Extraction of LHC Beam Parameters from Schottky Signals

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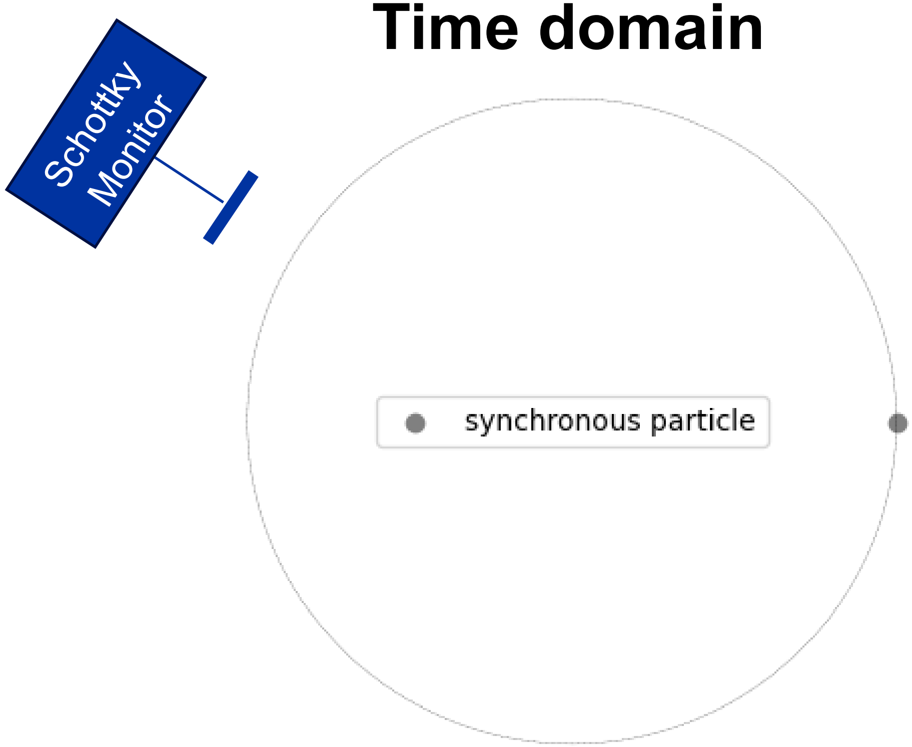
Outline

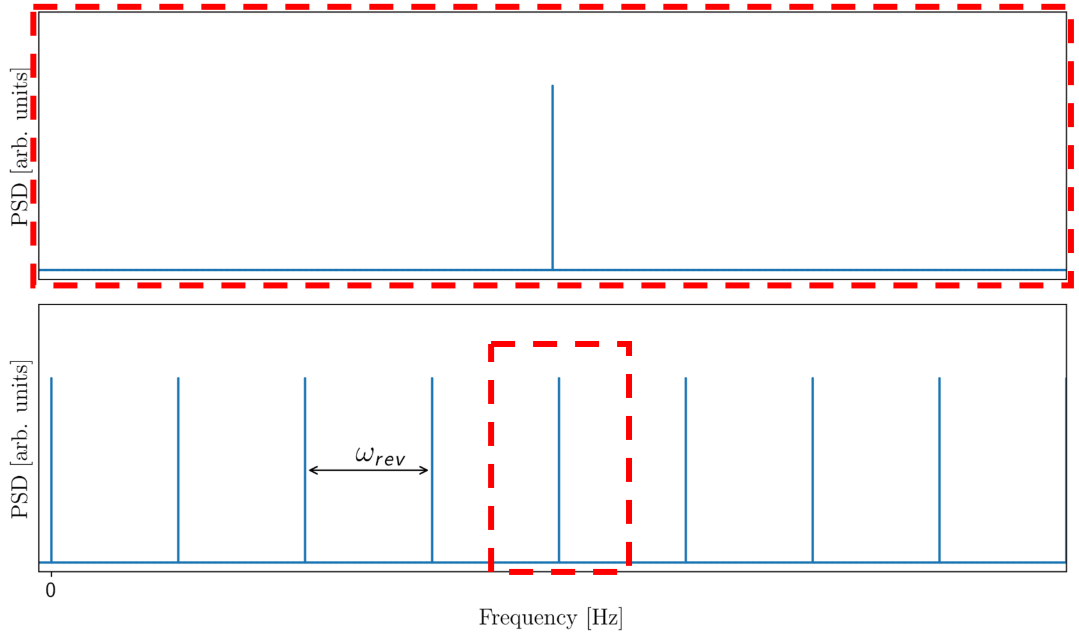
1. **Theory of Schottky signals of bunched beams**
2. **Schottky signals in the LHC**
3. **Extraction of the beam parameters**
    1. Synchrotron frequency
    2. Longitudinal bunch profiles
    3. Betatron tune
    4. Chromaticity
4. **Summary and future plans**

Schottky signals

Time domain



Frequency domain

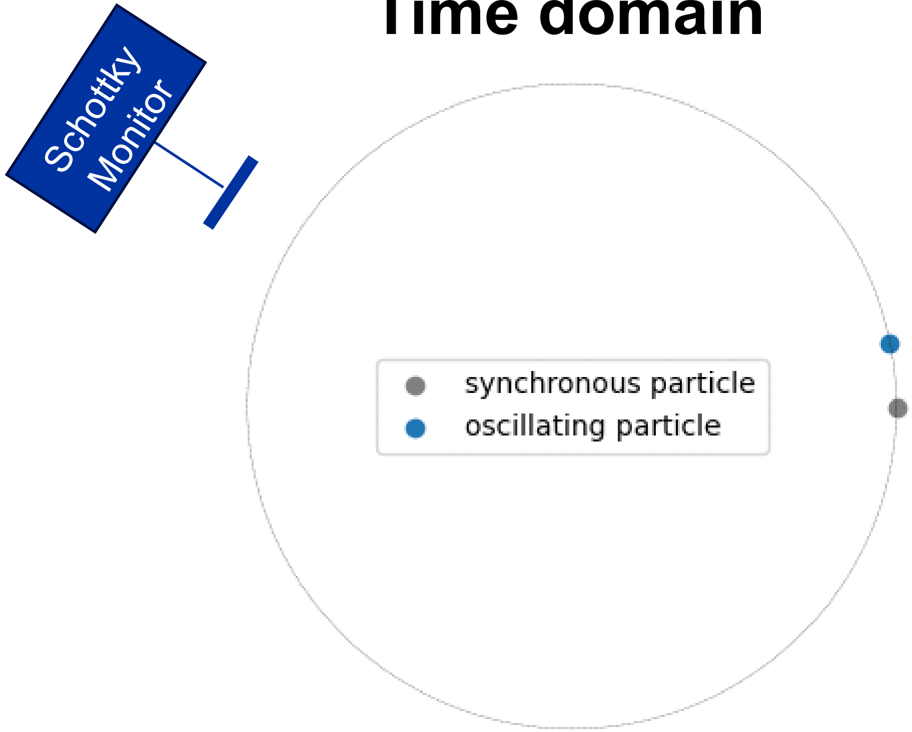


$$I_i(t) = \omega_0 q \sum_{n=-\infty}^{\infty} \delta[\underbrace{\omega_0 t - 2\pi n}_{\text{Synchronous particle arrival time}}] = \frac{\omega_0 q}{2\pi} \sum_{n=-\infty}^{\infty} e^{\overbrace{jn\omega_0 t}^{\text{frequency}}}$$

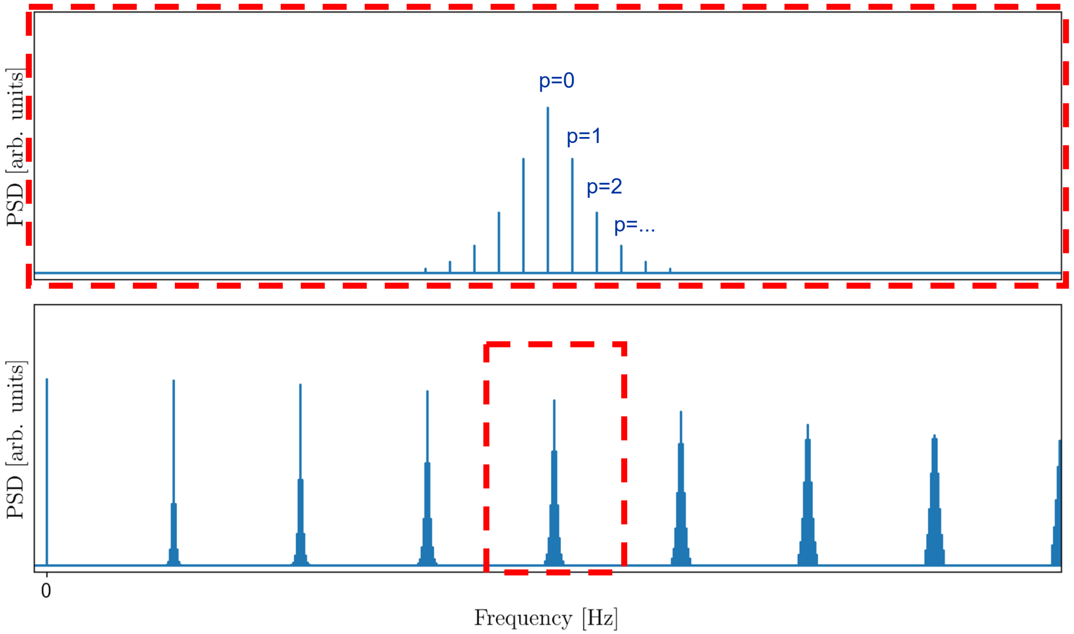
- *Fourier transform of a Dirac Comb is a Dirac Comb*

Schottky signals

Time domain



Frequency domain



$$\omega_0 q \sum_{n=-\infty}^{\infty} \delta[\omega_0(t+\tau_i(t)) - 2\pi n] = \frac{\omega_0 q}{2\pi}\sum_{n=-\infty}^{\infty} e^{jn\omega_0\left(t+\widehat{\tau}_i \sin\left(\Omega_{s_i}t+\varphi_{s_i}\right)\right)} = \frac{\omega_0 q}{2\pi}\sum_{n=-\infty}^{\infty}\sum_{p=-\infty}^{\infty} J_p\left(n\omega_0\widehat{\tau}_i\right) e^{j\left(n\omega_0 t + p\Omega_{s_i}t + p\varphi_{s_i}\right)}$$

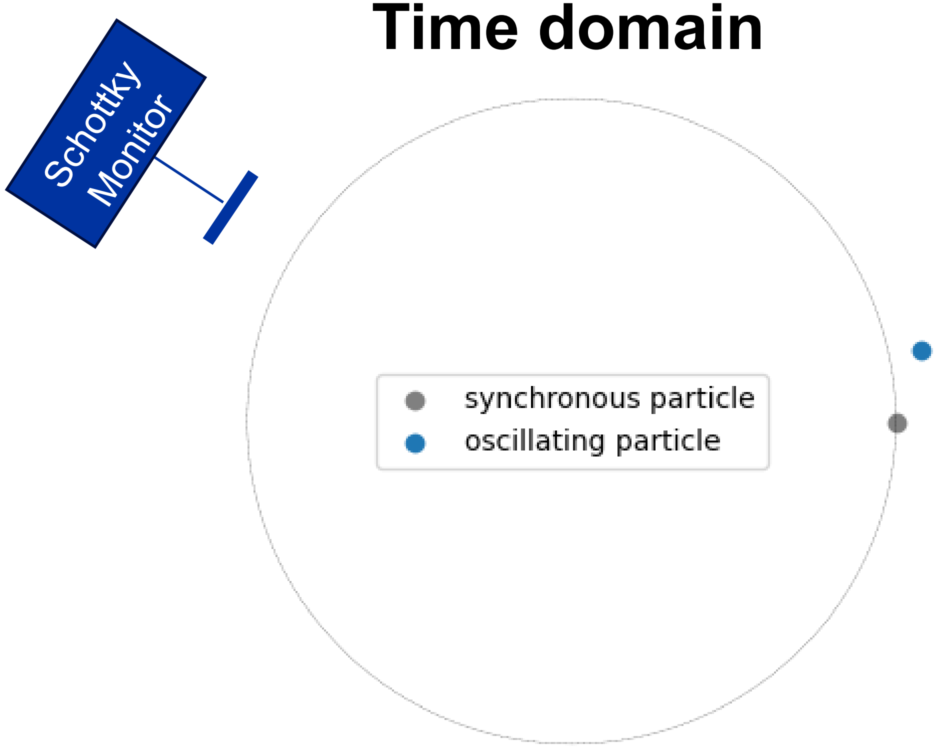
oscillating particle arrival time

amplitude — frequency — phase

Jacobi - Angers expansion,
aka frequency modulation

Schottky signals

Time domain



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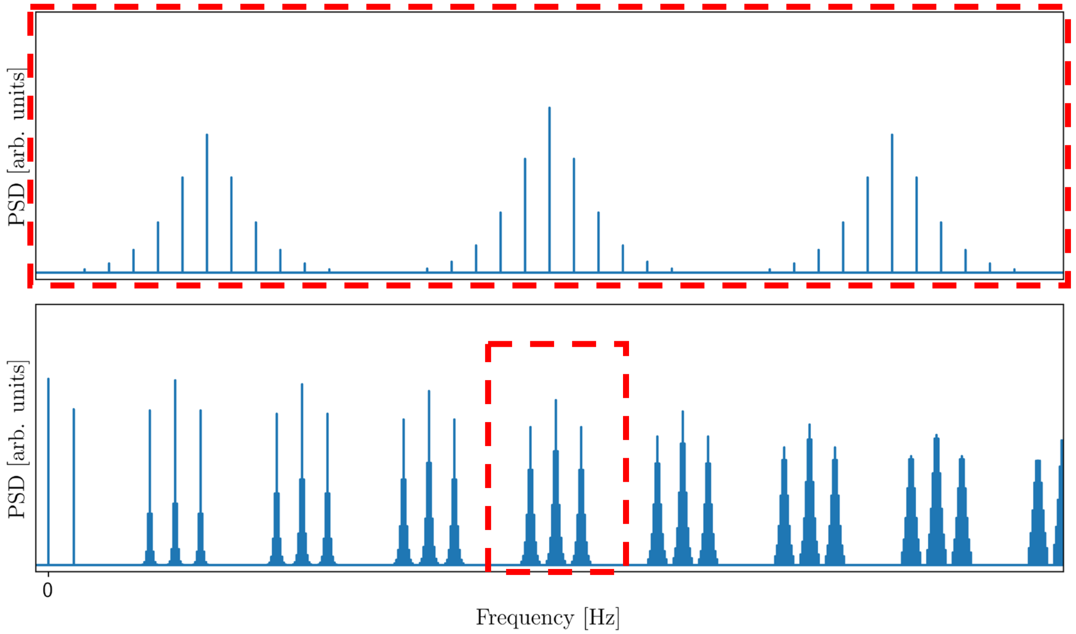


Additional terms described with:

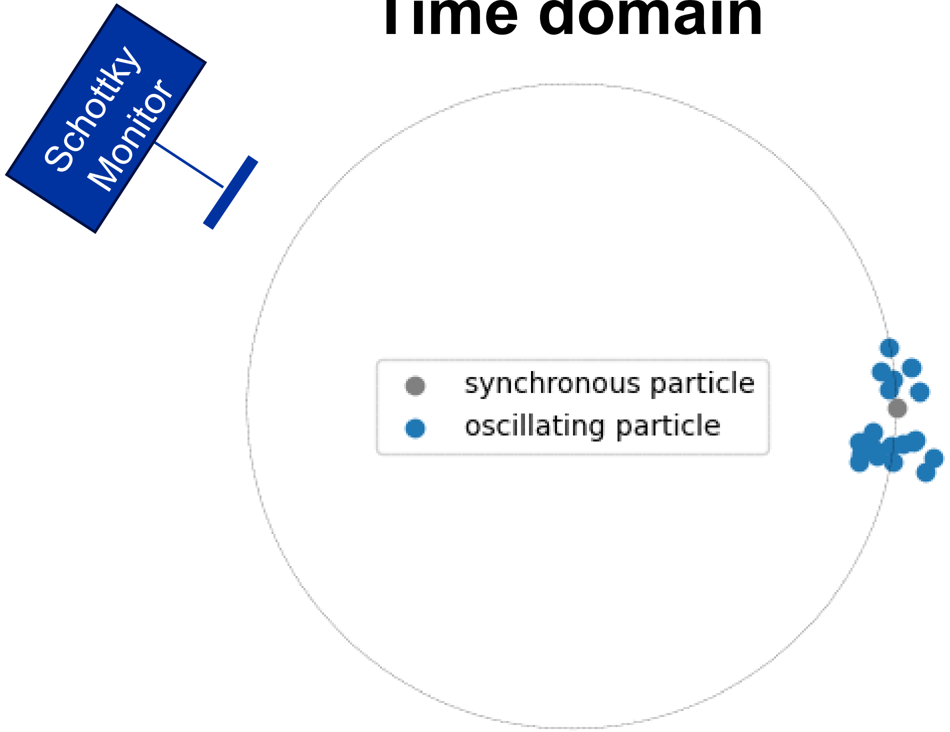
amplitude — frequency — phase

$$\sum_{n,p=-\infty}^{\infty} J_p\left(\chi^{\pm}_{\hat{\tau}_i,n\mp Q_I}\right) e^{j\left(\left[(n\pm Q_F)\omega_0+p\Omega_{s_i}\right]t+\varphi_{\beta_i}+p\varphi_{s_i}\right)}, \qquad \chi^{\pm}_{\hat{\tau}_i,n} = \left(n\hat{\tau}_i \pm \frac{\widehat{Q}_i}{\Omega_{s_i}}\right)\omega_0 = (n\eta \pm Q\xi)\,\frac{\omega_0\hat{p}_i}{\Omega_{s_i}p_0}$$

Identical sidebands for zero chromaticity

Schottky signals

Time domain



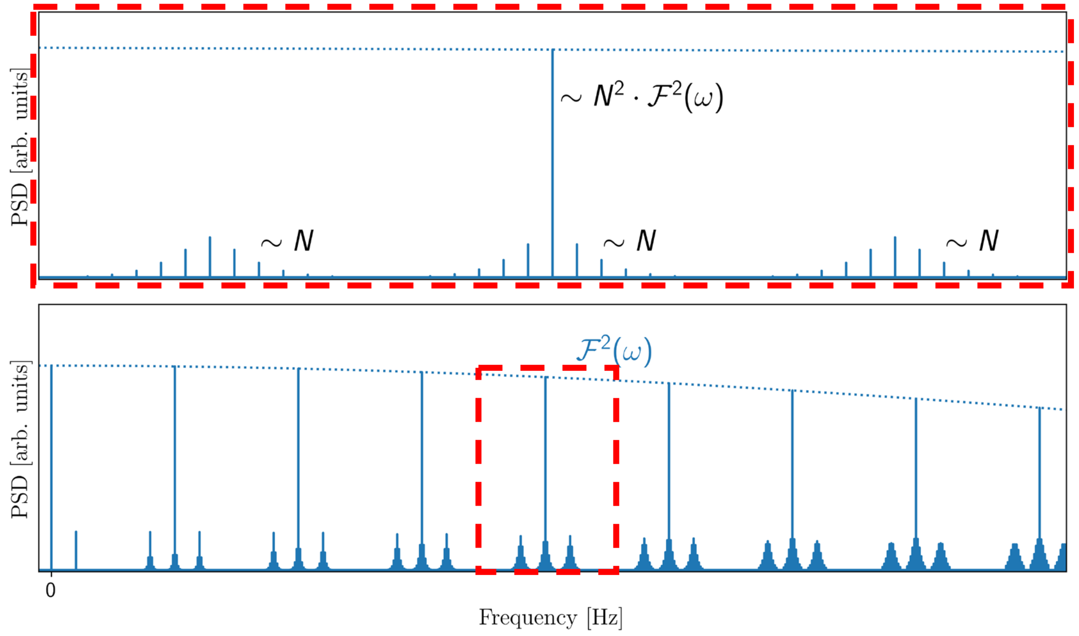
random phase

$$\sum_{n,p=-\infty}^{\infty} J_p\left(\chi^{\pm}_{\widehat{\tau}_i, n\mp Q_I}\right) e^{j\left(\left[(n\pm Q_F)\omega_0 + p\Omega_{s_i}\right]t + \varphi_{\beta_i} + p\varphi_{s_i}\right)}$$

random phase

$$\frac{\omega_0 q}{2\pi} \sum_{n=-\infty}^{\infty} \sum_{p=-\infty}^{\infty} J_p\left(n\omega_0\widehat{\tau}_i\right) e^{j\left(n\omega_0 t + p\Omega_{s_i} t + p\varphi_{s_i}\right)}$$

Frequency domain

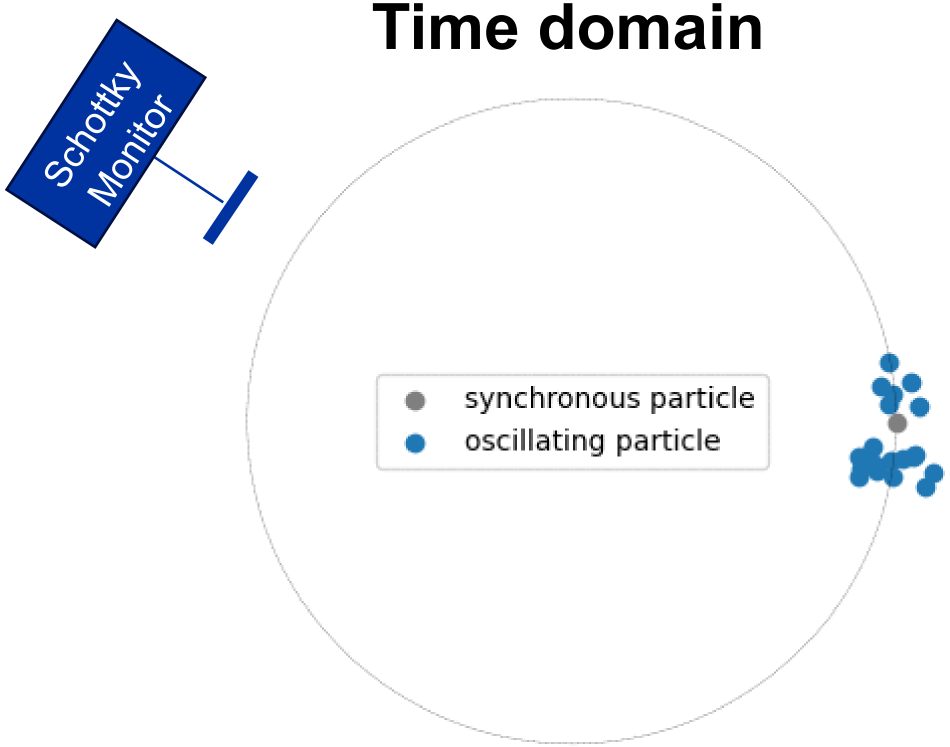


Power of central satelites proportional to the longitudinal form factor:

$$\mathcal{F}(\omega) = C_{norm} \int_{-\infty}^{\infty} e^{j\omega t} \mathcal{B}(t) dt$$

Schottky signals

Time domain



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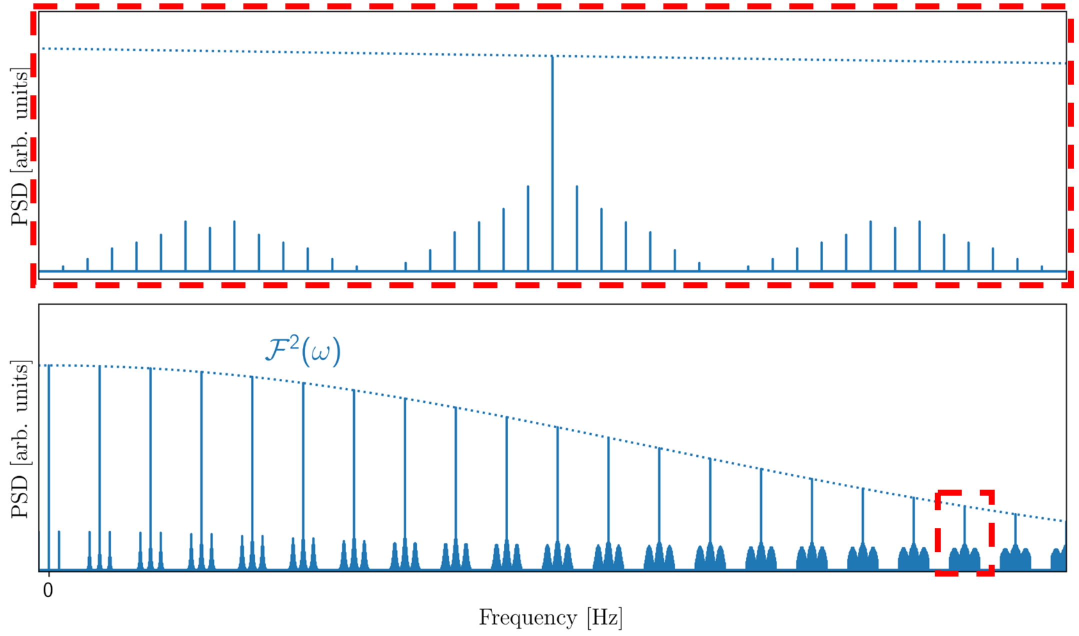


Power of central sattelites proportional to the longitudinal form factor:

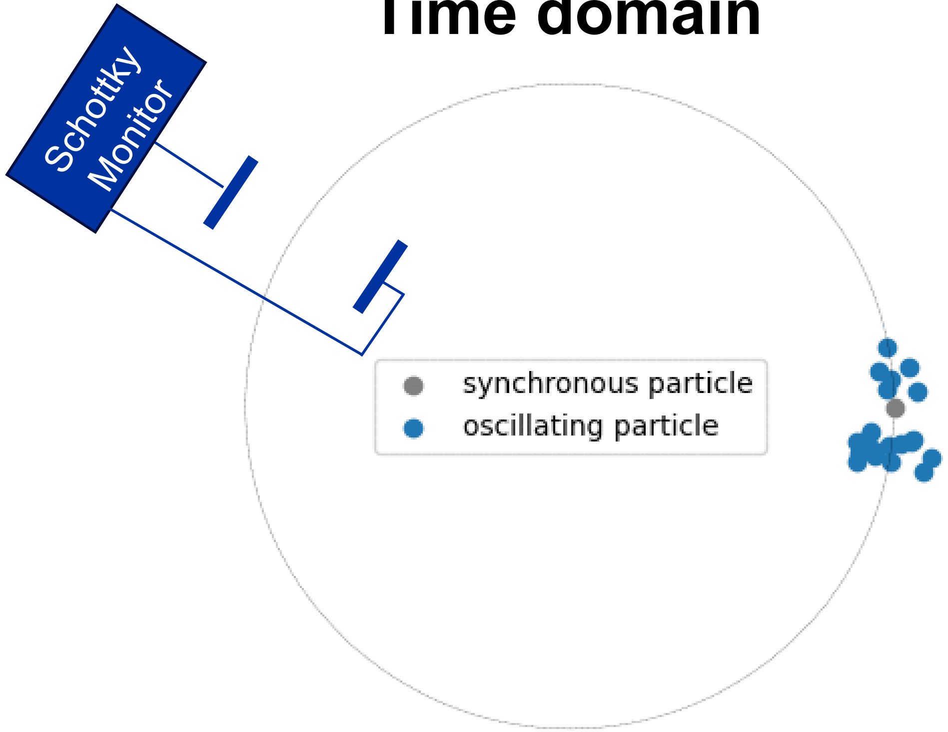
$$\mathcal{F}(\omega) = C_{norm} \int_{-\infty}^{\infty} e^{j\omega t} \mathcal{B}(t) dt$$

which vanishes at high frequencies:

$$\mathcal{F}(\omega) \xrightarrow[\omega \to \infty]{} 0$$

Schottky signals

Time domain



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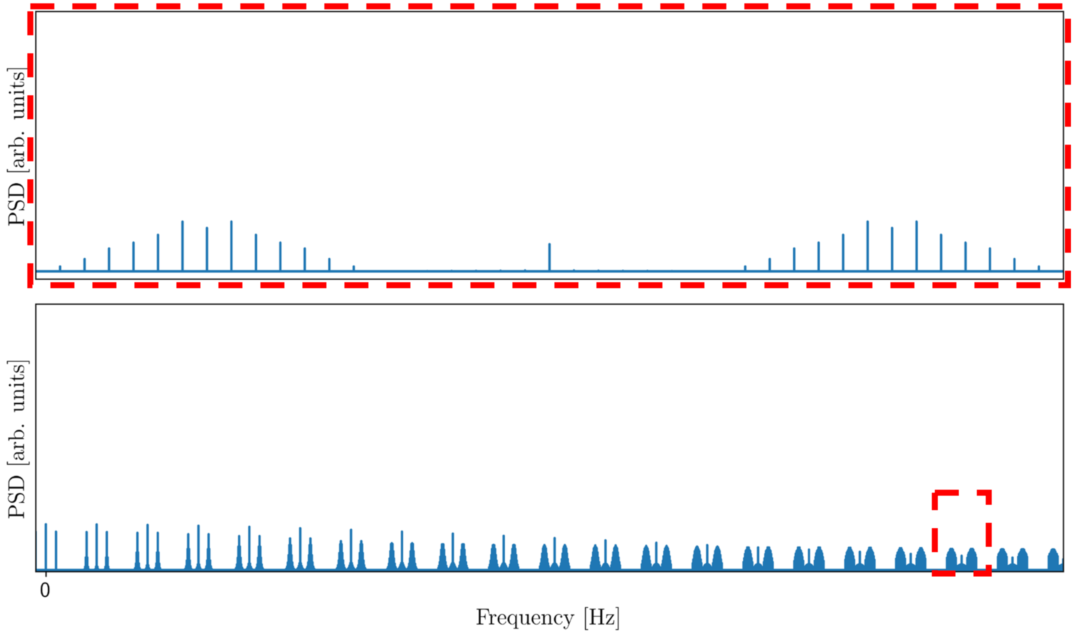
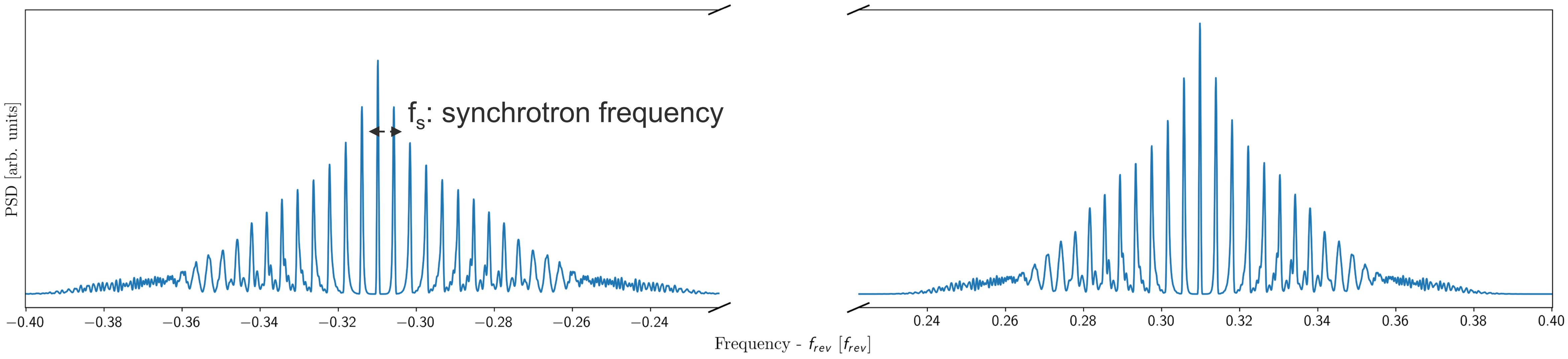


Transverse pickup is sensitive only to transverse Schottky component.
Assuming that common mode rejection is perfect...

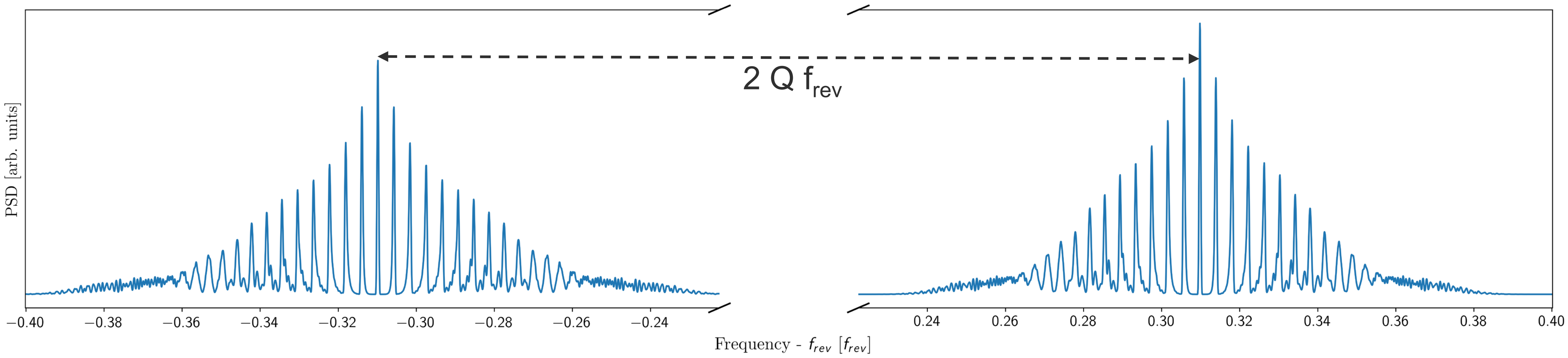
Beam parameters in Schottky signals



Synchroton frequency

Distance between two consecutive Bessel lines

Beam parameters in Schottky signals



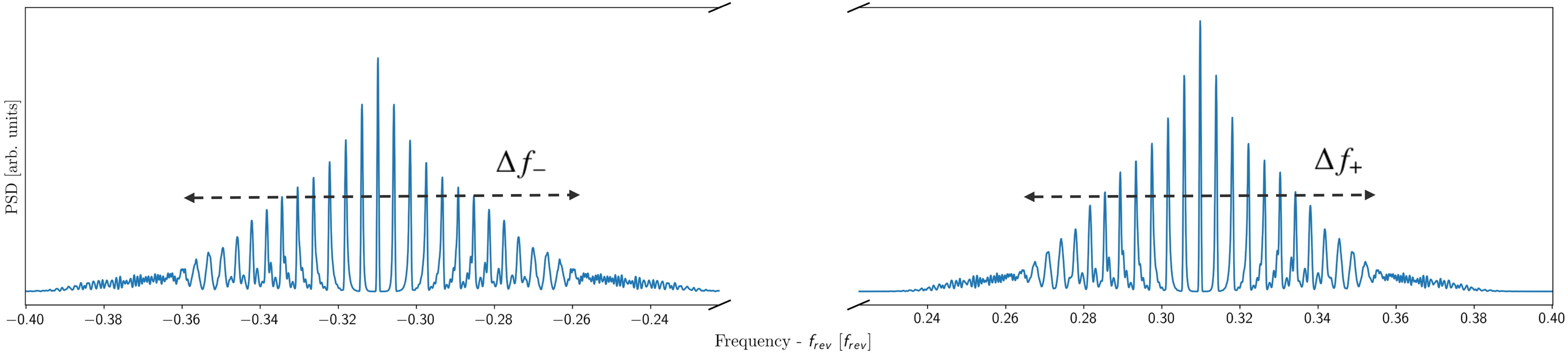
Betatron tune

Distance between the centers of two sidebands.

Also valid for non-zero chromaticity.

Correction may be required for non-negligible octupole current.

Beam parameters in Schottky signals



Chromaticity

$$Q\xi = -\eta \left(n \frac{\Delta f_- - \Delta f_+}{\Delta f_- + \Delta f_+} - Q_I \right)$$

$\Delta f_\pm$: RMS width of upper/lower sideband

In certain conditions +/- signs flip, see
K. Lasocha and D. Alves, Phys. Rev. Accel. Beams 25, 062801

Schottky signals: assumptions

Synchrotron and betatron motion

Synchrotron motion – harmonic, with amplitude dependent frequency:

$$\tau_i(t) = \widehat{\tau}_i \sin\left(\Omega_{s_i} t + \varphi_{s_i}\right)$$

$$\Omega_{s_i} = \frac{\pi}{2\mathcal{K}\left[\sin\left(\frac{h\omega_0\widehat{\tau}_i}{2}\right)\right]}\Omega_{s_0}$$

Betatron motion – harmonic, frequency changing linearly with momentum:

$$x_i(t) = \widehat{x}_i \cos\left[Q\omega_0 t + \frac{\widehat{Q_i}\omega_0}{\Omega_{s_i}} \sin\left(\Omega_{s_i} t + \varphi_{s_i}\right) + \varphi_{\beta_i}\right]$$

$$\widehat{Q_i} = Q\xi\frac{\widehat{p}_i}{p_0}$$

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Theory extensions

Arbitrary wave-form stationary voltage:
- V. Balbecov et al., EPAC’04, p. 791 (2004)

Effect of space charge:
- O. Boine-Frankenheim and V. Kornilov PRAB 12, 114201

Transverse and longitudinal impedance (early):
- C. Lannoy et al., HB'23 THBP47 (today evening!)
- C. Lannoy et al., WEP034, IBIC'23

Much more on unbunched beam...

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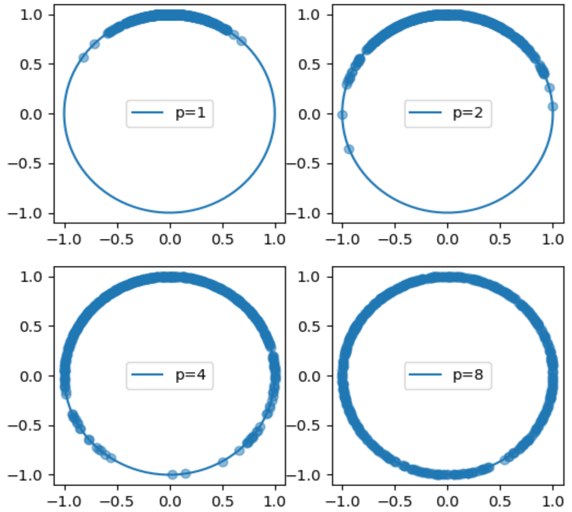
Uniform distribution of phases; no "coherent" components

Uniform distribution of $p\varphi_{s_i}$ and φ_{β_i} implies PSD proportional to the number of particles N.

Otherwise the power can be proportional to N^2.

Coherence is most pronounced in central satellites, at higher order p it smears out.



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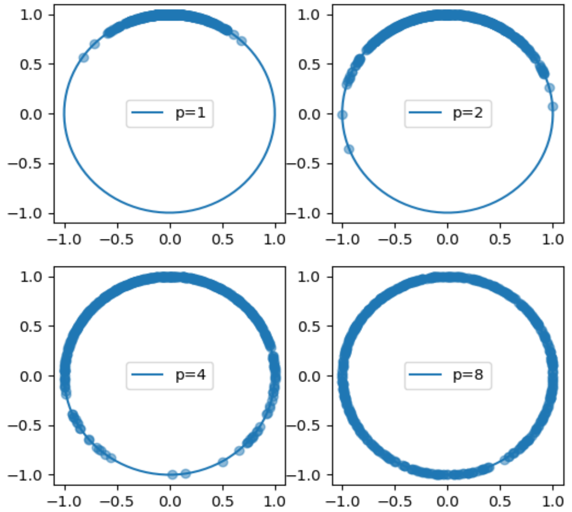
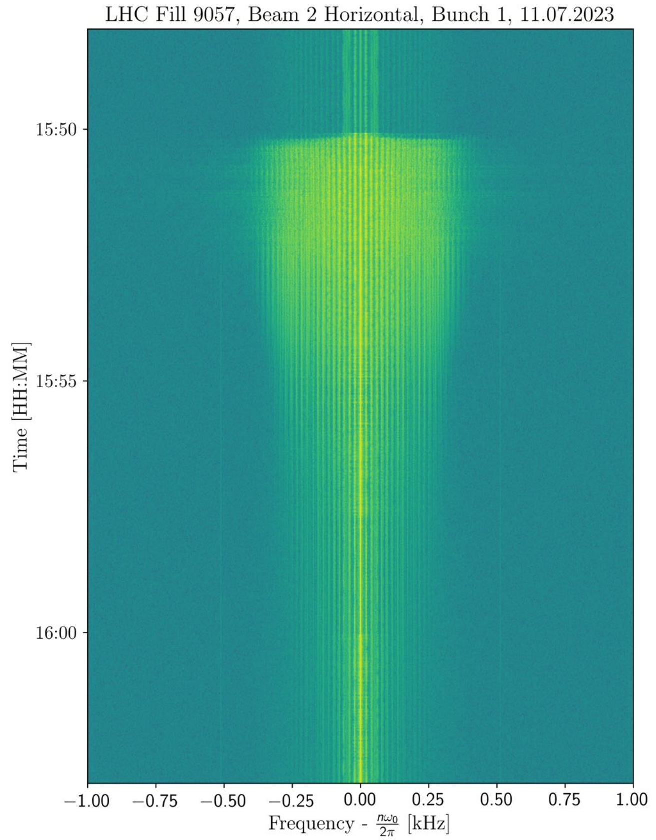


Coherence example: longitudinal blowup



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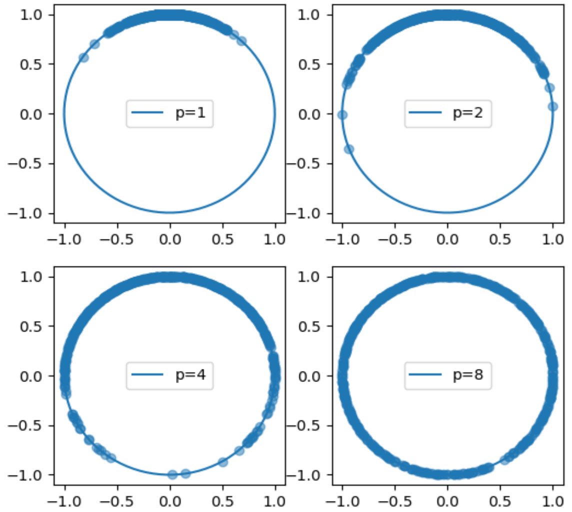
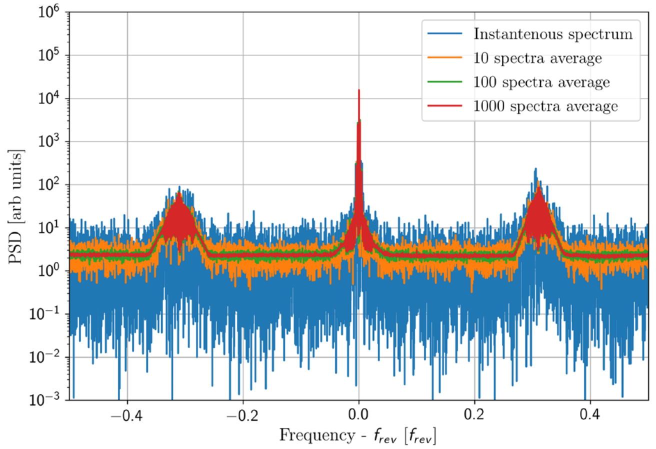


Sufficiently long time averaging

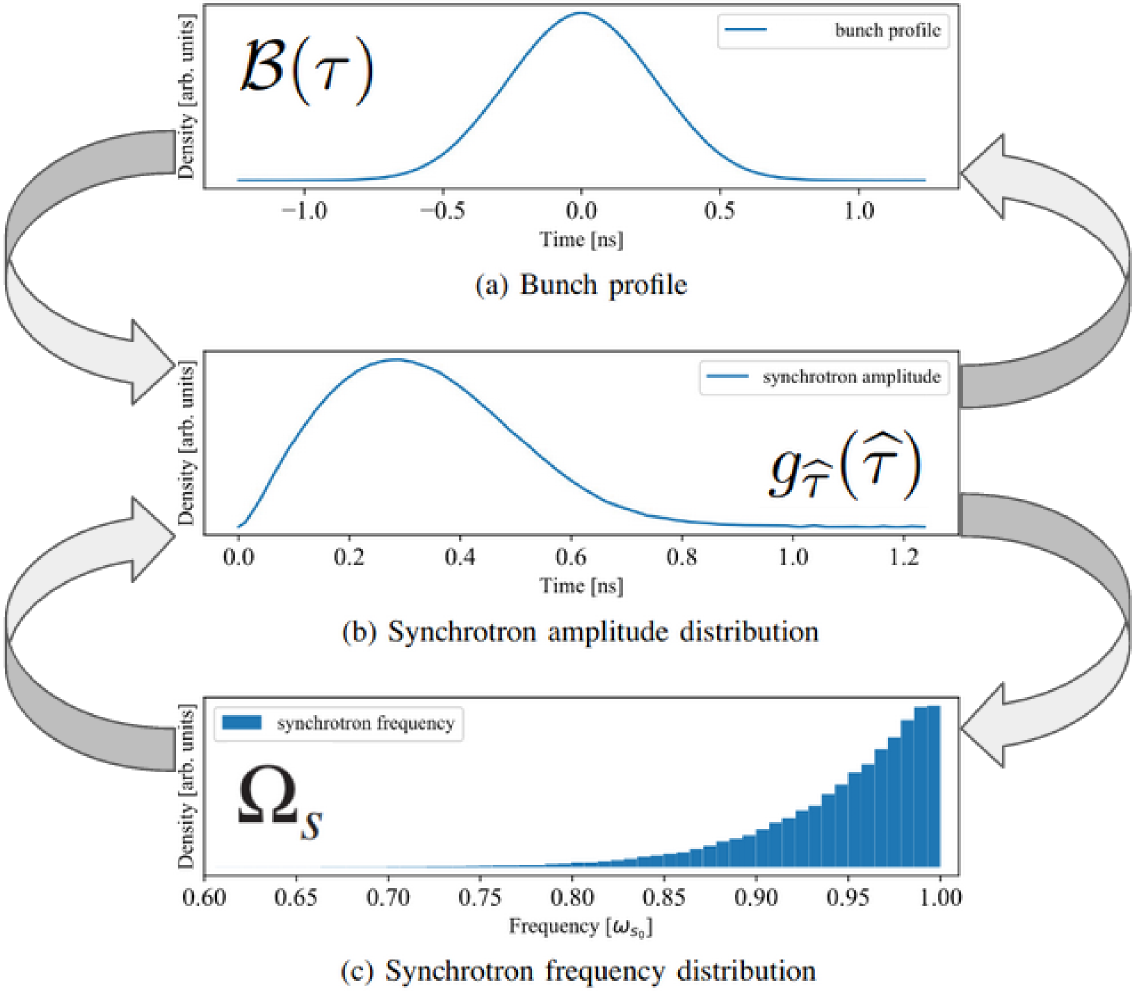
The theory predicts only the expected, ensemble averaged spectrum. Time averaging required to have a correspondence.

Analyzed LHC spectra are averaged for 100 s.



See C. Lannoy et al., WEP035, IBIC'23

Equivalence of longitudinal characteristics



From probabilistic principles (or Abel transform):

$$\mathcal{B}(\tau) = \int_{|\tau|}^{\infty} \frac{g_{\widehat{\tau}}(\widehat{\tau})}{\pi\sqrt{\widehat{\tau}^2 - \tau^2}} d\widehat{\tau}$$

From the theory of mathematical pendulum:

$$\Omega_s = \frac{\pi}{2\mathcal{K}[\sin(\frac{h\omega_0\hat{\tau}}{2})]}\Omega_{s_0}$$

Matrix form of Schottky spectra

Mathematically, incoherent Schottky spectra are given as a function of:

- Synchrotron amplitude distribution - at least 2 parameters
- Nominal synchrotron frequency - 1 parameter

And:

- Betatron tune - 1 parameter
- Chromaticity - 1 parameter

Longitudinal:

$$\frac{\omega_0 q}{2\pi}\sum_{n=-\infty}^{\infty}\sum_{p=-\infty}^{\infty} J_p\left(n\omega_0\widehat{\tau}_i\right) e^{j\left(n\omega_0 t+p\Omega_{s_i}t+p\varphi_{s_i}\right)}$$

Transverse:

$$\sum_{n,p=-\infty}^{\infty} J_p\left(\chi^{\pm}_{\widehat{\tau}_i,n\mp Q_I}\right) e^{j\left(\left[(n\pm Q_F)\omega_0+p\Omega_{s_i}\right]t+\varphi_{\beta_i}+p\varphi_{s_i}\right)}$$

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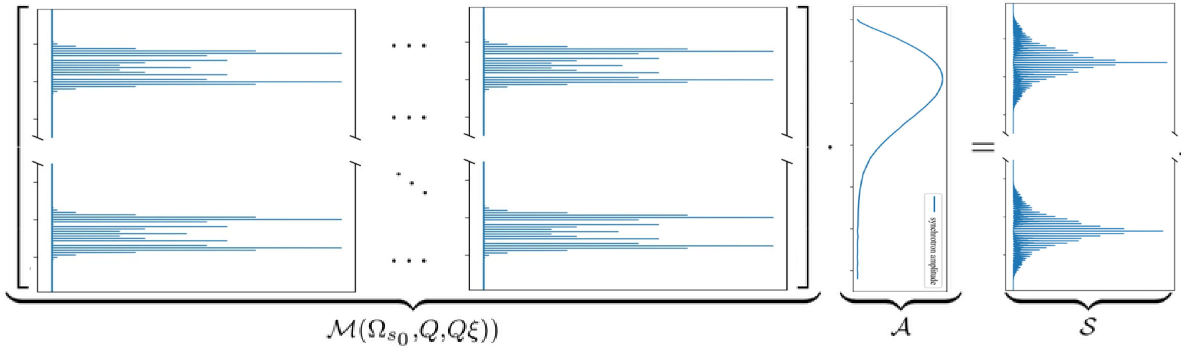
For a given set of parameters multiparticle spectrum can be calculated with a simple matrix transform.

$$\underbrace{\begin{bmatrix} P_T^{\pm}(\omega_1, \widehat{\tau}_1, \Omega_{s0}, Q, Q\xi) & \cdots & P_T^{\pm}(\omega_1, \widehat{\tau}_n, \Omega_{s0}, Q, Q\xi) \\ P_T^{\pm}(\omega_2, \widehat{\tau}_1, \Omega_{s0}, Q, Q\xi) & \cdots & P_T^{\pm}(\omega_2, \widehat{\tau}_n, \Omega_{s0}, Q, Q\xi) \\ \vdots & \ddots & \vdots \\ P_T^{\pm}(\omega_m, \widehat{\tau}_1, \Omega_{s0}, Q, Q\xi) & \cdots & P_T^{\pm}(\omega_m, \widehat{\tau}_n, \Omega_{s0}, Q, Q\xi) \end{bmatrix}}_{\mathcal{M}(\Omega_{s0}, Q, Q\xi)} \cdot \underbrace{\begin{bmatrix} \widetilde{g}(\widehat{\tau}_1) \\ \widetilde{g}(\widehat{\tau}_2) \\ \vdots \\ \widetilde{g}(\widehat{\tau}_n) \end{bmatrix}}_{\mathcal{A}} = \underbrace{\begin{bmatrix} P_T^{\pm}(\omega_1) \\ P_T^{\pm}(\omega_2) \\ \vdots \\ P_T^{\pm}(\omega_m) \end{bmatrix}}_{\mathcal{S}}.$$



Details in: K. Lasocha and D. Alves, Phys. Rev. Accel. Beams 23, 062803
K. Lasocha and D. Alves, Phys. Rev. Accel. Beams 25, 062801

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Use case 1: fast Schottky spectra simulation.

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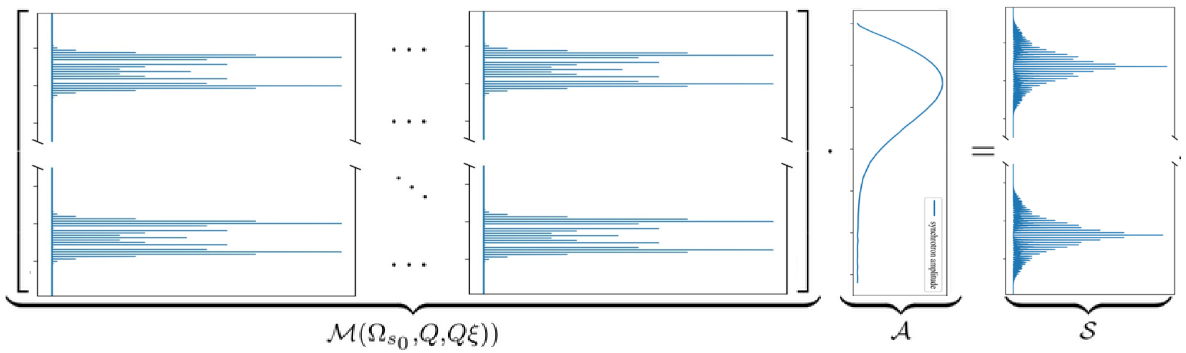

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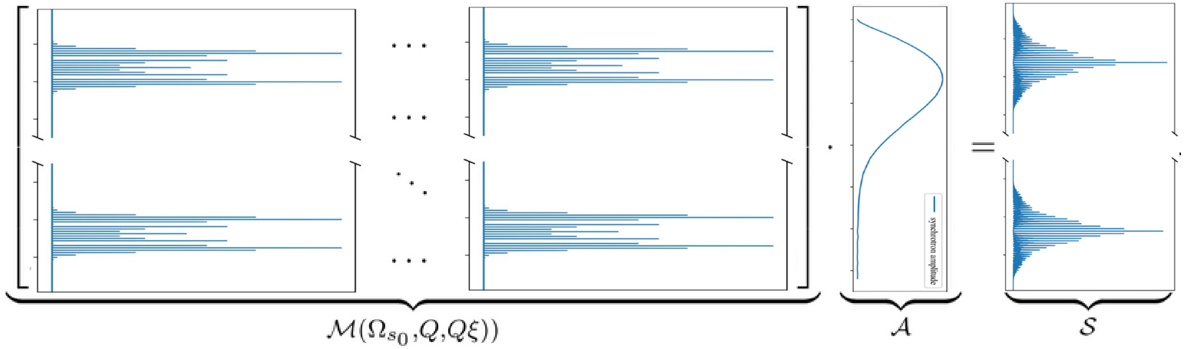
Use case 1: fast Schottky spectra simulation



Use case 2 (spectral fitting): given an experimentally measured spectrum, true parameters would minimize the cost function:

$$C\left(\Omega_{s0}, Q, Q\xi, \mathcal{A}\right) = \left|\mathcal{M}\left(\Omega_{s0}, Q, Q\xi\right) \cdot \mathcal{A} - \left[\mathcal{S}_{exp}\right]\right|^2$$

Minimizing routines iteratively simulate Schottky spectra and compare them with the measurement.

Details in: K. Lasocha and D. Alves, Phys. Rev. Accel. Beams 23, 062803
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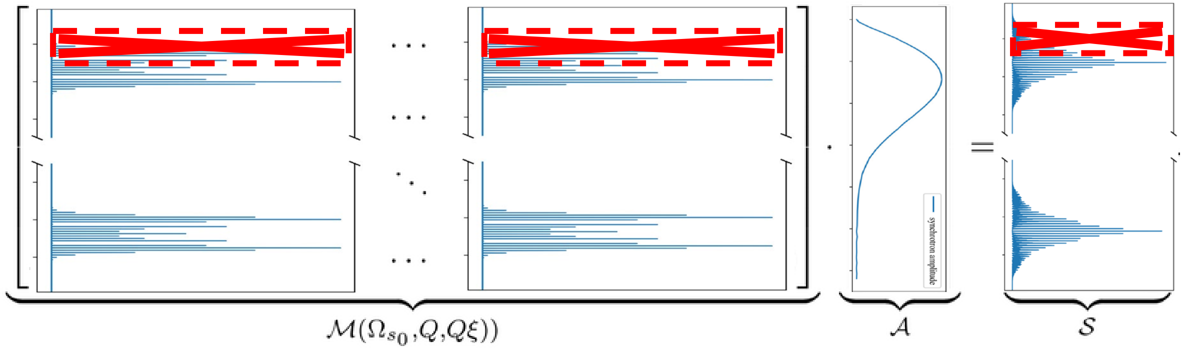
Matrix form: excluding frequency bins

Fitting procedure also allows to exclude the spectral regions with undesired components.

$$\underbrace{\begin{bmatrix} P_T^{\pm}(\omega_1, \widehat{\tau}_1, \Omega_{s0}, Q, Q\xi) & \cdots & P_T^{\pm}(\omega_1, \widehat{\tau}_n, \Omega_{s0}, Q, Q\xi) \\ P_T^{\pm}(\omega_2, \widehat{\tau}_1, \Omega_{s0}, Q, Q\xi) & \cdots & P_T^{\pm}(\omega_2, \widehat{\tau}_n, \Omega_{s0}, Q, Q\xi) \\ \vdots & \ddots & \vdots \\ P_T^{\pm}(\omega_m, \widehat{\tau}_1, \Omega_{s0}, Q, Q\xi) & \cdots & P_T^{\pm}(\omega_m, \widehat{\tau}_n, \Omega_{s0}, Q, Q\xi) \end{bmatrix}}_{\mathcal{M}(\Omega_{s0}, Q, Q\xi))} \cdot \underbrace{\begin{bmatrix} \widetilde{g}(\widehat{\tau}_1) \\ \widetilde{g}(\widehat{\tau}_2) \\ \vdots \\ \widetilde{g}(\widehat{\tau}_n) \end{bmatrix}}_{\mathcal{A}} = \underbrace{\begin{bmatrix} P_T^{\pm}(\omega_1) \\ P_T^{\pm}(\omega_2) \\ \vdots \\ P_T^{\pm}(\omega_m) \end{bmatrix}}_{\mathcal{S}} .$$



LHC Schottky Monitor

- One system for two particle species: protons and Pb^{82+} ions, one device per beam and per plane

Typical LHC beam parameters:

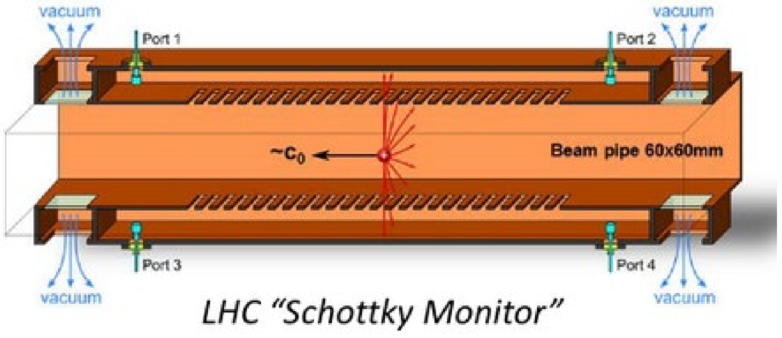
	p^+	Pb^{82+}
$N_{particles}$ (per bunch)	10^{11}	10^{8}
Bunch length (4σ)	1-1.4 ns	
Normalized transverse emittance	1.5-2.5 μm	
Energy Inj/Flattop (per nucleon)	0.45 - 6.8 TeV	0.18 - 2.6 TeV

LHC Schottky Monitor

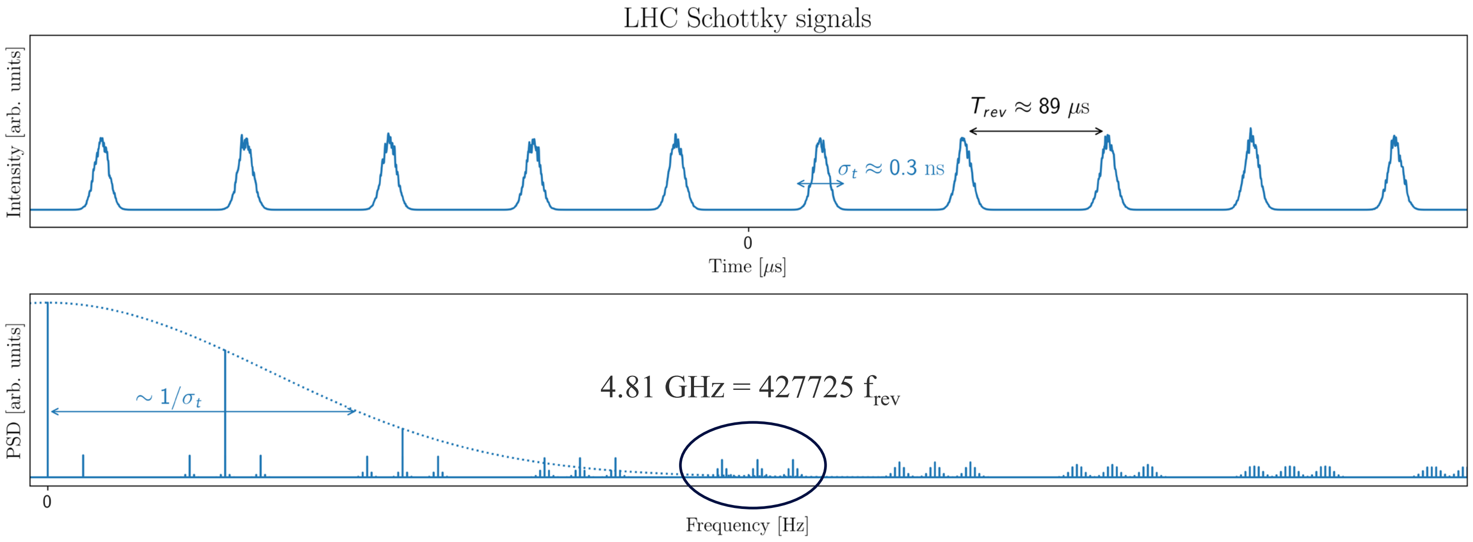
- One system for two particle species: protons and Pb^{82+} ions, one device per beam and per plane
- Pair of slotted waveguides, probing beam field at 4.81 GHz, followed by filtering and down mixing to 11.2 kHz

Typical LHC beam parameters:

	p^+	Pb^{82+}
$N_{particles}$ (per bunch)	10^{11}	10^{8}
Bunch length (4σ)	1-1.4 ns	
Normalized transverse emittance	1.5-2.5 μm	
Energy Inj/Flattop (per nucleon)	0.45 - 6.8 TeV	0.18 - 2.6 TeV



Details on the LHC Schottky system in
M. Betz et al., NIM, vol. 874, pp 113-126, 2017



LHC Schottky Monitor

- One system for two particle species: protons and Pb^{82+} ions, one device per beam and per plane
- Pair of slotted waveguides, probing beam field at 4.81 GHz, followed by filtering and down mixing to 11.2 kHz
- Gating system enables observation of single bunches

Typical LHC beam parameters:

	p^+	Pb^{82+}
$N_{particles}$ (per bunch)	10^{11}	10^{8}
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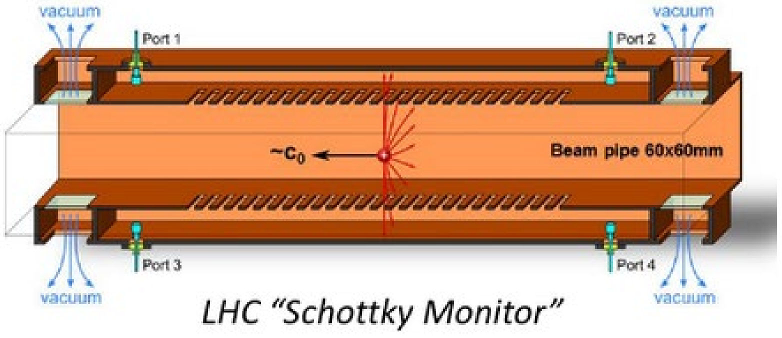

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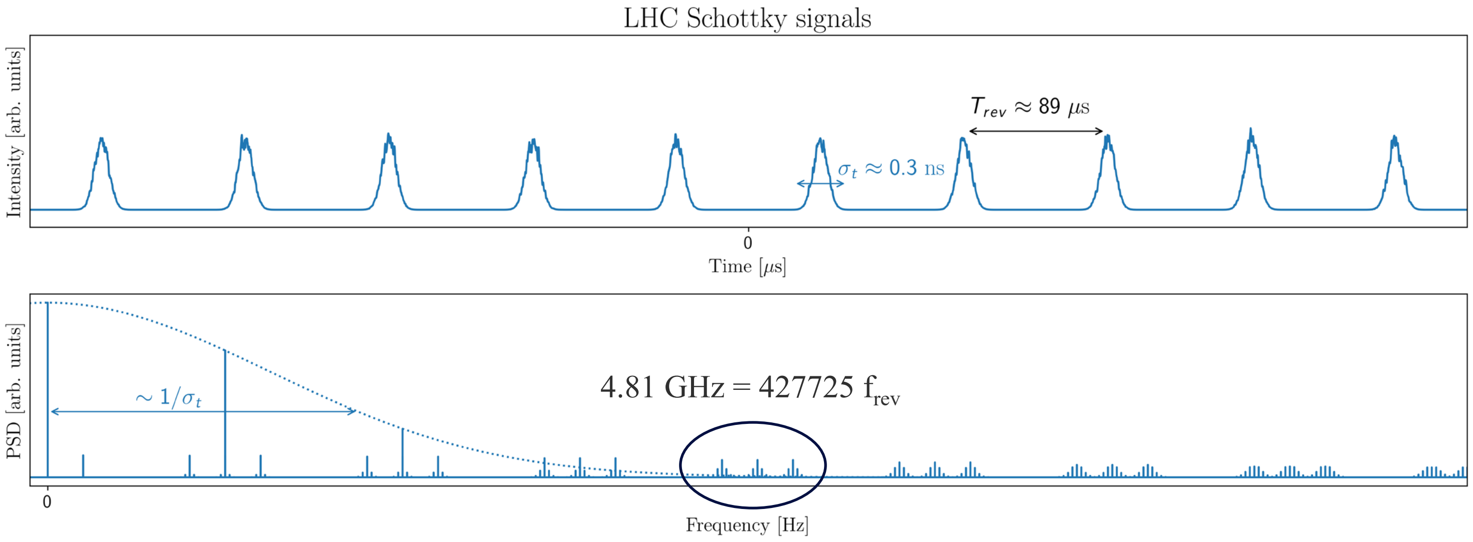

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- **The only instrument with the potential of measuring the chromaticity in the LHC in a non-invasive way**

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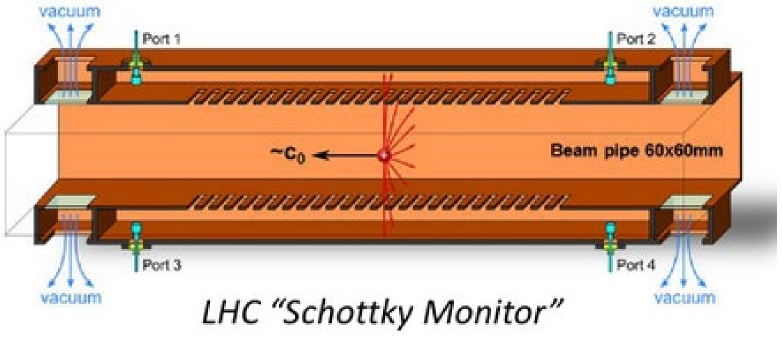

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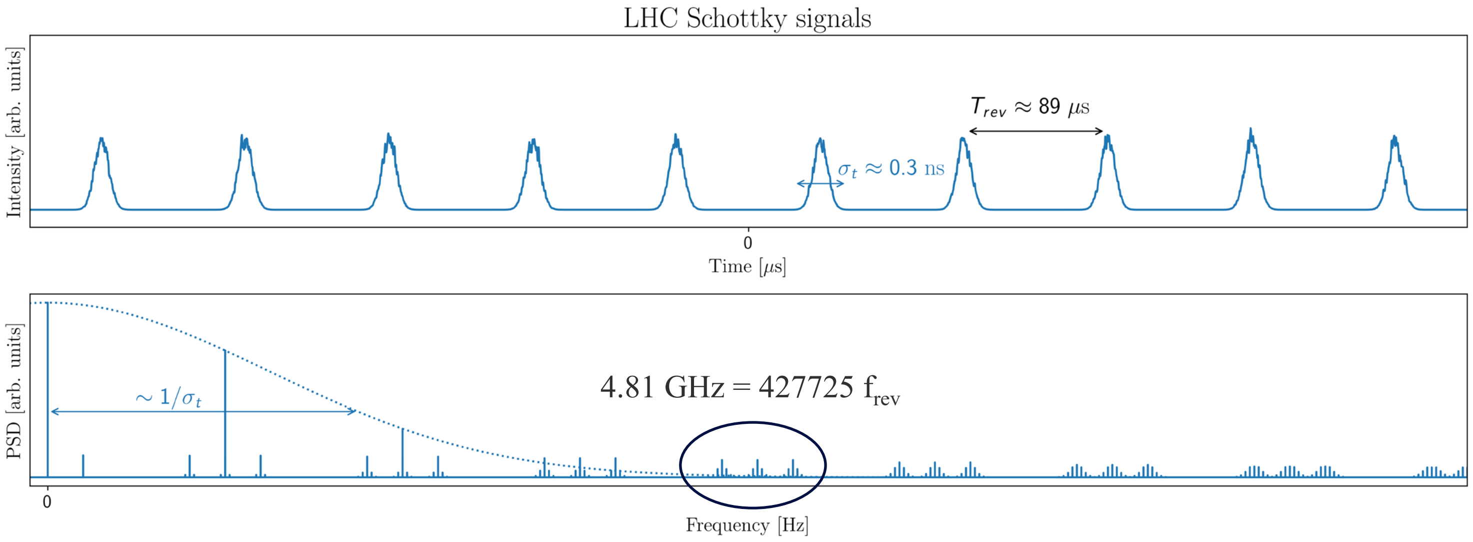

LHC Schottky Spectra

(upper transverse sideband)

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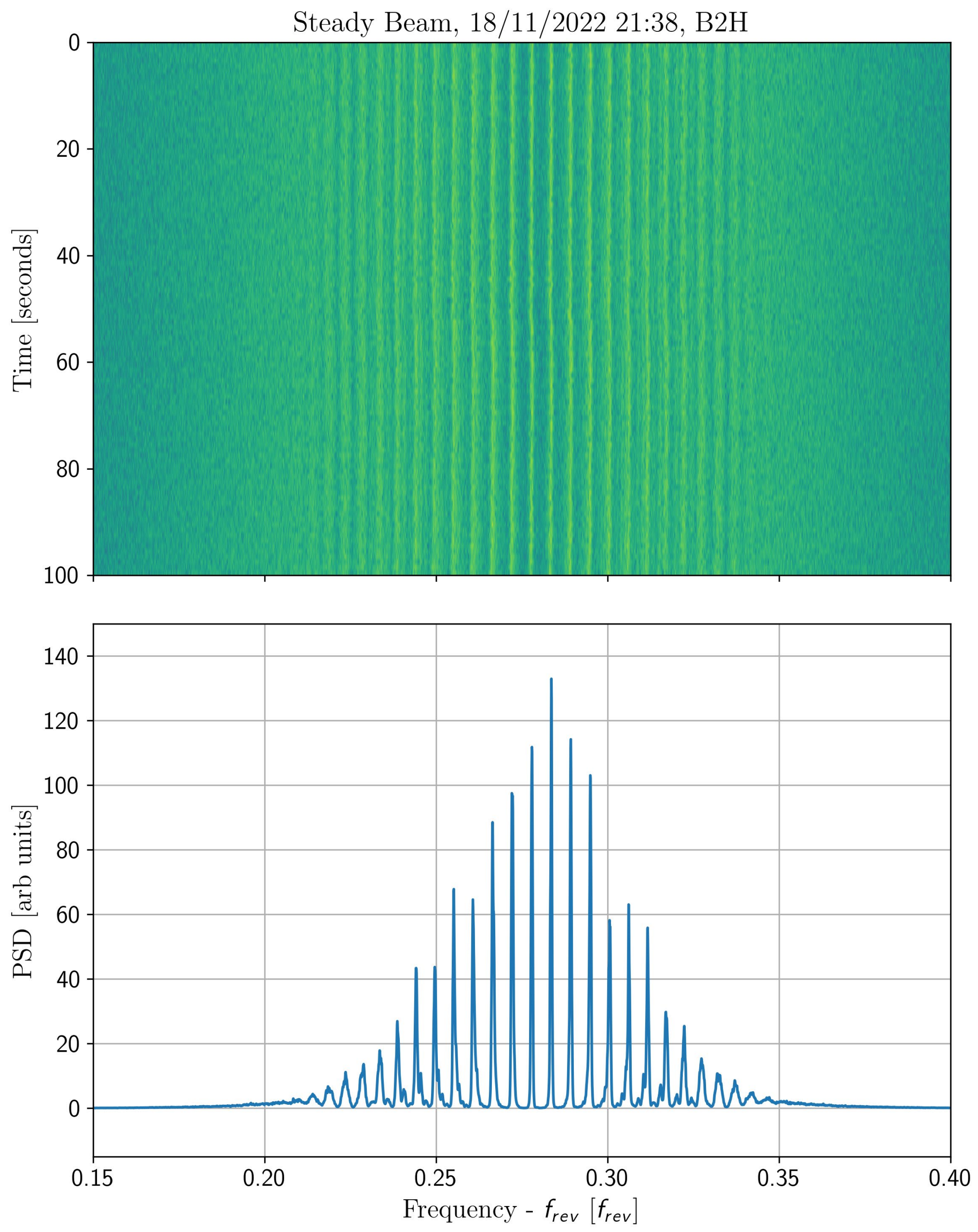

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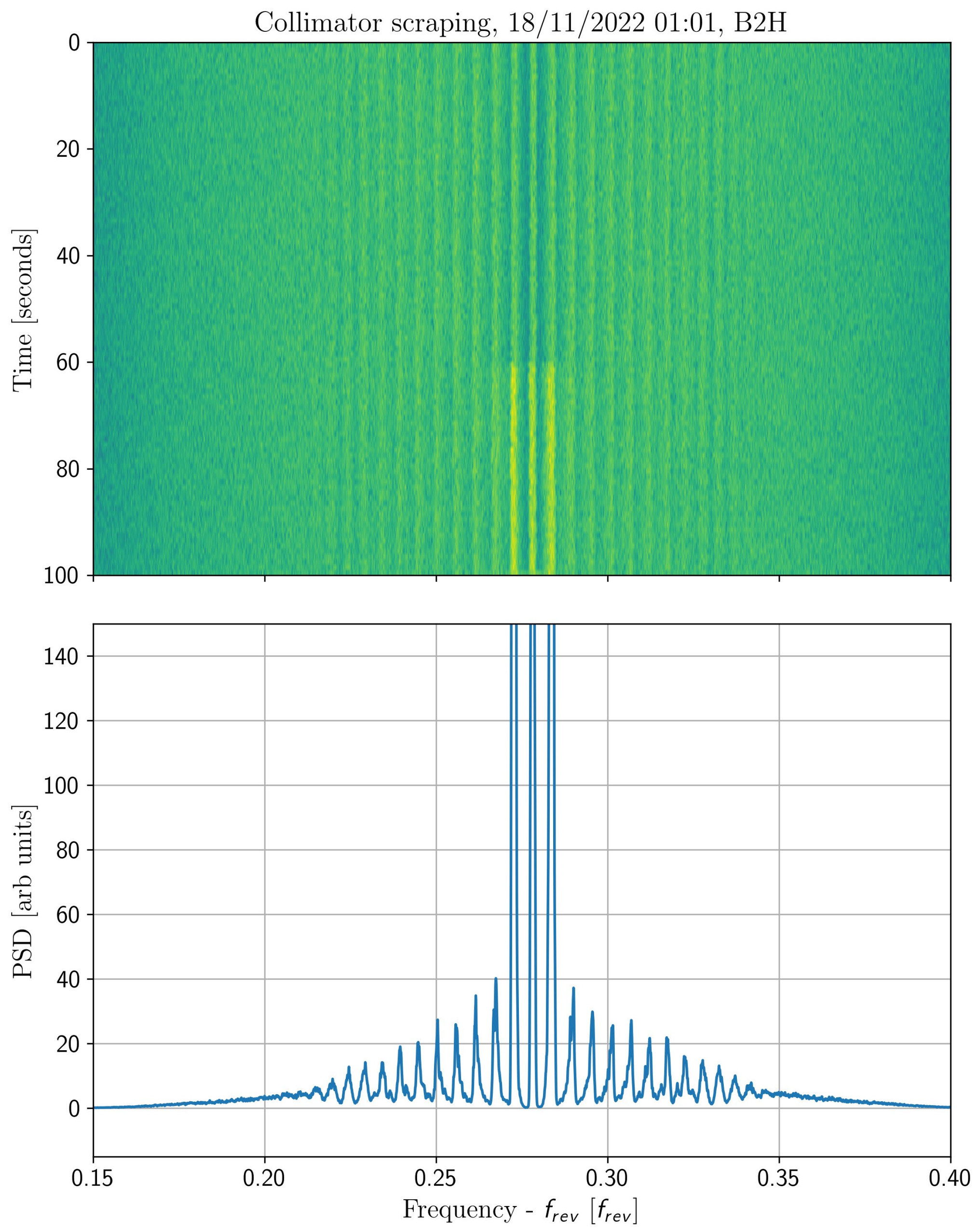

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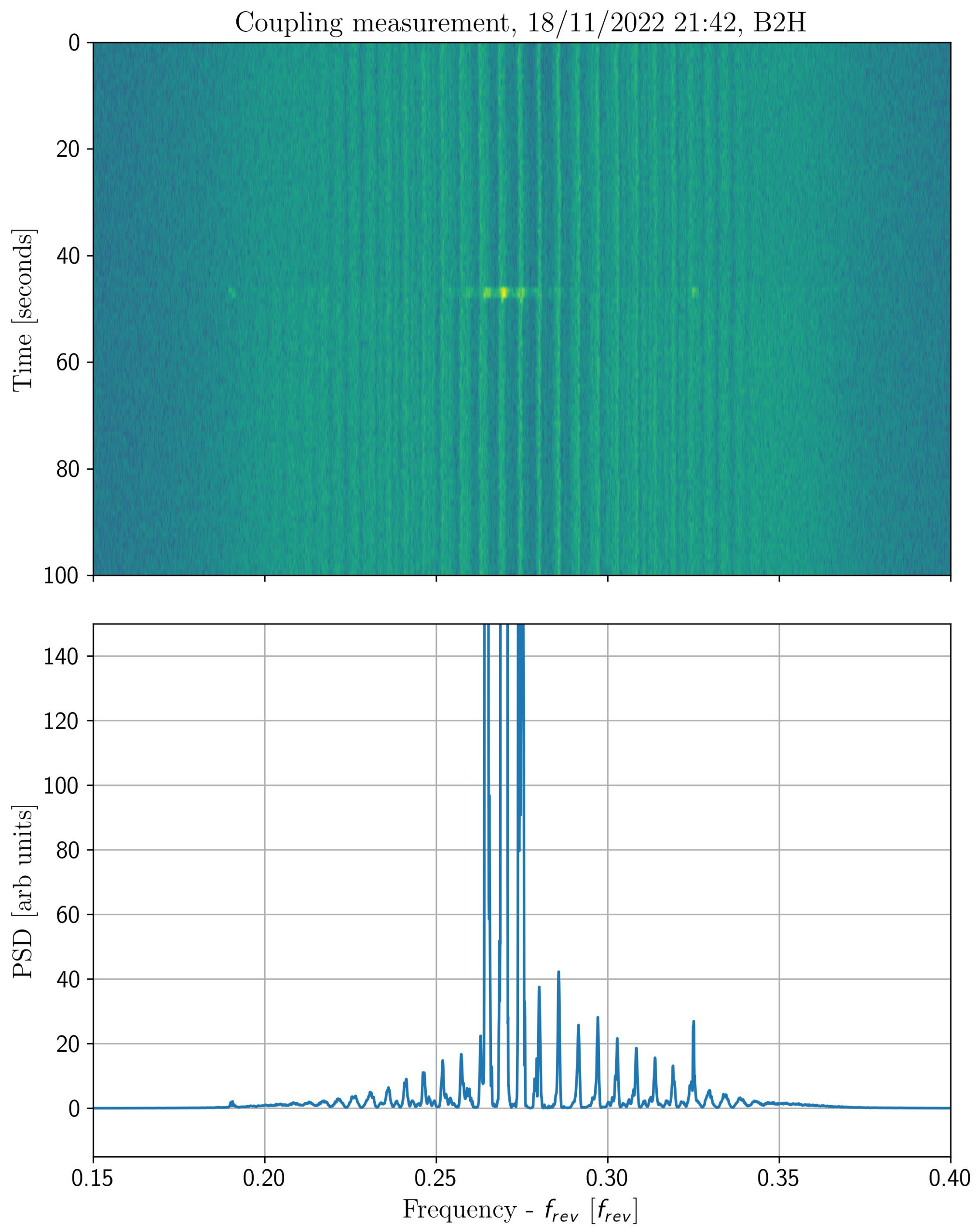

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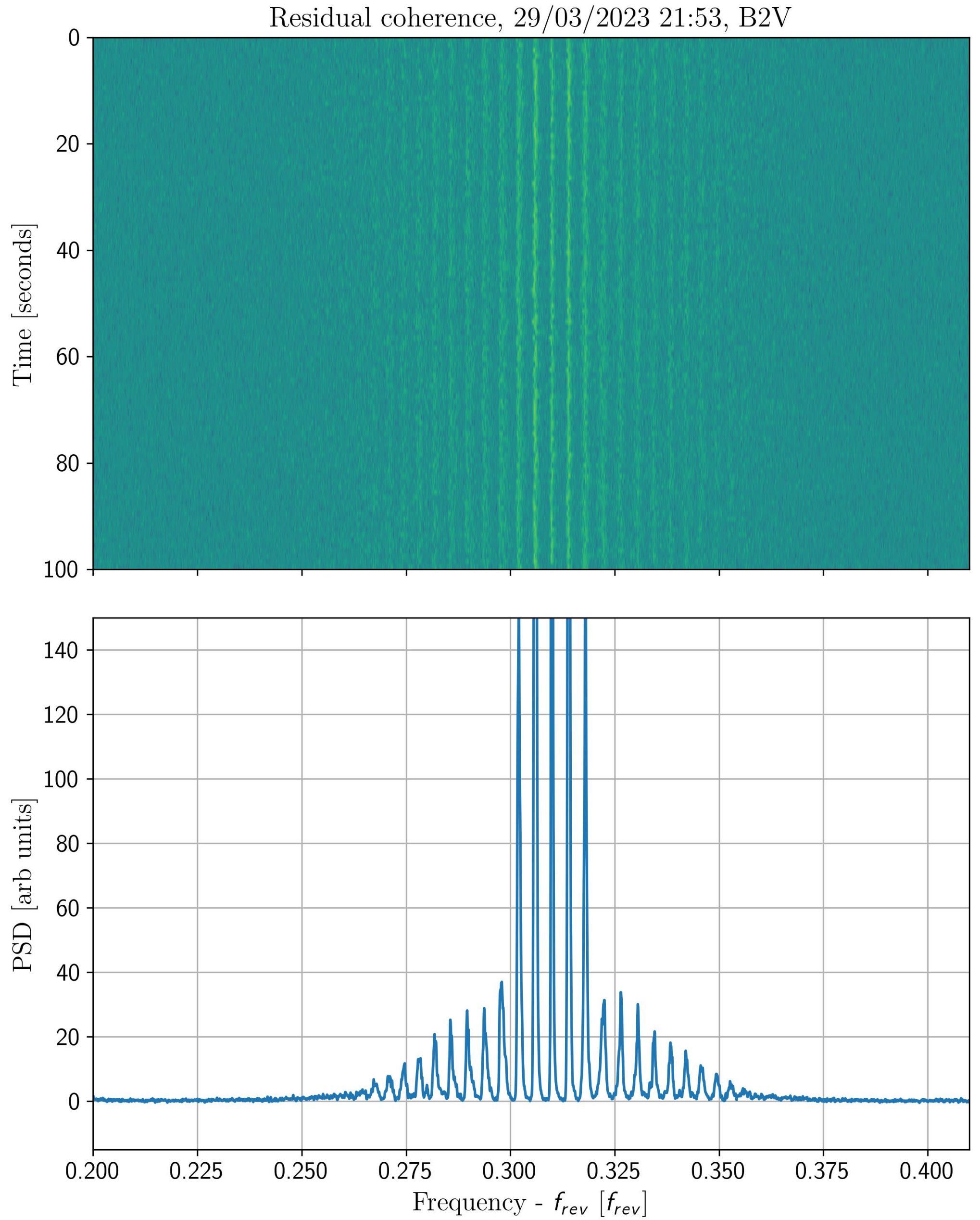

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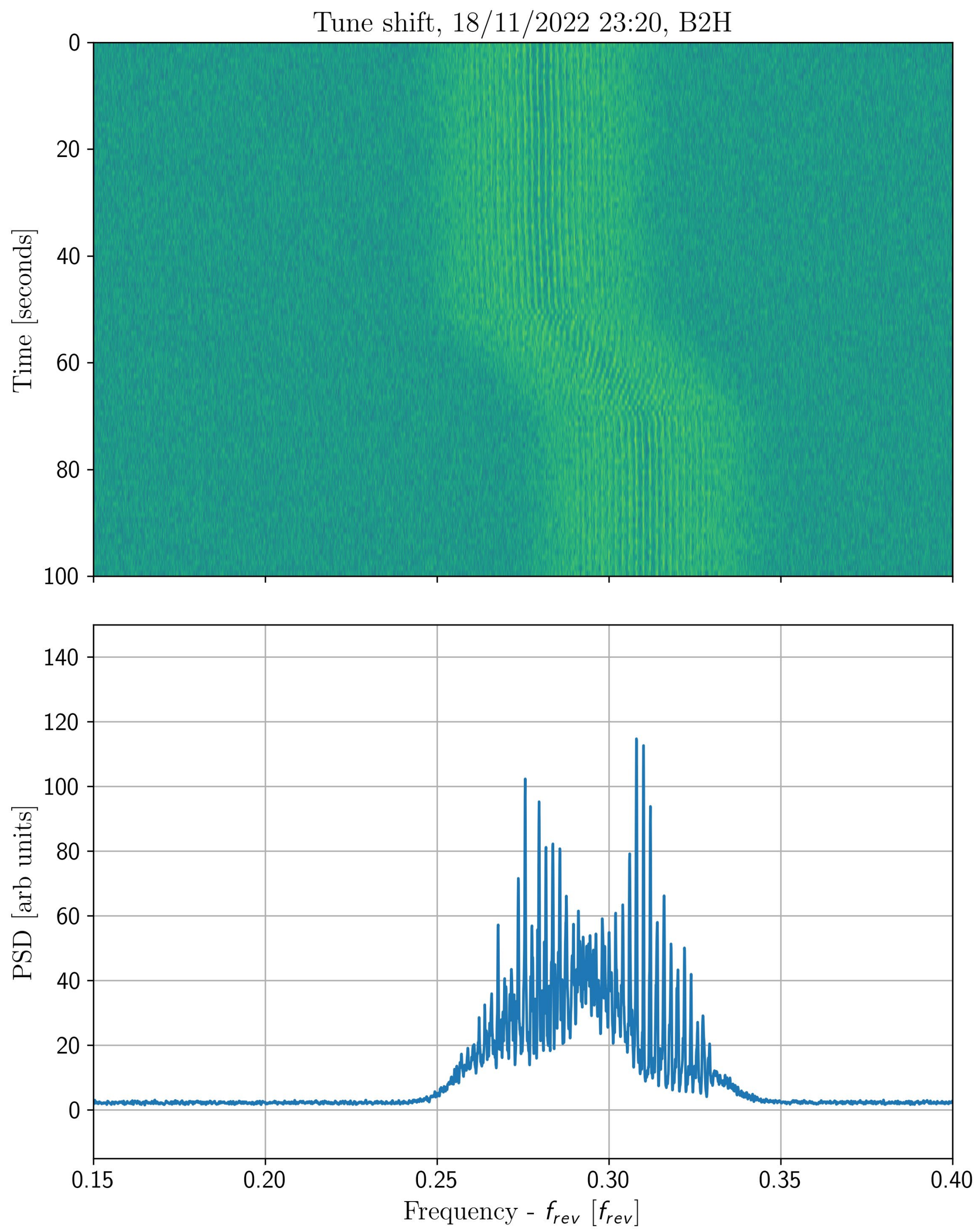

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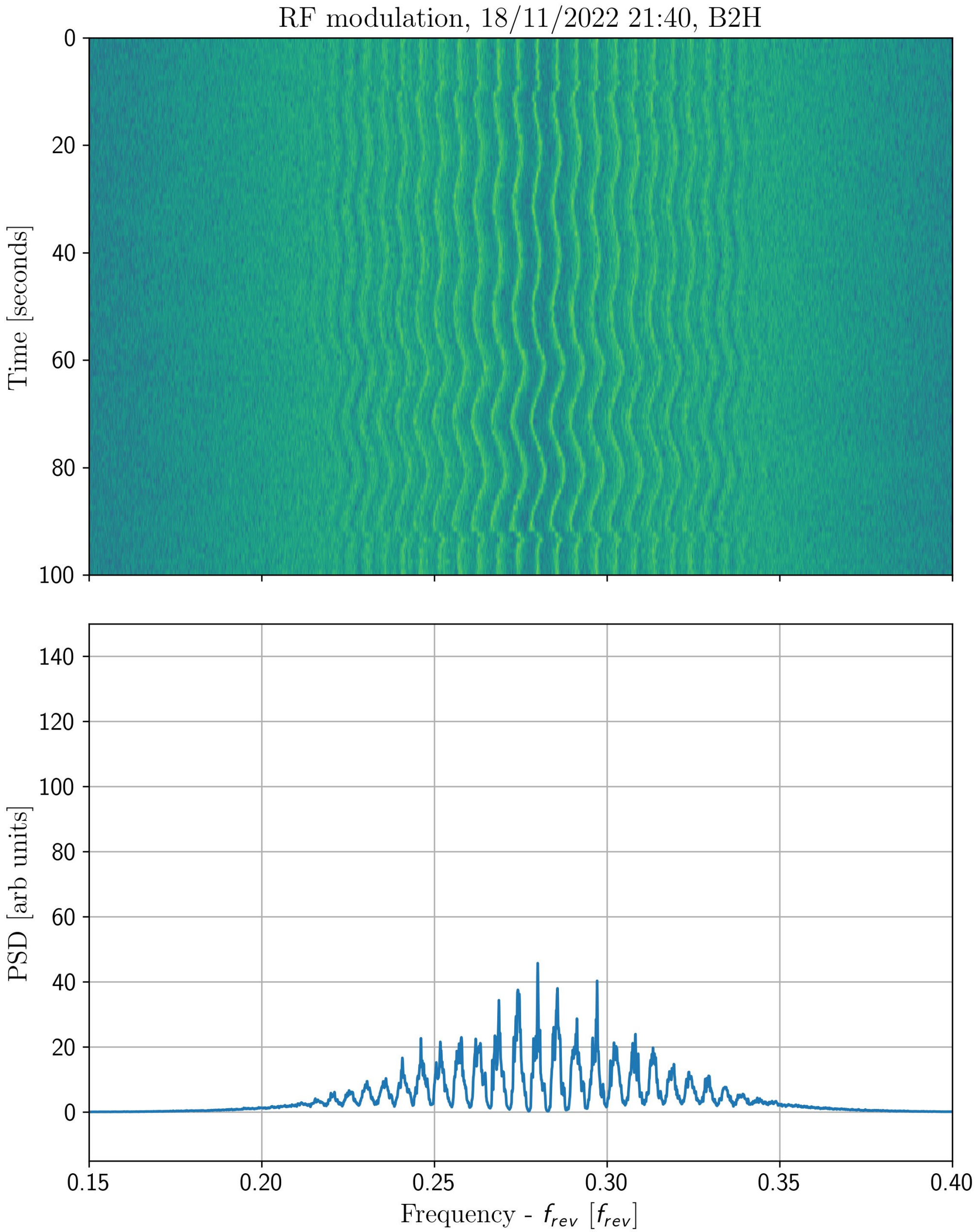

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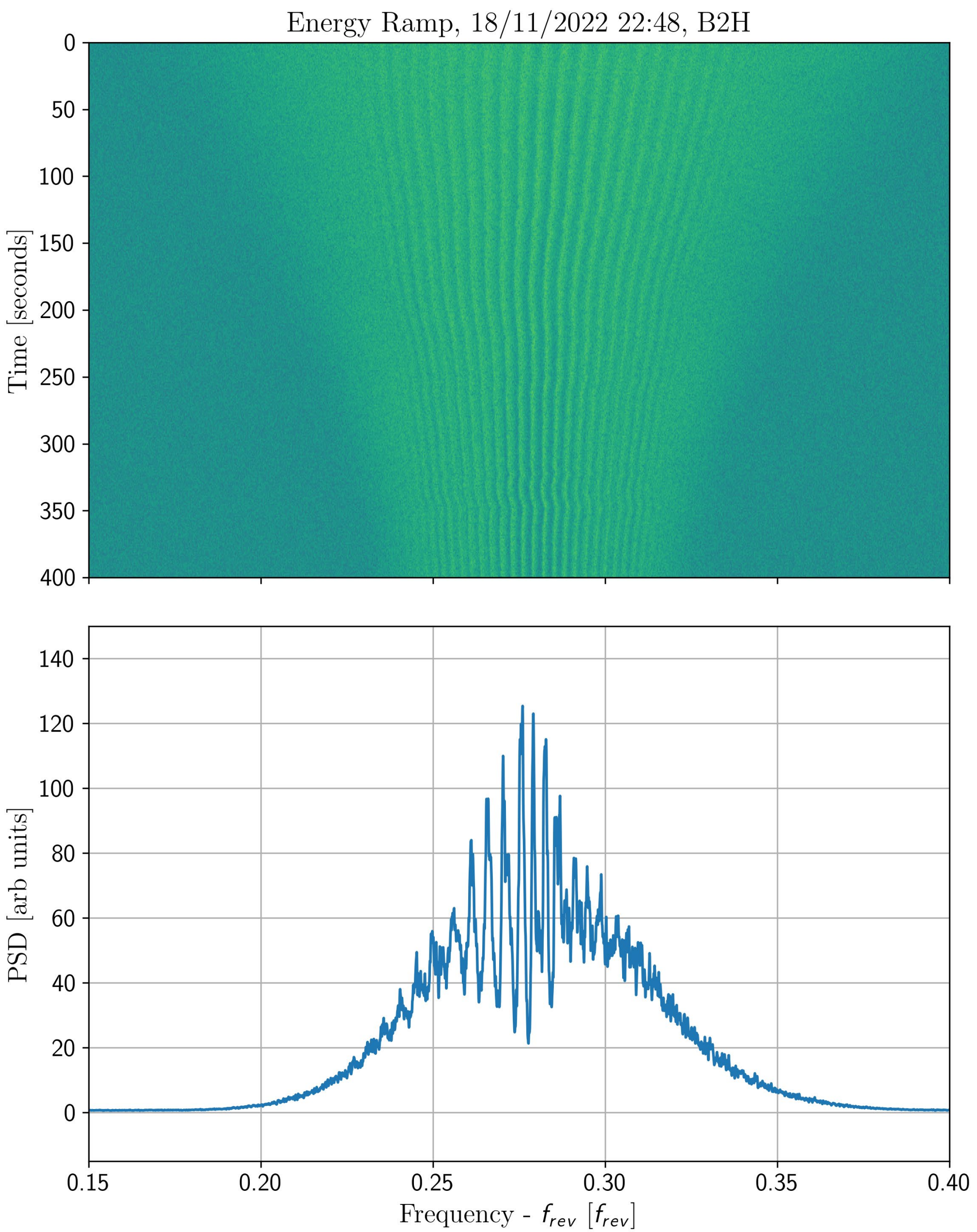

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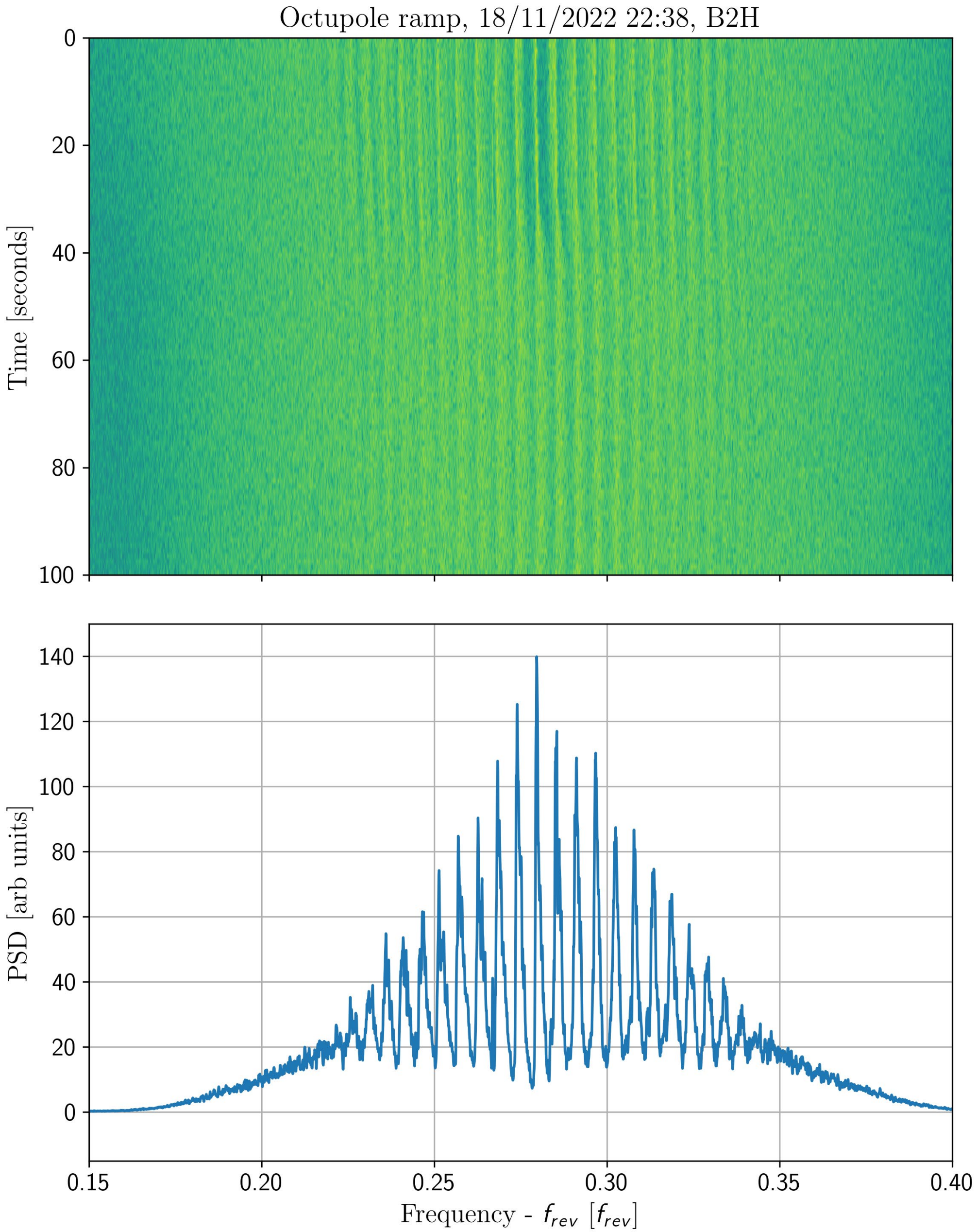

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Ion beam

Proton beam

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Ion beam

Proton beam

saturation, low signal to noise ratio

LHC Schottky Spectra

Analysis techniques known

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Under development

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Extraction of beam parameters: nominal synchrotron frequency

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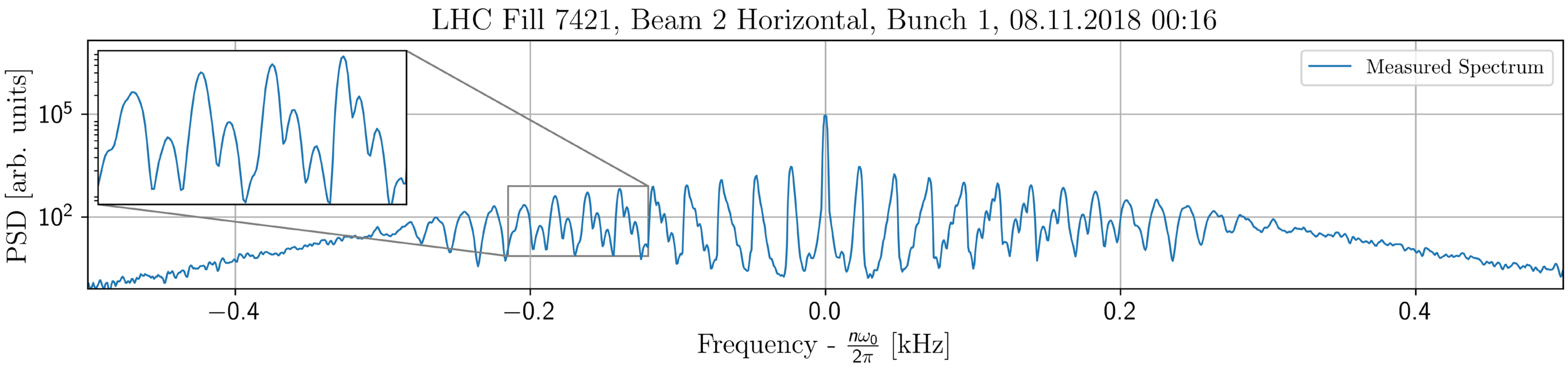

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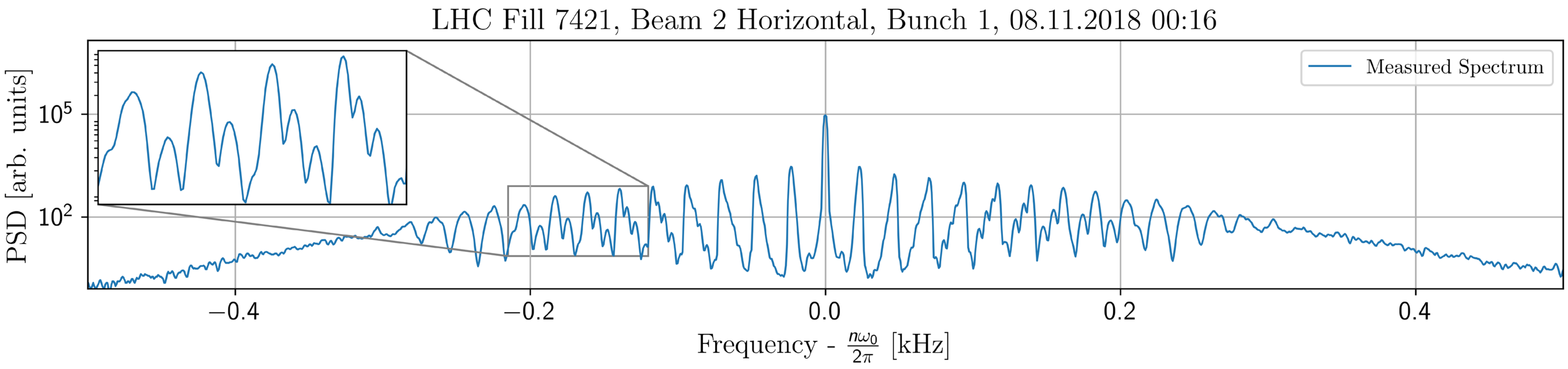

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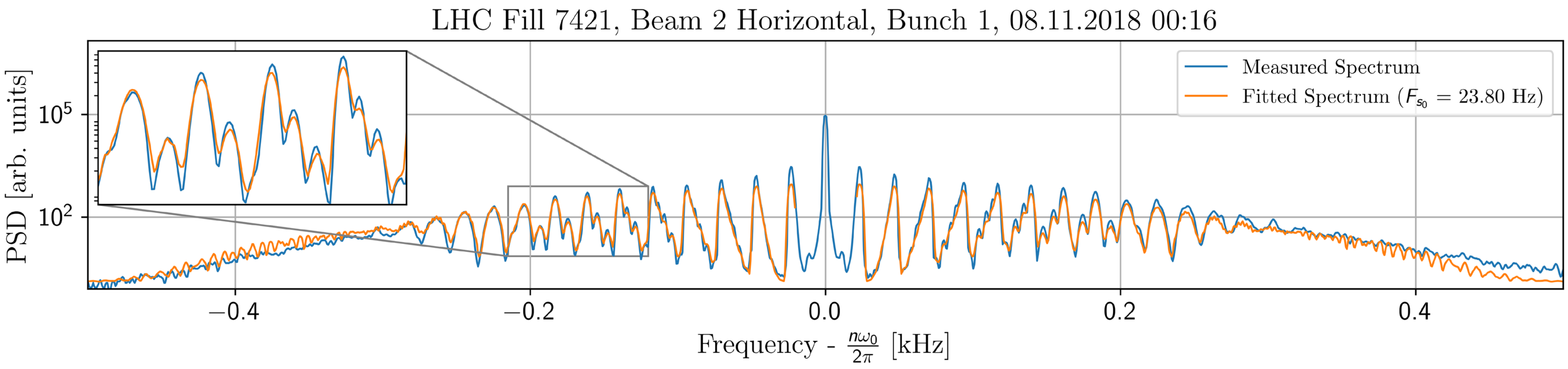
- **Minimization performed using the Differential Evolution algorithm from SciPy library.**



Details in: K. Lasocha and D. Alves, Phys. Rev. Accel. Beams 23, 062803

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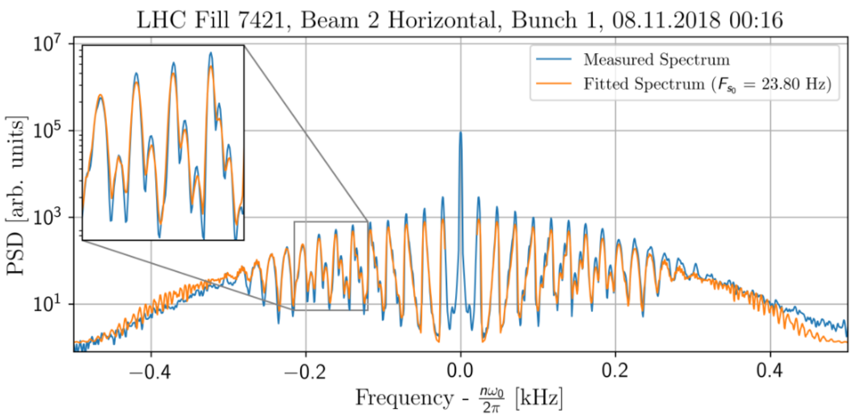

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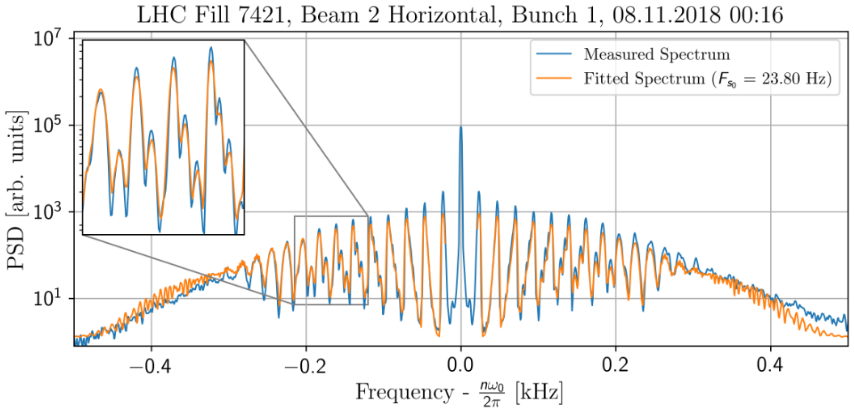
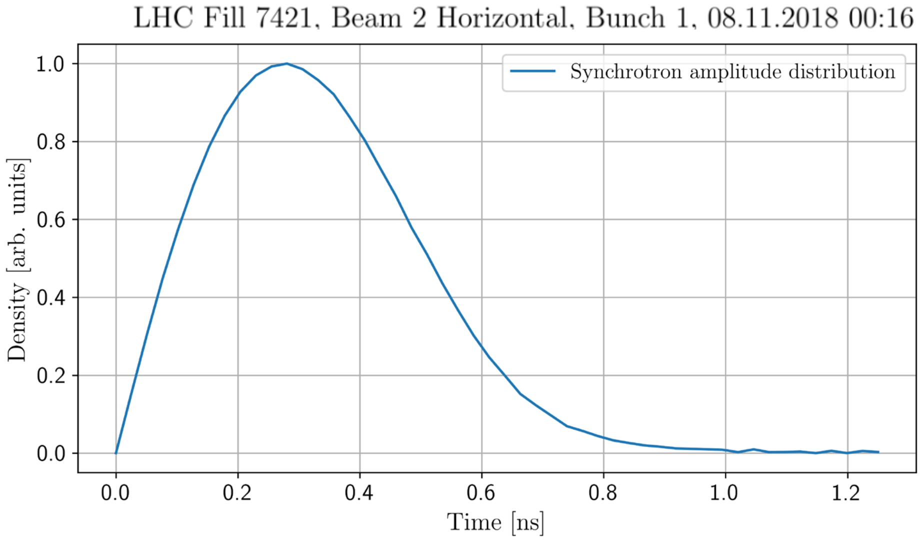


- **Obtained synchrotron amplitude distribution can be transformed into longitudinal bunch profile**



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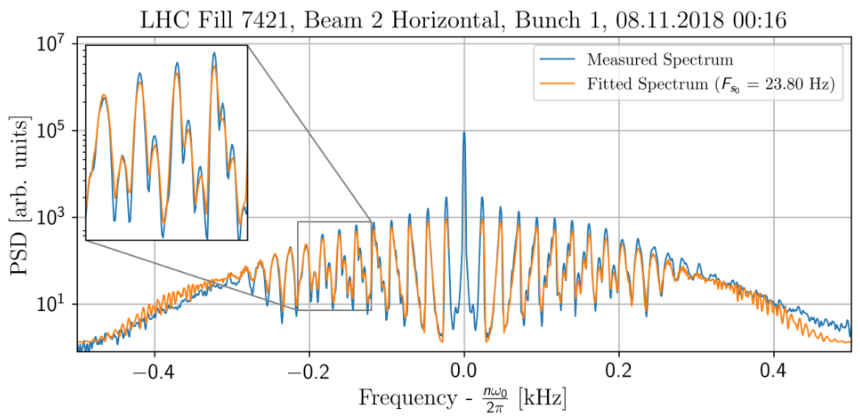

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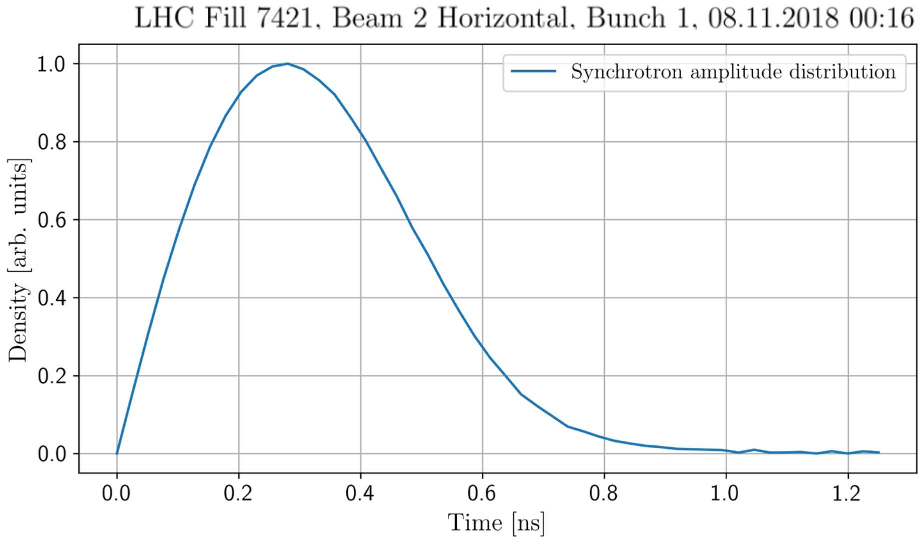

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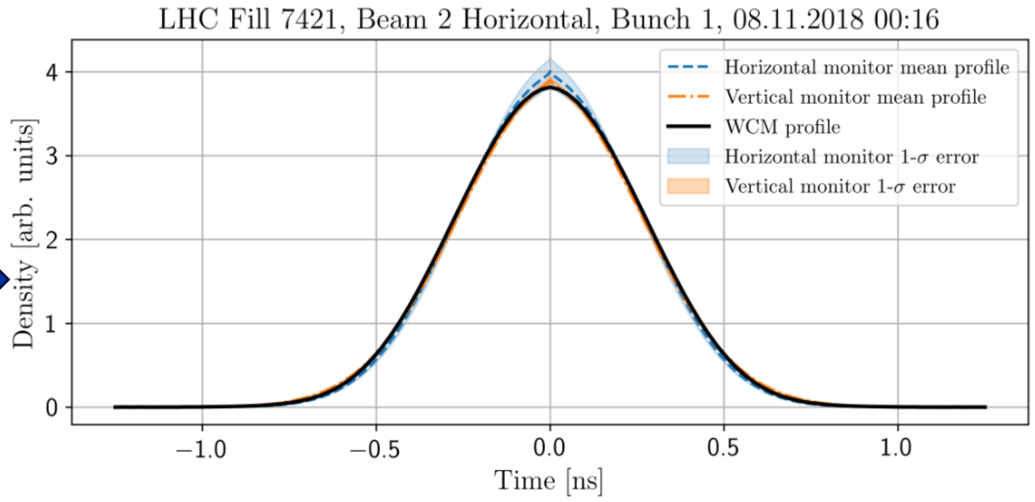

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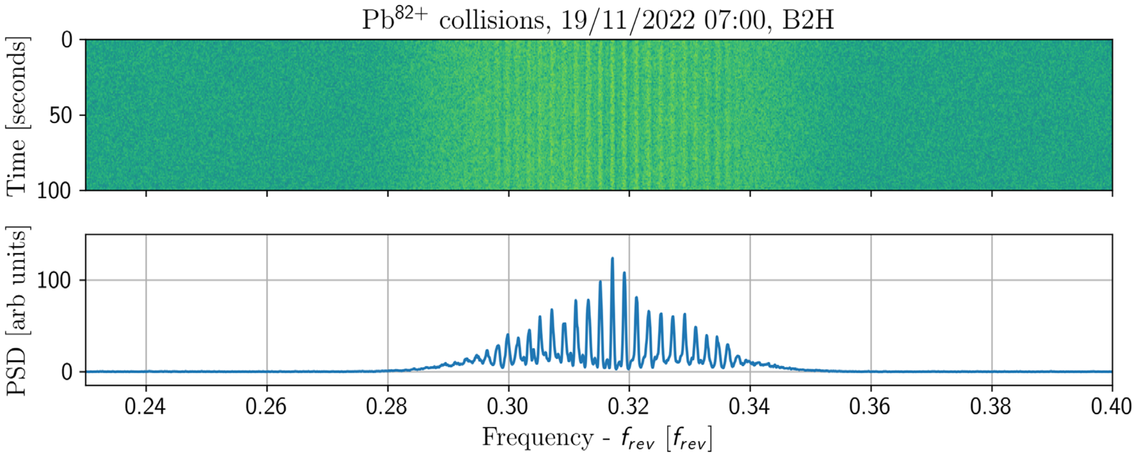
Extraction of beam parameters: tune & chromaticity

Example: 8 hour long ion collisions in Nov 2022



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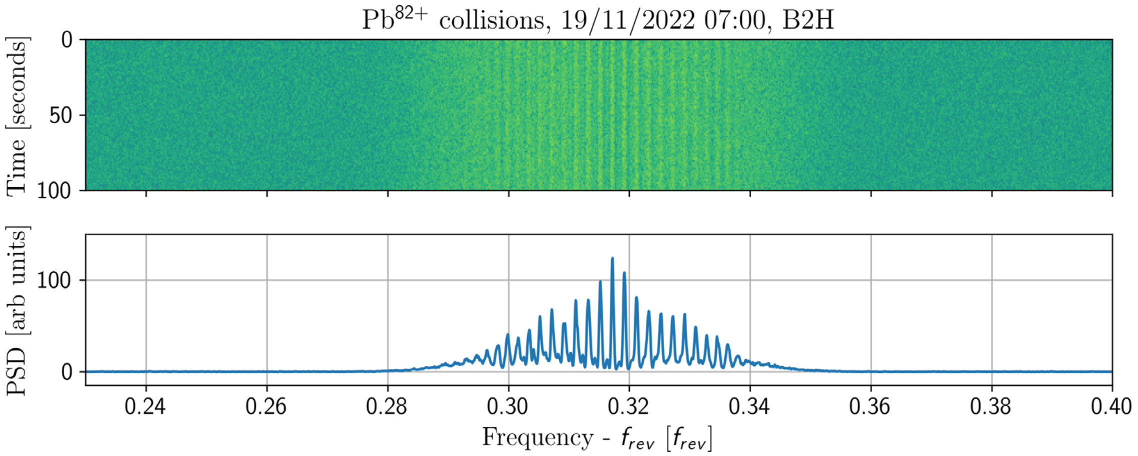

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$$C_{MD}(k) = \sum_{i=1}^{i=N} \left| P_T^{\pm}(\omega_{k-i}) - P_T^{\pm}(\omega_{k+i}) \right|$$

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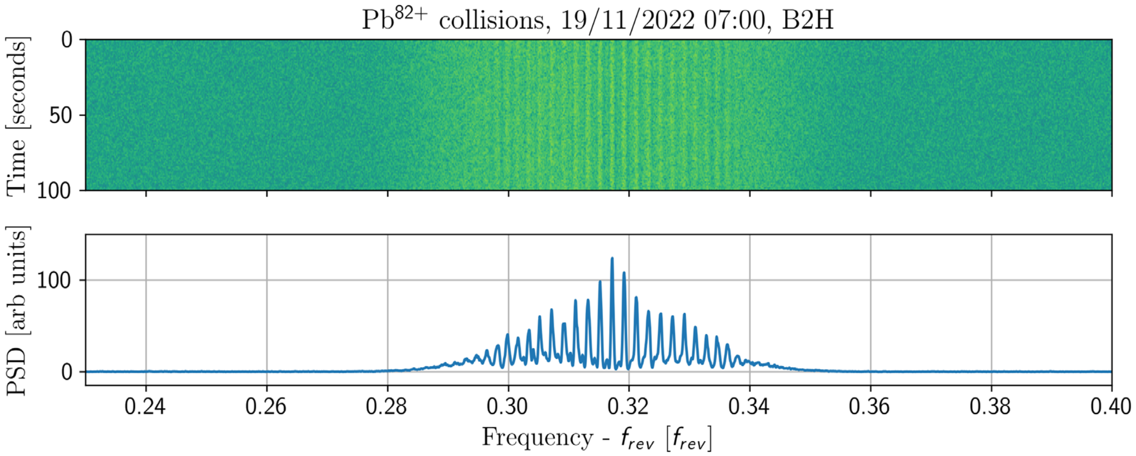

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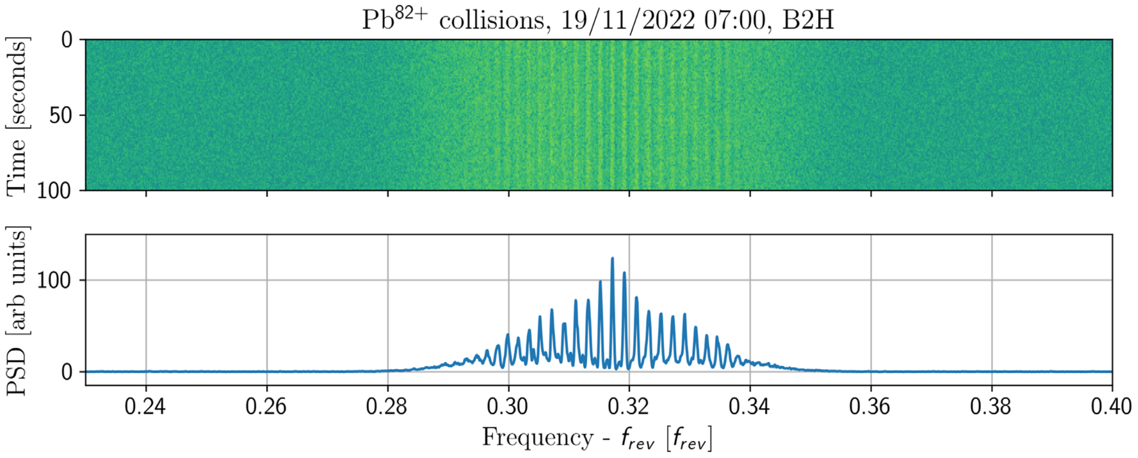

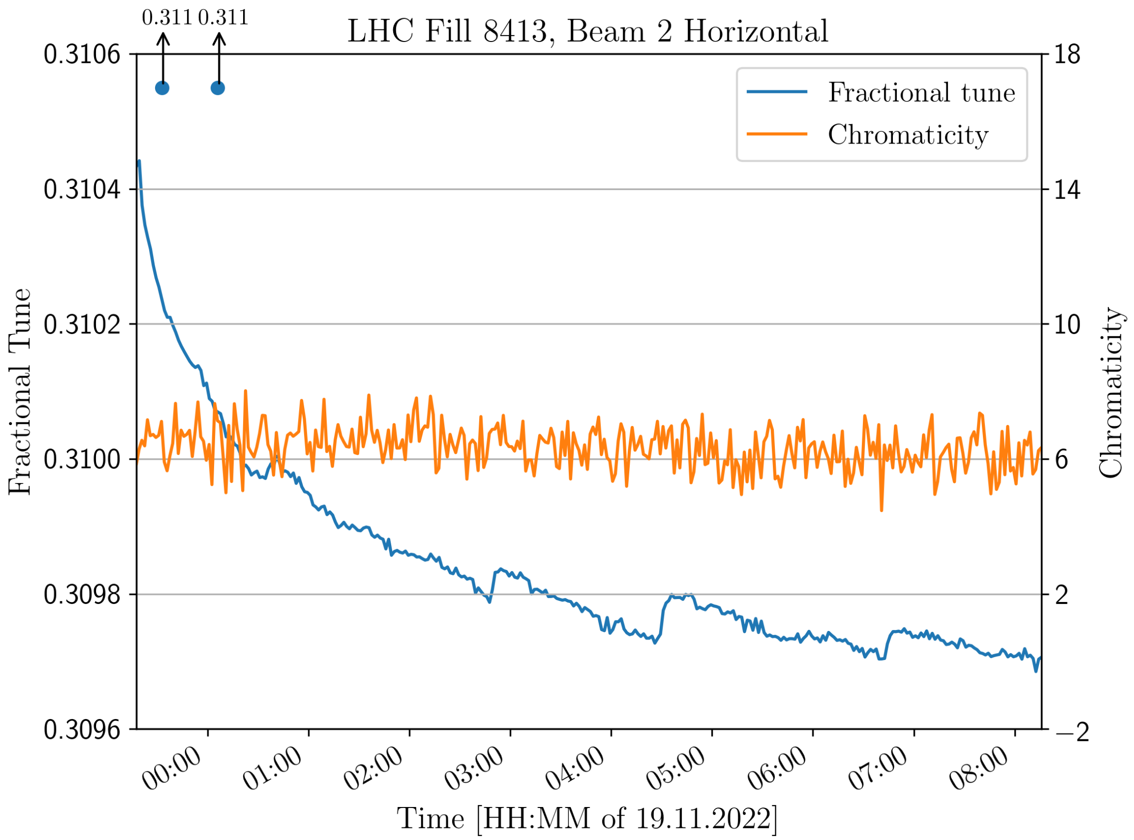

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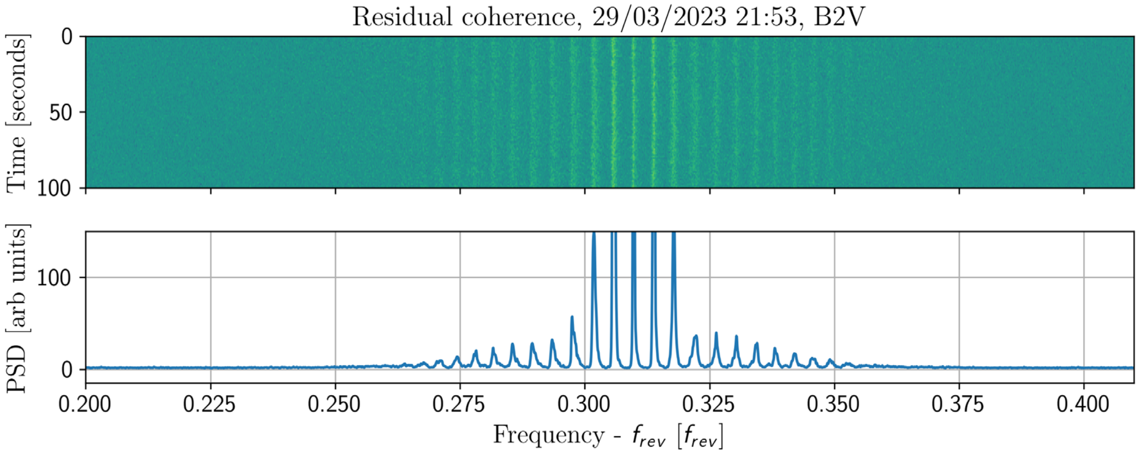

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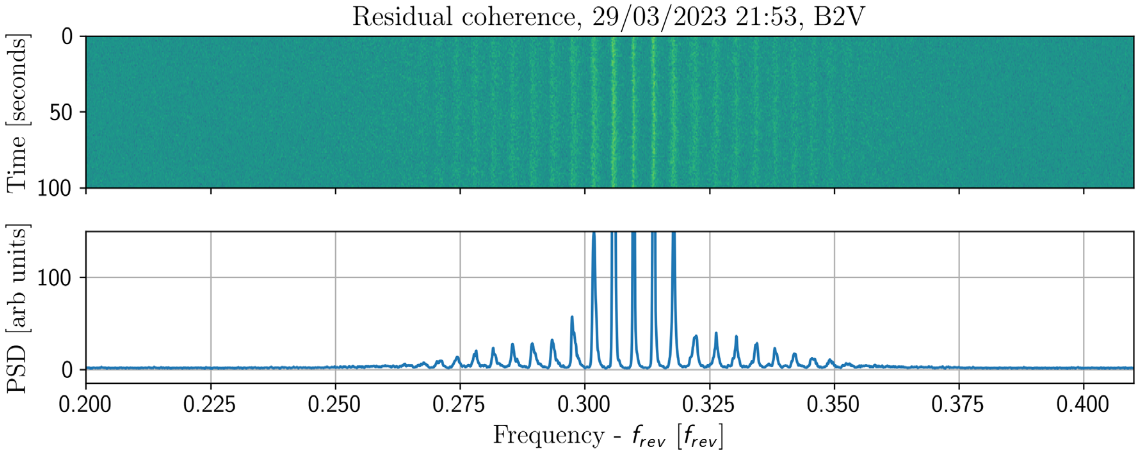

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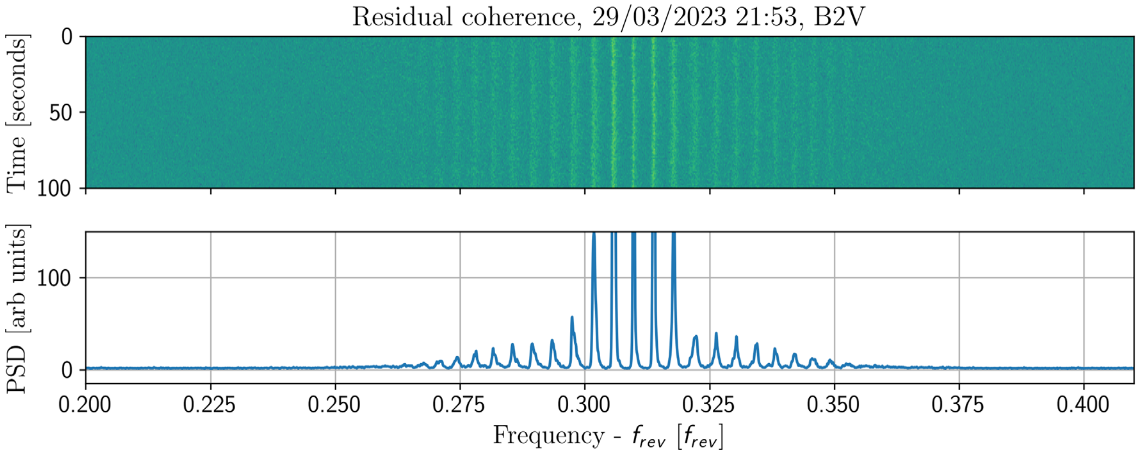

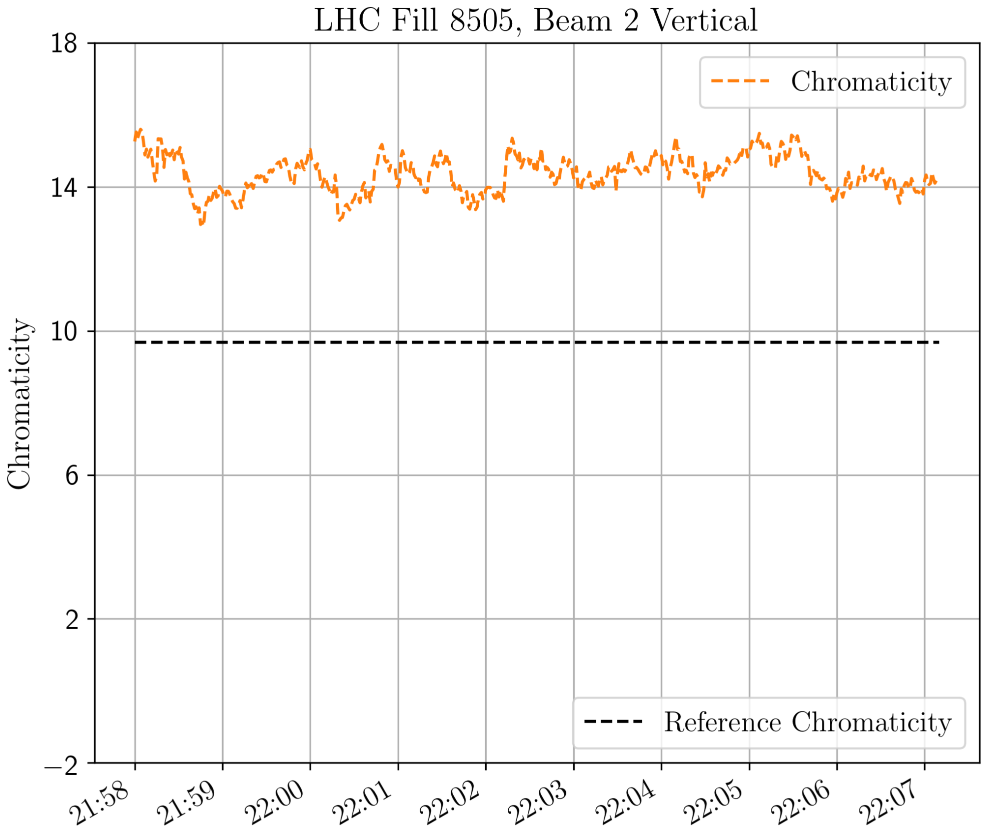

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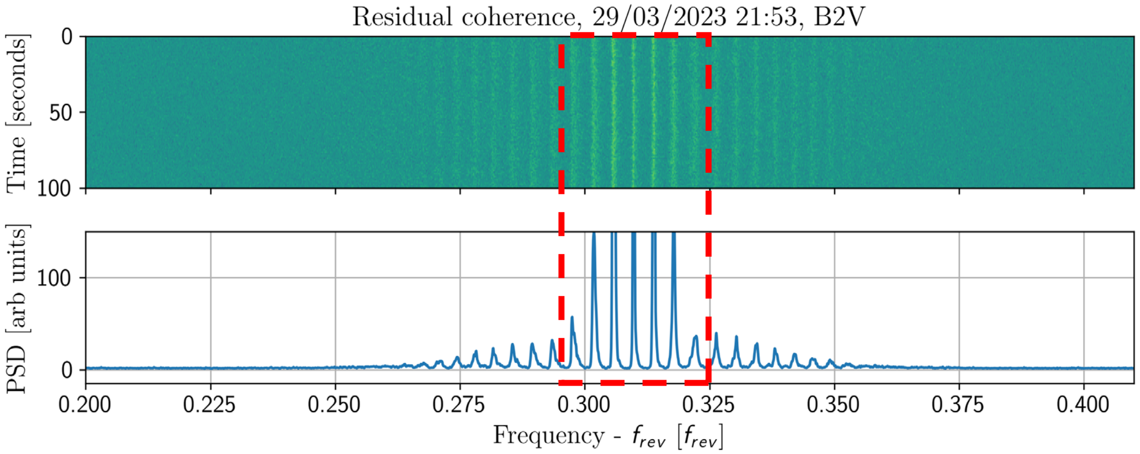

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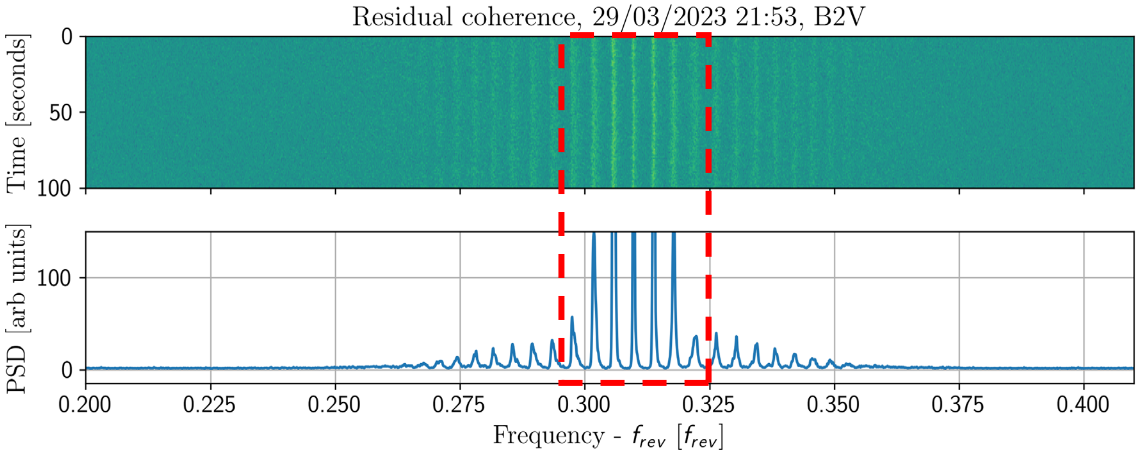

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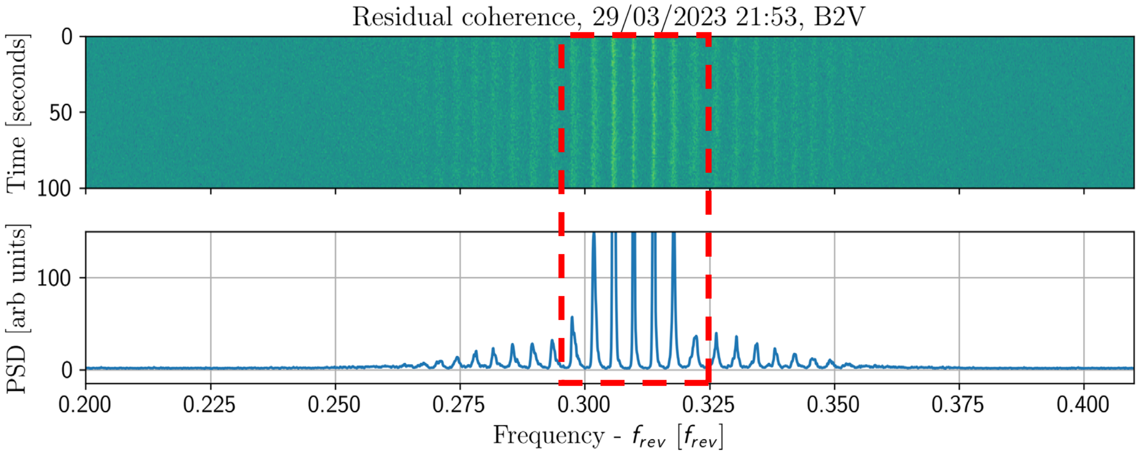

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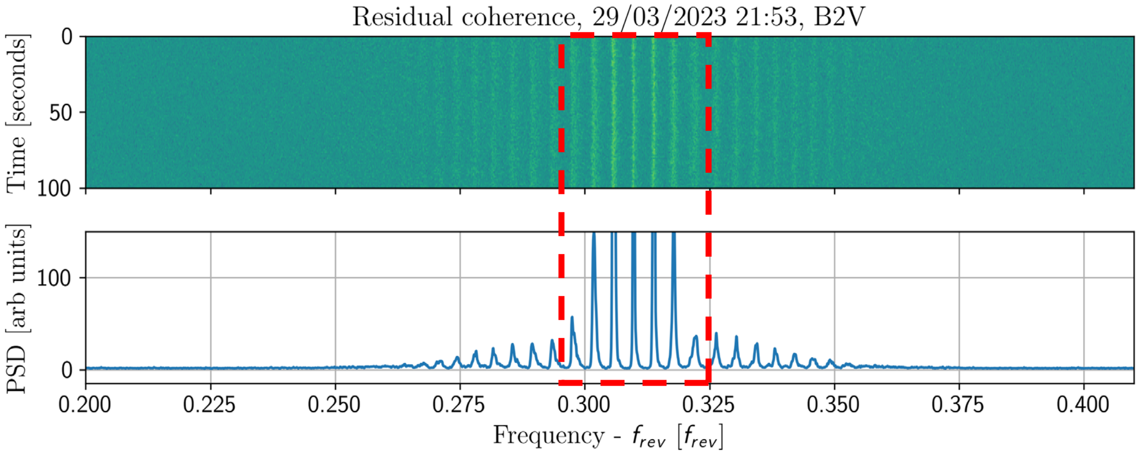

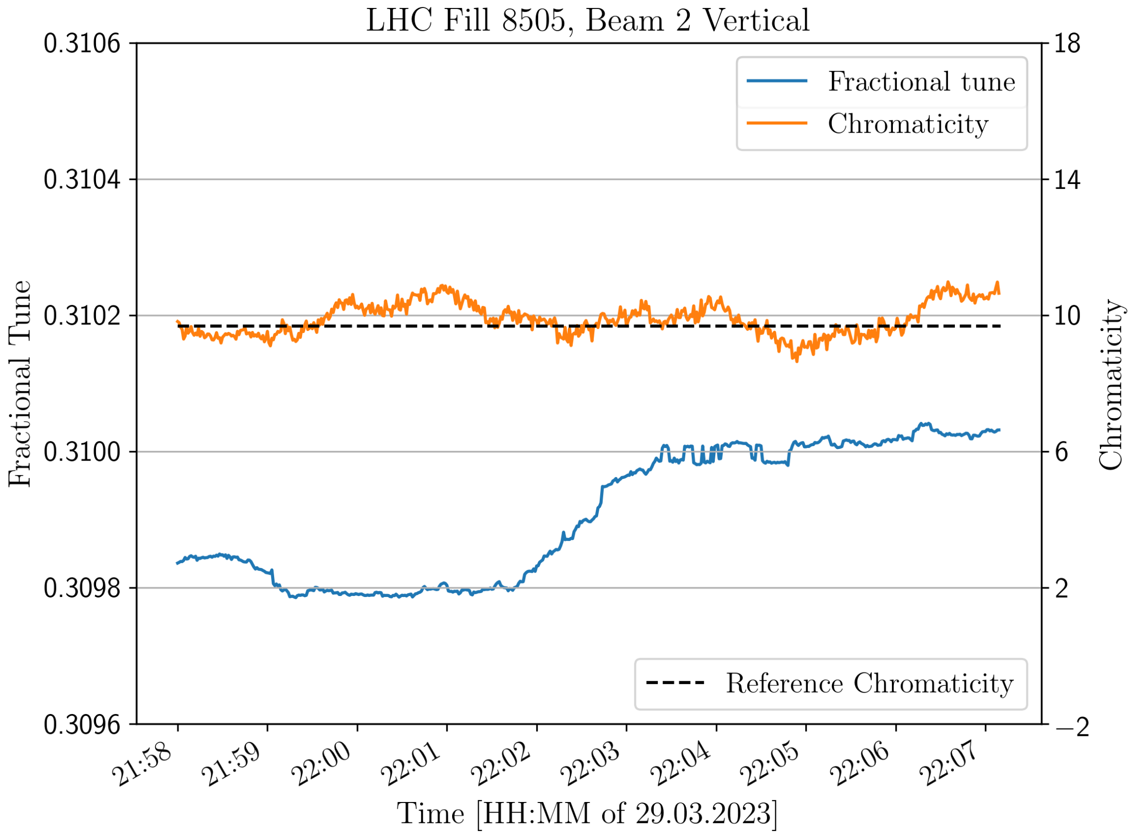

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LHC Schottky online signal analysis pipeline

Obtain a Schottky spectrum

→

Get the distribution of **synchrotron amplitudes** from longitudinal bunch profile

→

C. Lannoy et al., WEP035, IBIC'23

Determine valid frequency bins:
1) Statistical properties
2) Correlation between planes

↓

Calculate nominal $\mathbf{f_s}$ based on RF voltage and longitudinal Schottky spectrum

←

Calculate **betatron tune** using MD algorithm

←

Determine **chromaticity** by fitting the transverse spectrum

Implementation in the final stage of development, planned be in use in the end of 2023

Further plans

Instrument & Beam:

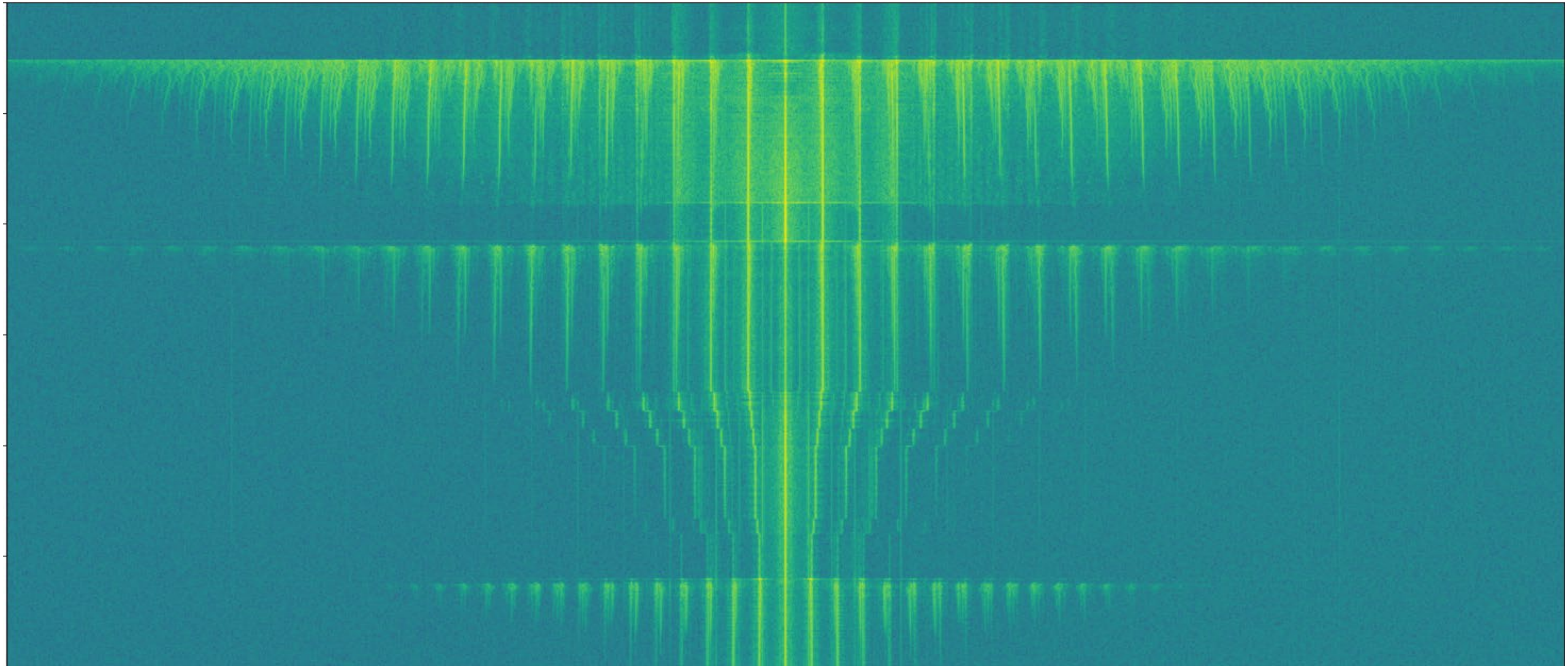
- Full automation of Schottky signal analysis, applying both "traditional" estimates and spectral fitting procedures
- Investigation on the source of coherent component in Schottky spectra: effects of abort gap cleaning, orbit feedback, ...

Theory:

- Quantify the effect of octupoles on Schottky spectra, assess their impact on tune and chromaticity estimates
- Study the modifications of Schottky spectra due to the beam-coupling impedance

See C. Lannoy et al., HB'23 THBP47

Thank you for your attention!



RF manipulations, 24.04.2023, LHC Beam 2 Horizontal