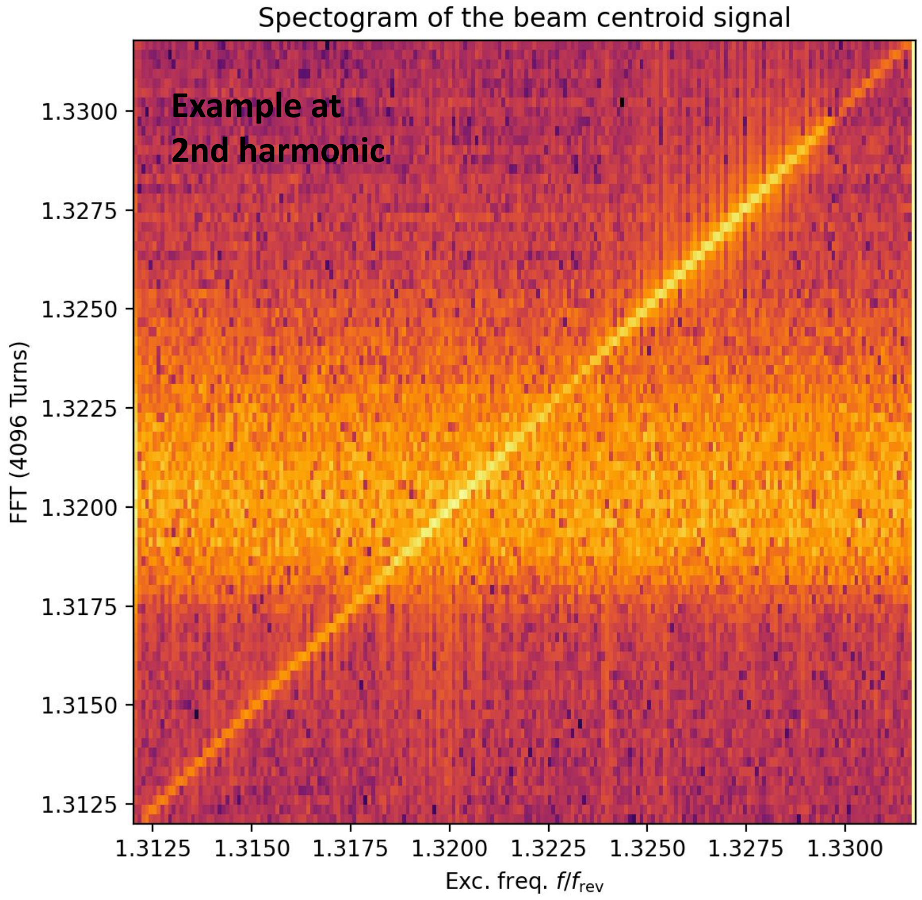
Simulation

Simulation results for the Heidelberg machine

Typical signal from simulation



FFT with 4096 (x 20 samples) turns windows



Simulation

Simulation results for the Heidelberg machine

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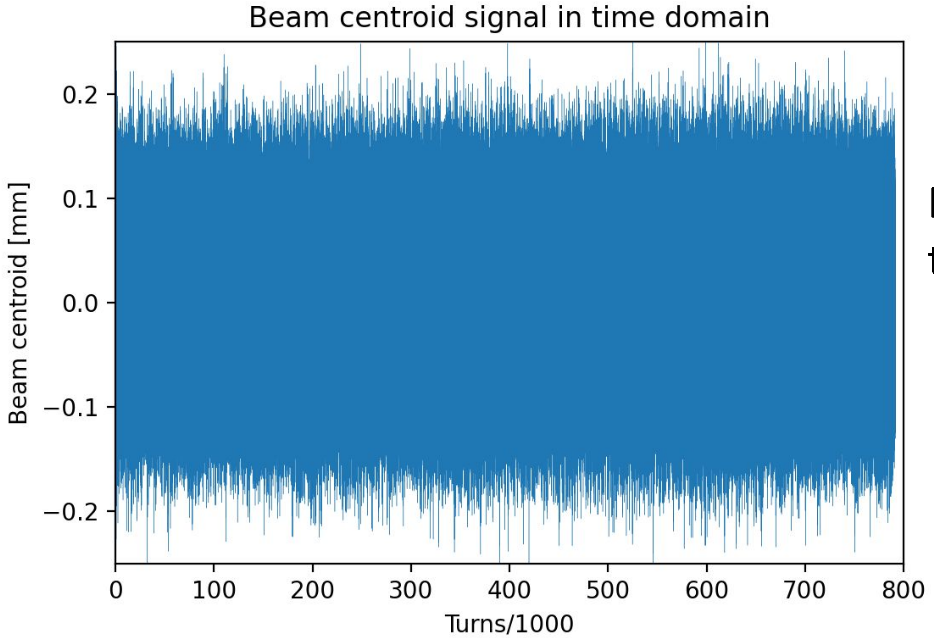
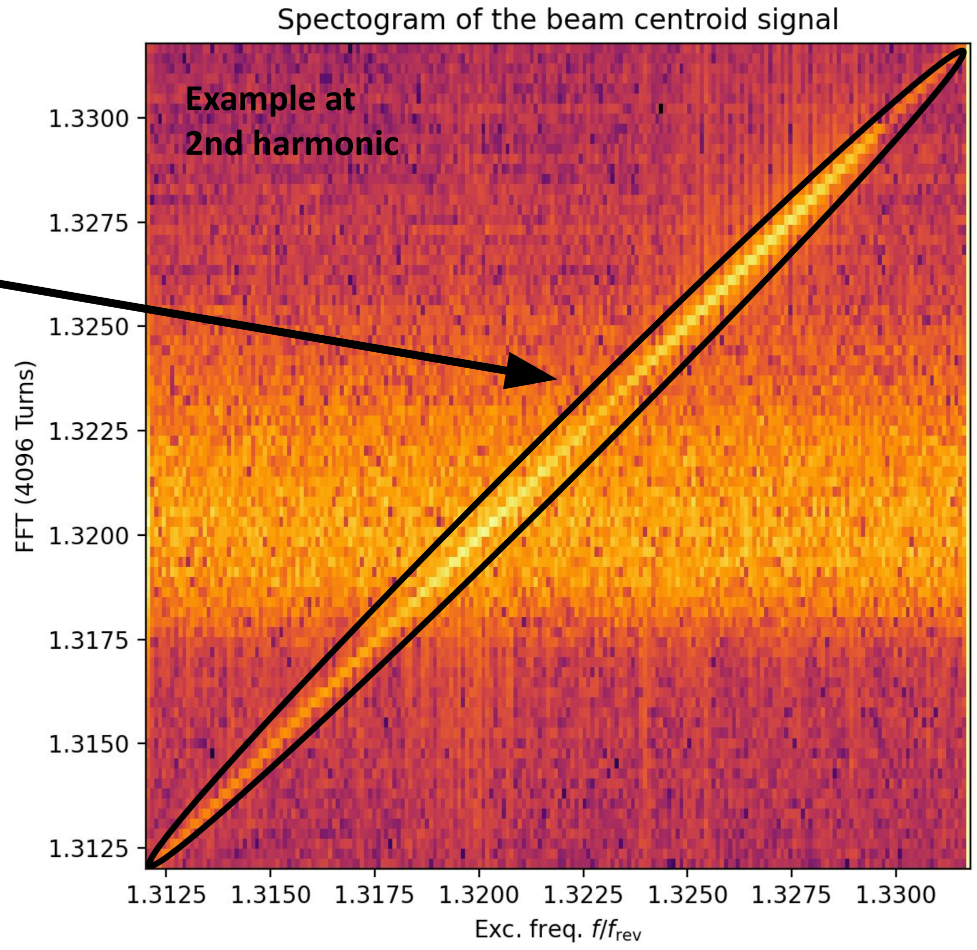


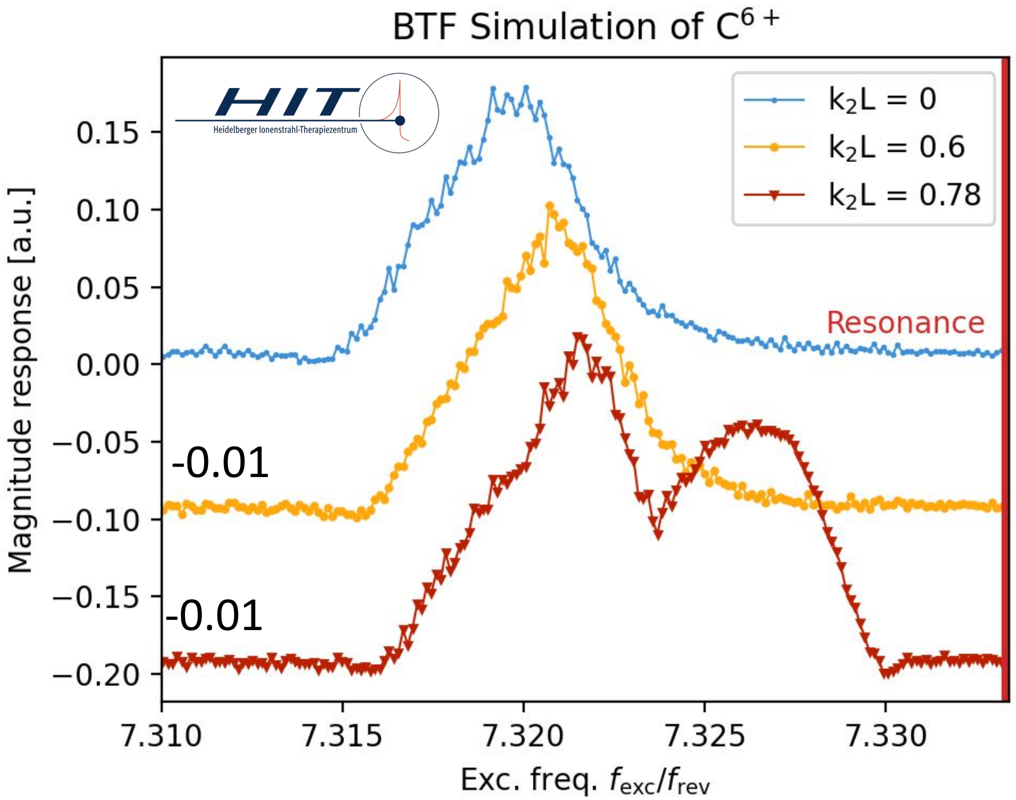
BTF signal

FFT with 4096 (x 20 samples) turns windows



Simulation

Simulation results for the Heidelberg machine



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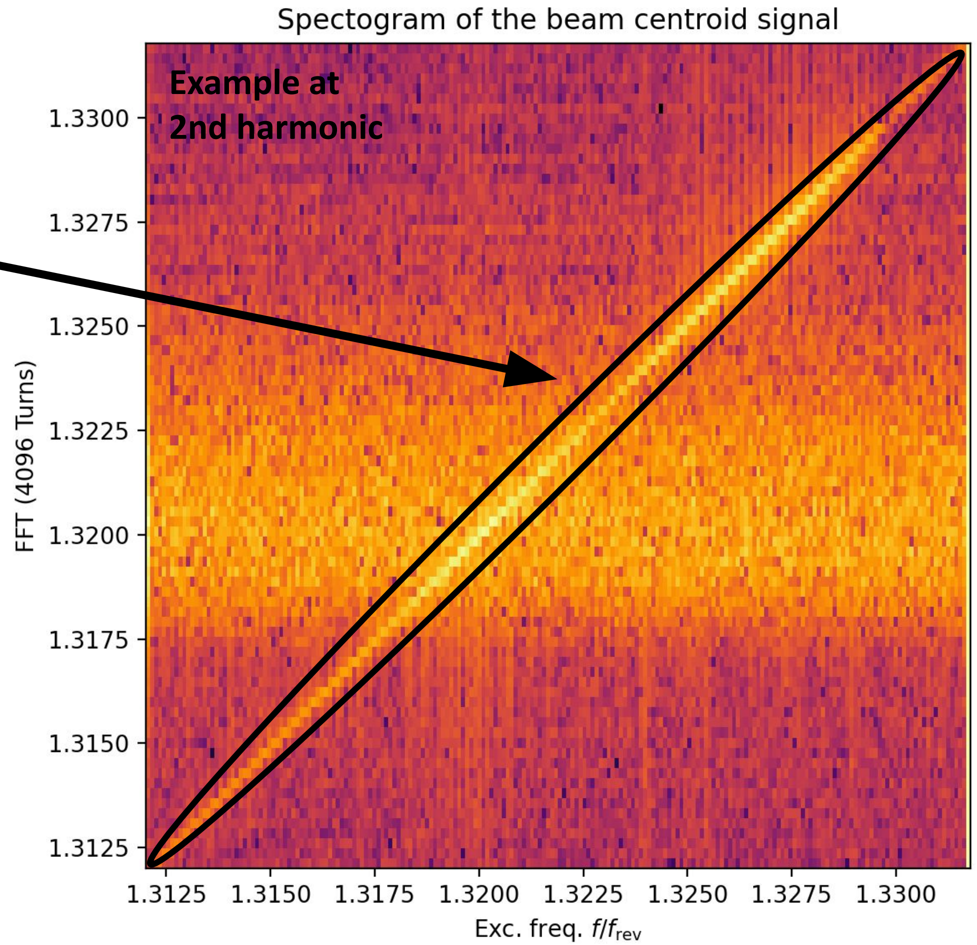

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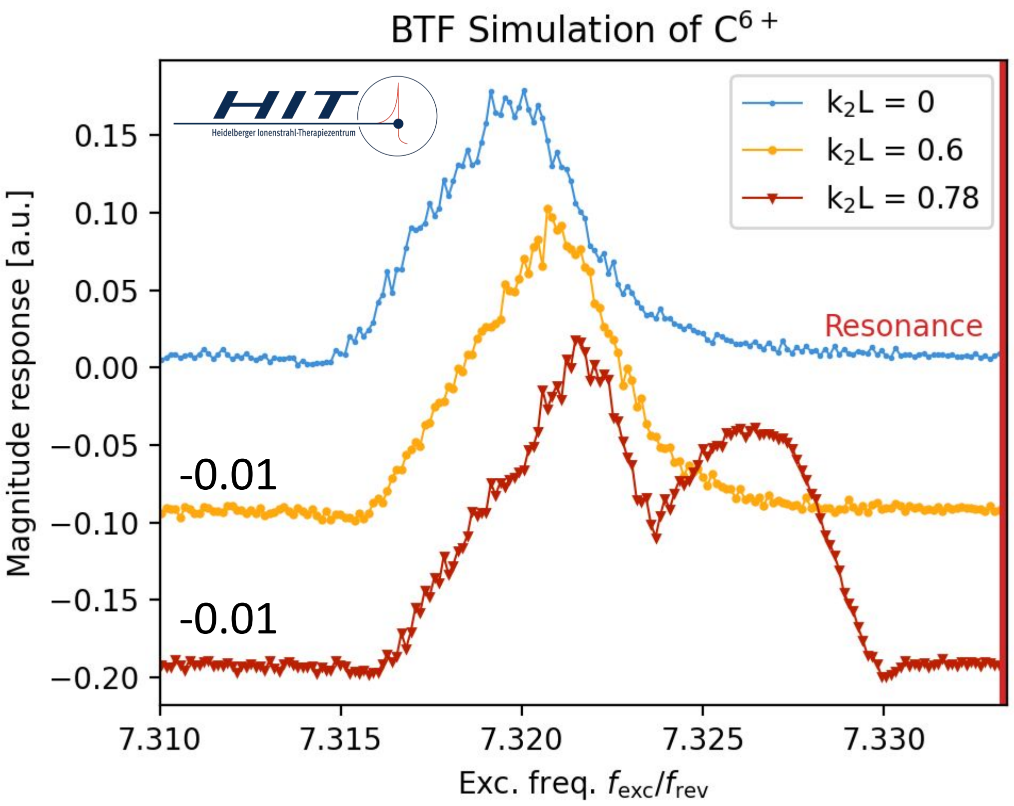
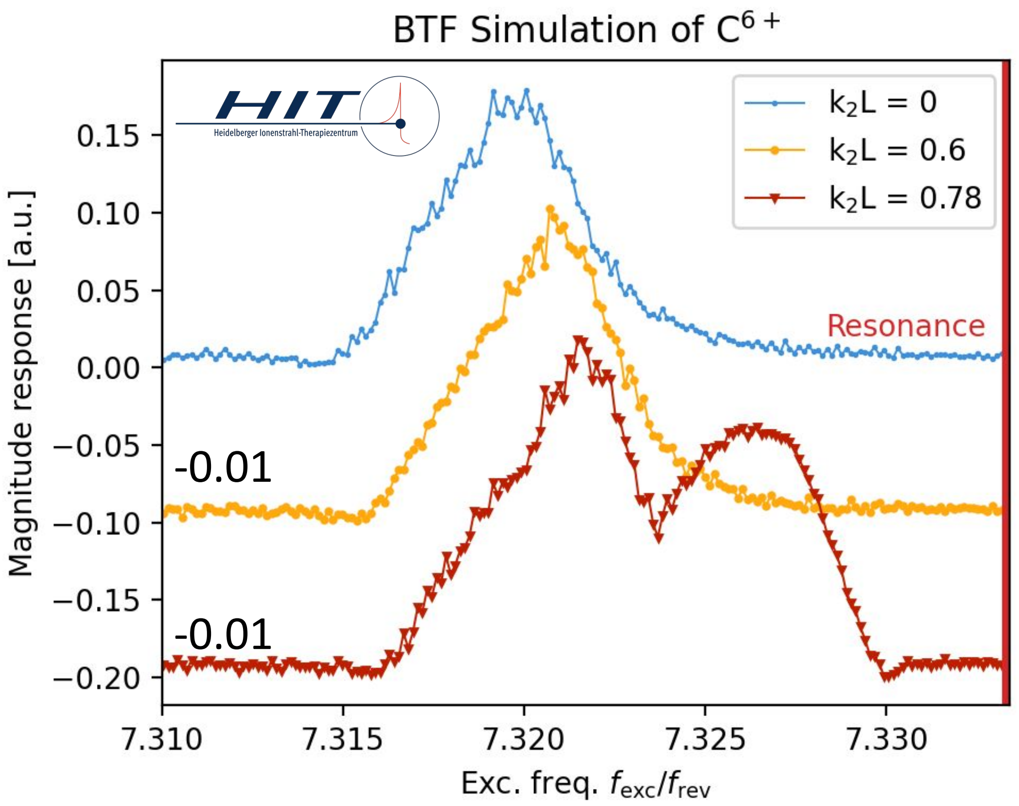
Simulation results for the Heidelberg machine



- Emittance is a free parameter but constrained
- Momentum spread is constrained but decapture influence is unknown (bunched -> coasting beam)
- Qualitative results agree with the simulation
- Excitation history plays an important role
- Studies are ongoing

Simulation

Simulation results for the Heidelberg machine



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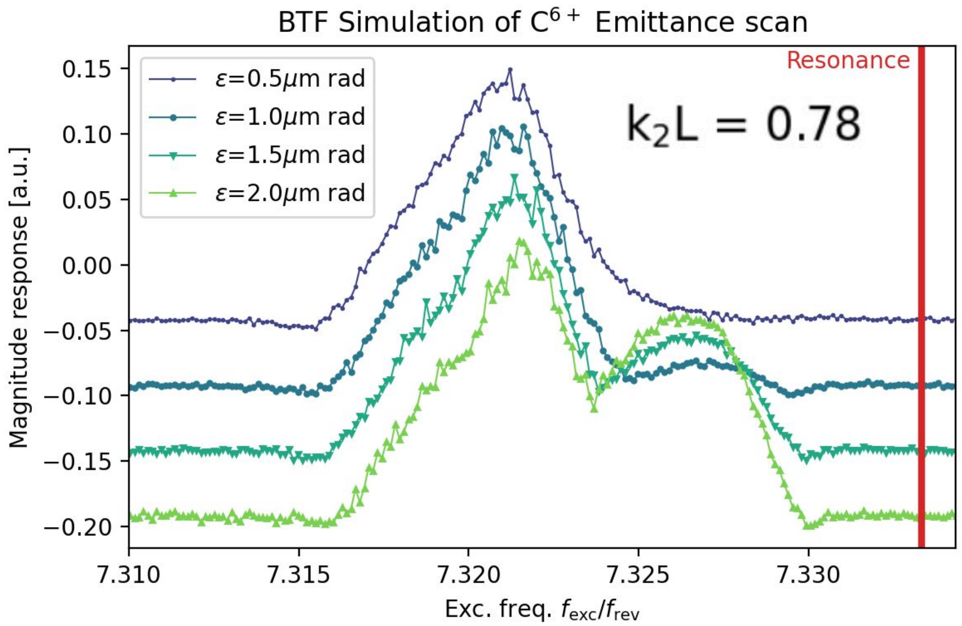

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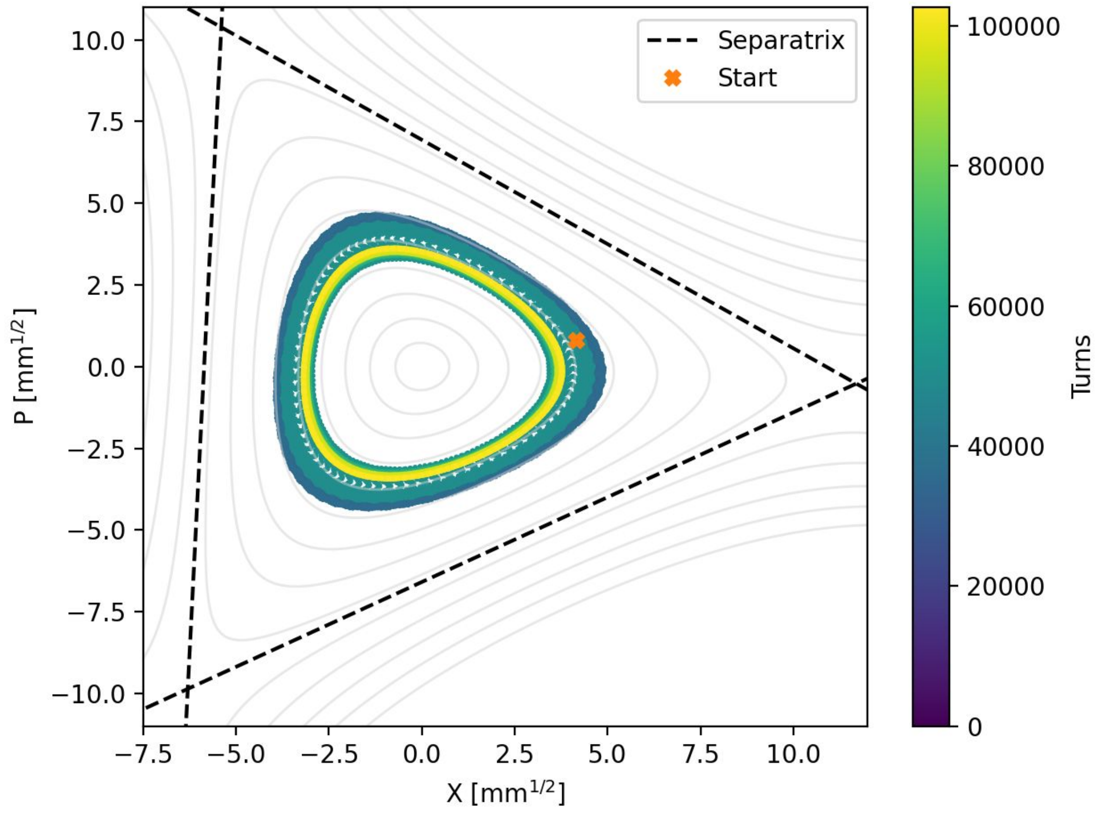
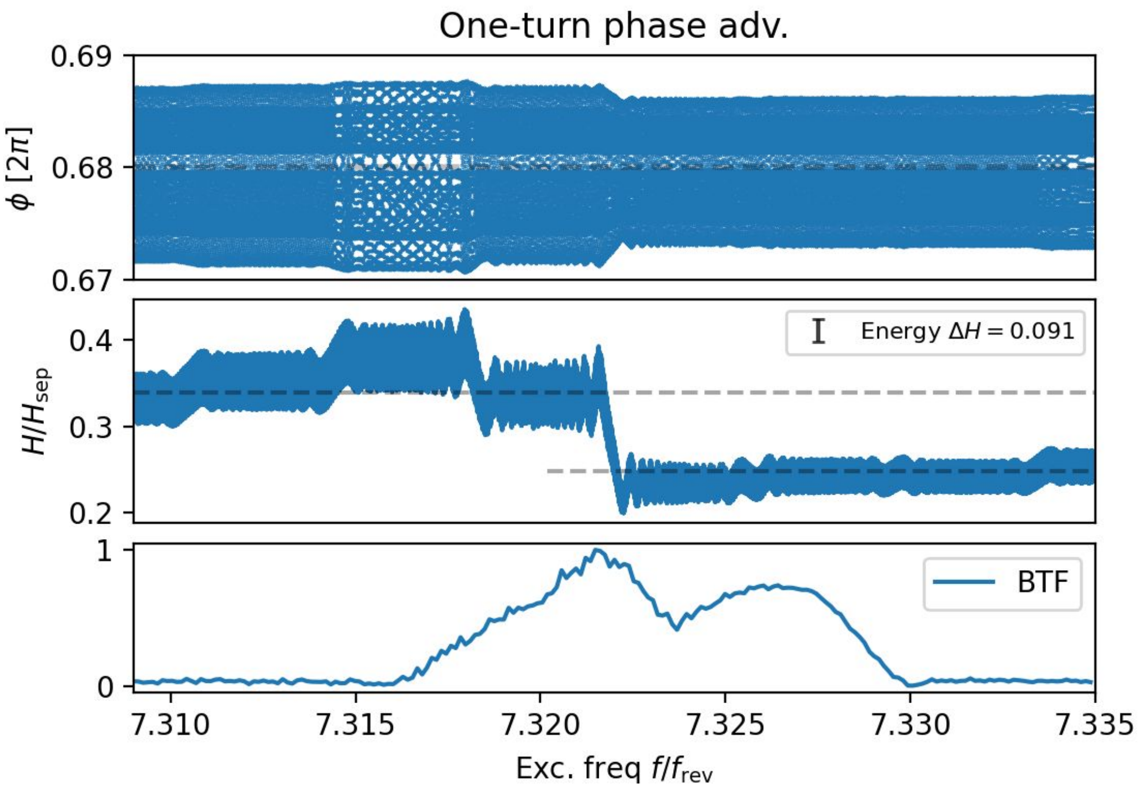
Single particle dynamics

Energy change through excitation

$$q_{x,1} = \frac{3S}{\sqrt{2^5 \pi}} J^{1/2} \sin 3\phi$$

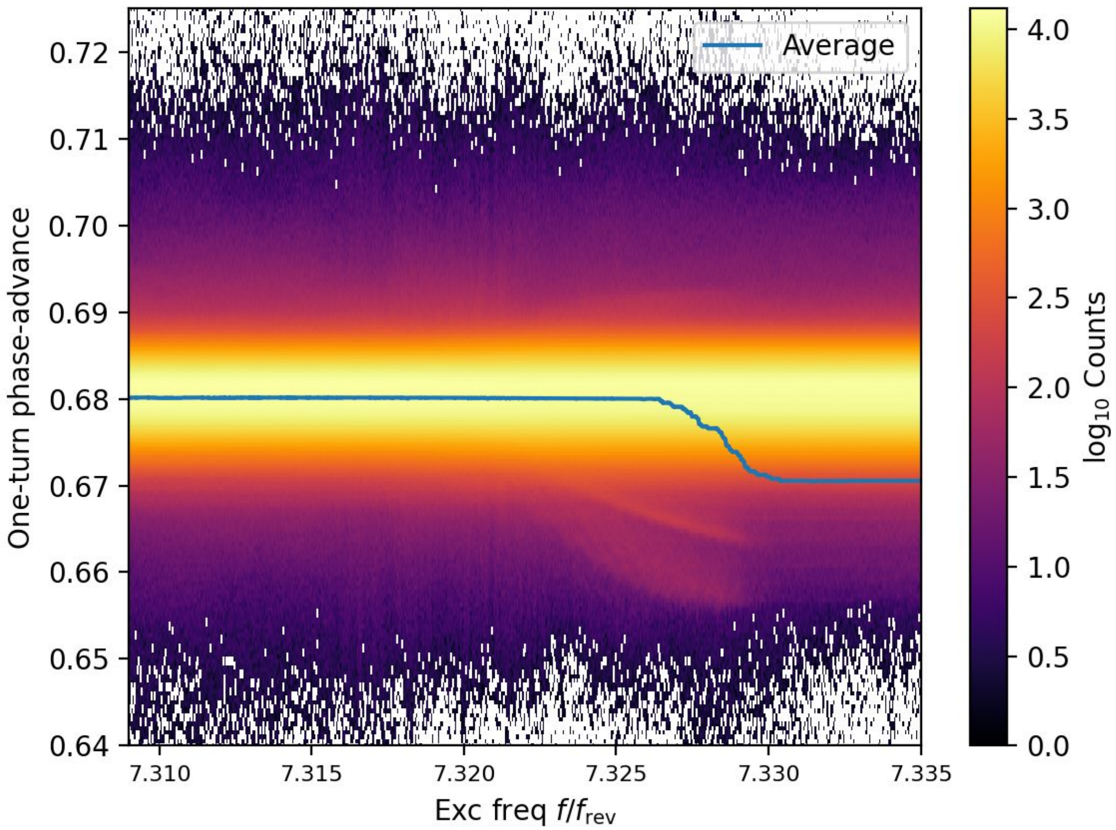




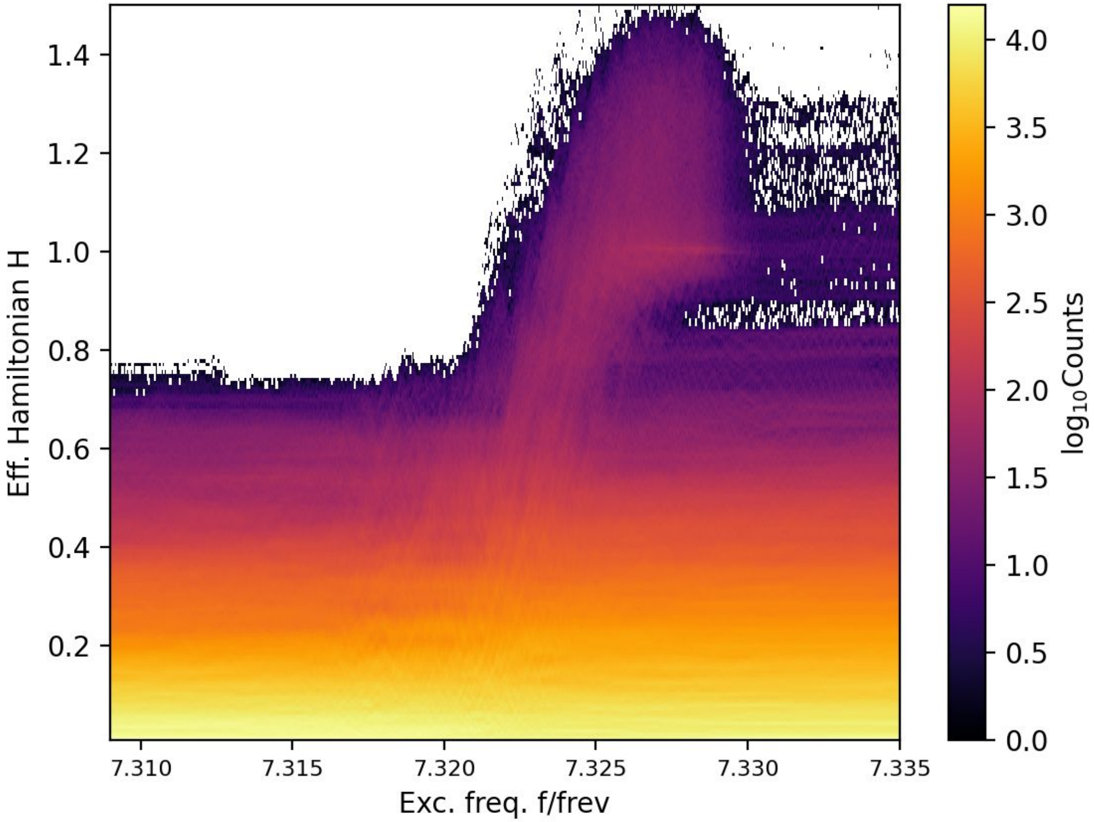
Multi-particle dynamics

Energy and one-turn phase-advance distributions

One-turn phase-adv distribution during excitation process



Eff. Hamiltonian distribution during excitation process



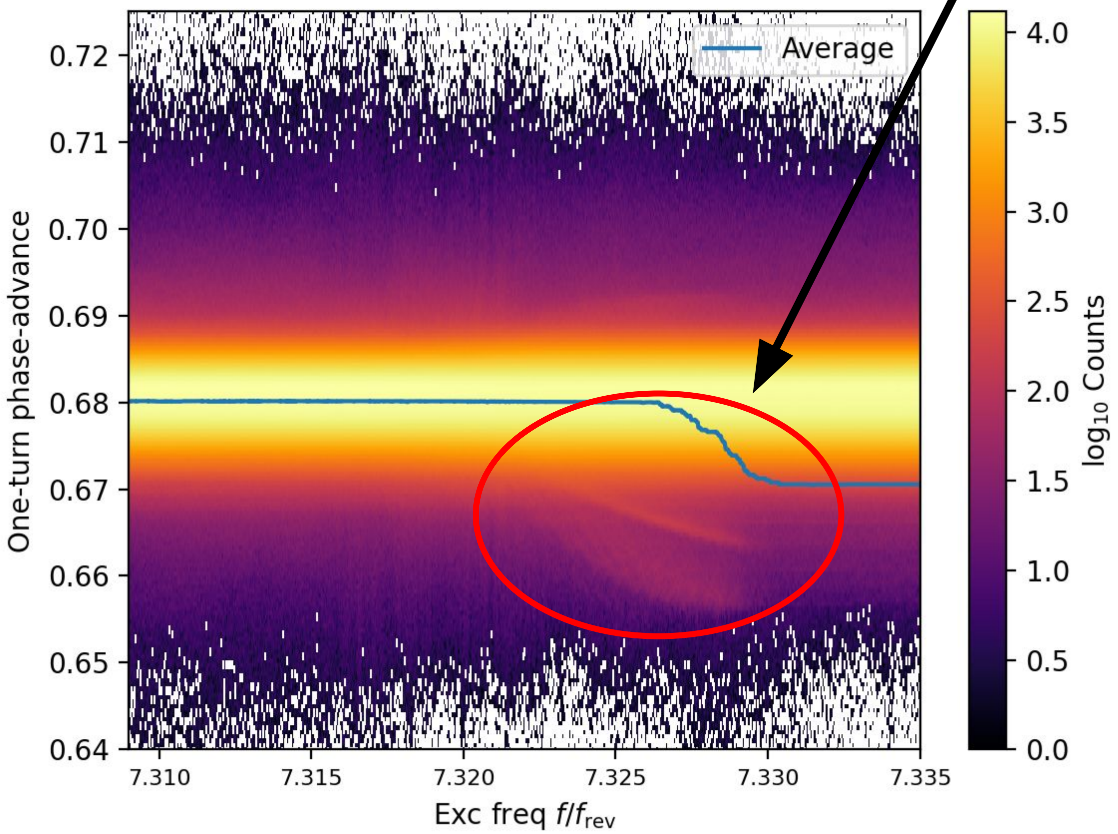
Multi-particle dynamics

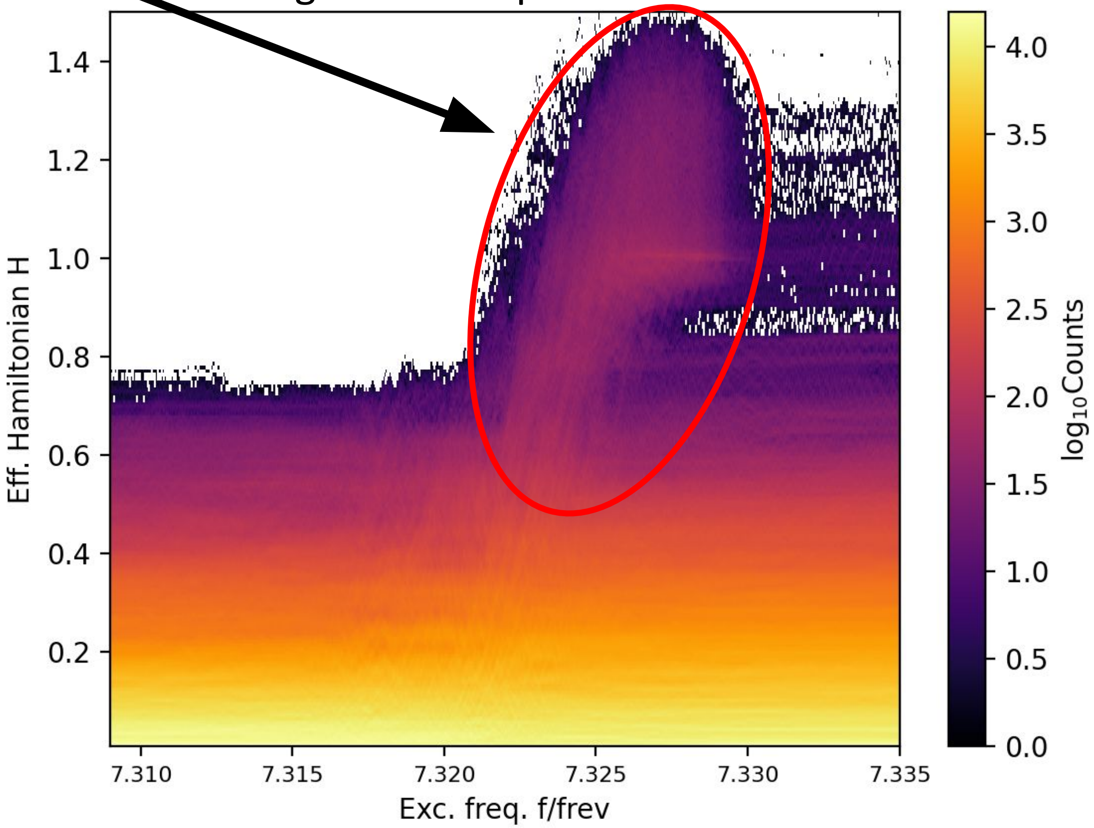
Energy and one-turn phase-advance distributions

One-turn phase-adv distribution during excitation process

Shift and broadening of the distribution

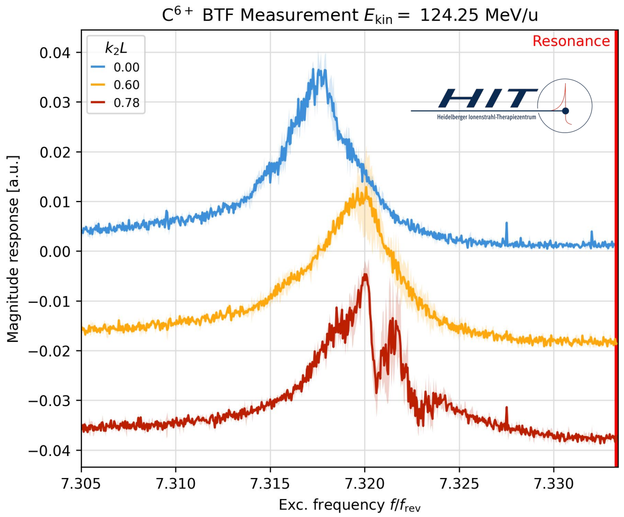
Eff. Hamiltonian distribution during excitation process

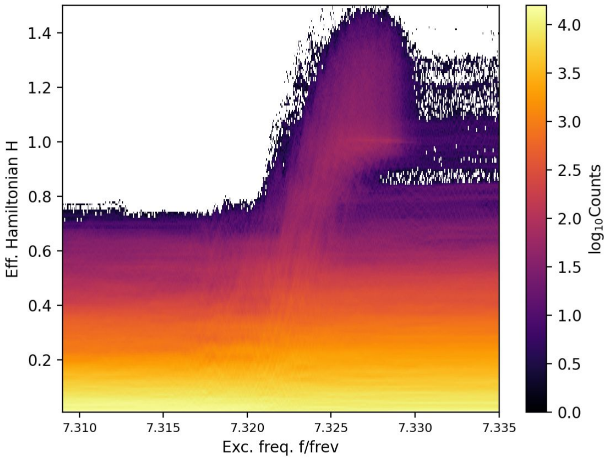


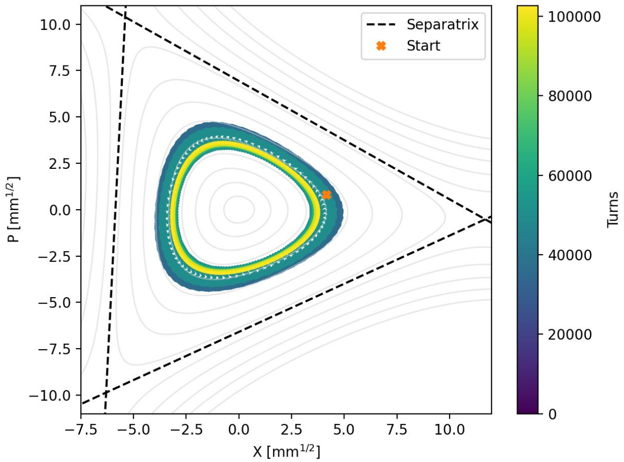


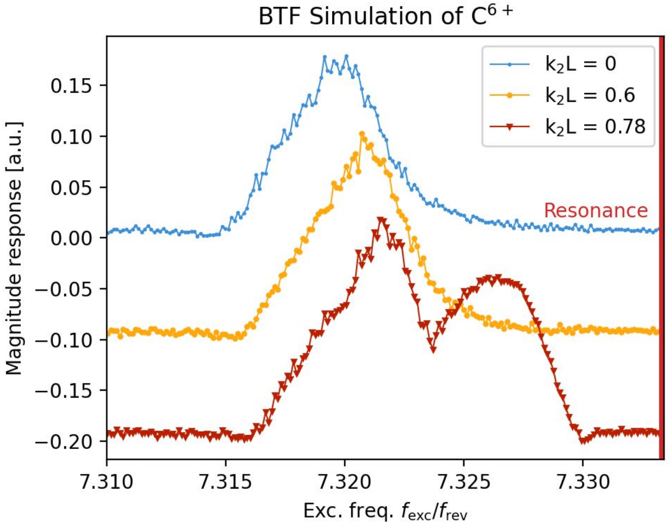
Summary

- The beam dynamics near the third integer resonance are well described by the Kobayashi Hamiltonian
- The measured BTF signal splits asymmetrically towards the resonance into two peaks
- The simulation shows that energy gain/loss induces a phase-amplitude detuning
- Initial conditions are key to understanding the underlying non-linear dynamics









Many thanks to
P. Forck, the HIT and GSI machine operation teams

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