Periodic solution for transport of intense and coupled coasting beams through quadrupole channels

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Outline

- Introduction
- Periodic solution with space charge and coupling
- Benchmarking with multi-particle tracking
- Conclusion

Introduction

$$C := \begin{bmatrix} \langle xx \rangle & \langle xx' \rangle & \langle xy \rangle & \langle xy' \rangle \\ \langle x'x \rangle & \langle x'x' \rangle & \langle x'y \rangle & \langle x'y' \rangle \\ \langle yx \rangle & \langle yx' \rangle & \langle yy \rangle & \langle yy' \rangle \\ \langle y'x \rangle & \langle y'x' \rangle & \langle y'y \rangle & \langle y'y' \rangle \end{bmatrix}$$

eigen-emittances: determined by ten independent moments

$$\varepsilon_{1,2} = \frac{1}{2}\sqrt{-\mathrm{tr}\,(CJ)^2 \pm \sqrt{\mathrm{tr}^2\,(CJ)^2 - 16\det(C)}}$$

4d-rms-emittance:

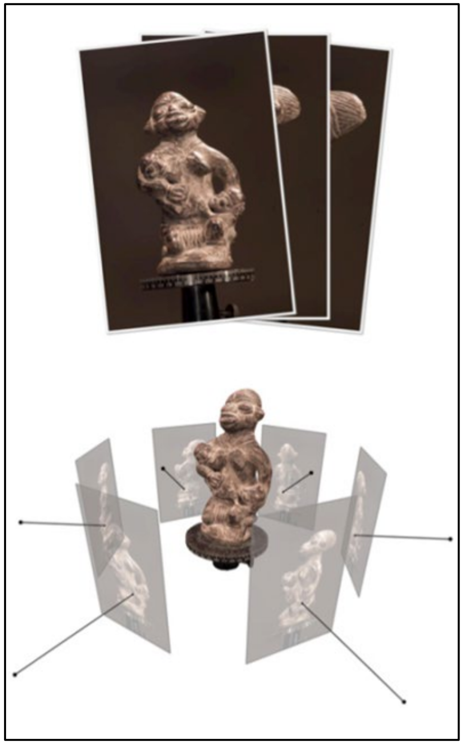
$$\varepsilon_{4d} = \sqrt{\det|C|} = \varepsilon_1 \cdot \varepsilon_2$$

$$\varepsilon_u = \sqrt{\langle uu \rangle \langle u'u' \rangle - \langle uu' \rangle^2}$$

projected rms-emittances: determined by three independended moments each

$$J = M^{\mathrm{T}} \cdot J \cdot M\,, \qquad J := \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

symplectic transformations preserve respective emittance



Introduction

- as of today:
  - periodic rms-envelopes for intense beams can be determined if there is no hor./ver. coupling
  - periodicity → both transverse envelopes match the external lattice periodicity

- hor./ver. coupled beams:
  - periodicity must include all 10 rms-moments; just two envelopes are not sufficient
  - periodic envelopes do not imply periodic <xy>, <xy'>, <x'y>, ….

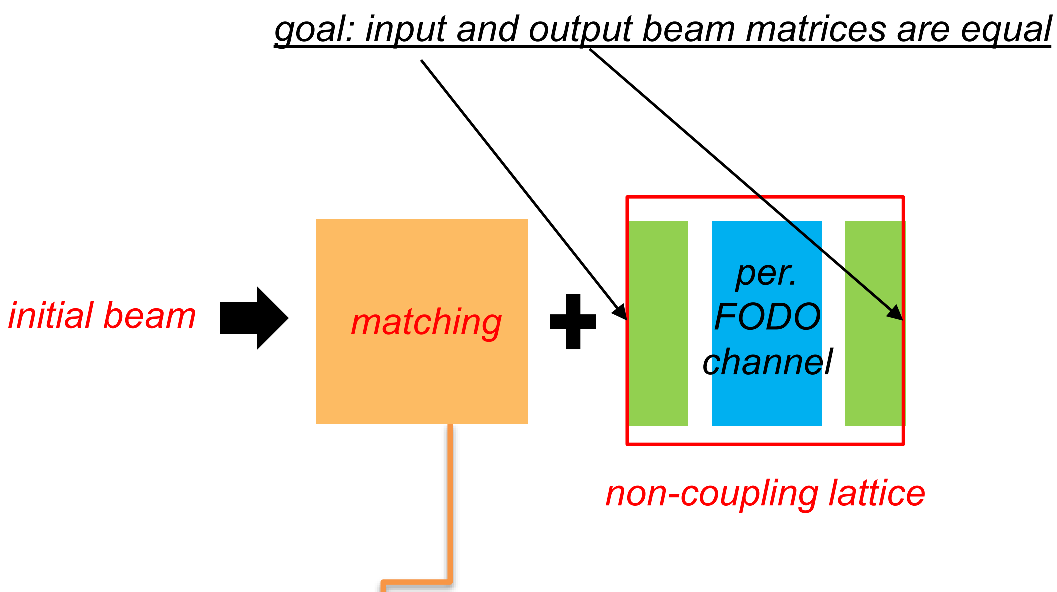
- hor./ver. coupled (spinning) beams may serve to suppress emittance growth
  - see recent PRAB:

Effects of beam spinning on the fourth-order particle resonance of 3D bunched beams in high-intensity linear accelerators

Yoo-Lim Cheon, Seok-Ho Moon, and Moses Chung*

- to fully explore this potential (for instance) → issue of full 4d-periodicity to be addressed

Concept of obtaining periodic solution



matching transport matrix couples:

$$\Re\left(m_1, m_2, \ldots, m_{16}\right) = \begin{bmatrix} m_1 & m_2 & m_3 & m_4 \\ m_5 & m_6 & m_7 & m_8 \\ m_9 & m_{10} & m_{11} & m_{12} \\ m_{13} & m_{14} & m_{15} & m_{16} \end{bmatrix}$$

method:

- impose R to be symplectic
- impose det |R| = 1
- vary its elements such that …
- beam matrix matches FODO channel

Concept of obtaining periodic solution

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for zero current:

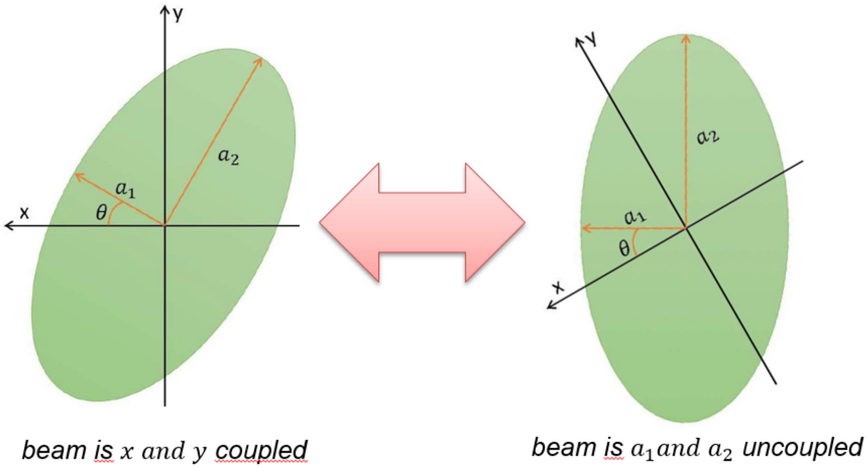
- method is straight forward since FODO transport matrix is determined by lattice only
- FODO-lattice does not depend on beam shape
- method seems an overkill
- however, apply it in order to adapt it for intense beams

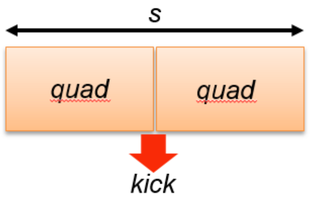
for current:

- use rms-tracking (KV) with space charge to calculate an effective, linear, and symplectic FODO-transport-matrix
- this FODO-matrix depends on current and initial beam shape
- however, it is a matrix and one can apply the "method" as seen in the following

Space-charge kicks with coupling

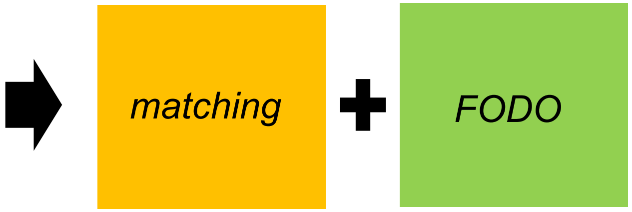
- rms-tracking w/ space charge (sc) for uncoupled beams is state-of-the-art
- issue with coupled beams are non-upright space sc forces
- solution:
  - rotate beam in space
  - calculate upright forces
  - re-rotate forces
  - apply sc-kicks, modeled by matrices
- rms-tracking is sequence of matrices: drift, quad, sc-kick
- with these tools, the effective FODO-matrix can be calculated for a given beam shape at the channel‘s entrance
- FODO-transport-matrix is effectivelly used as for the zero-current matching





Periodic solution with current

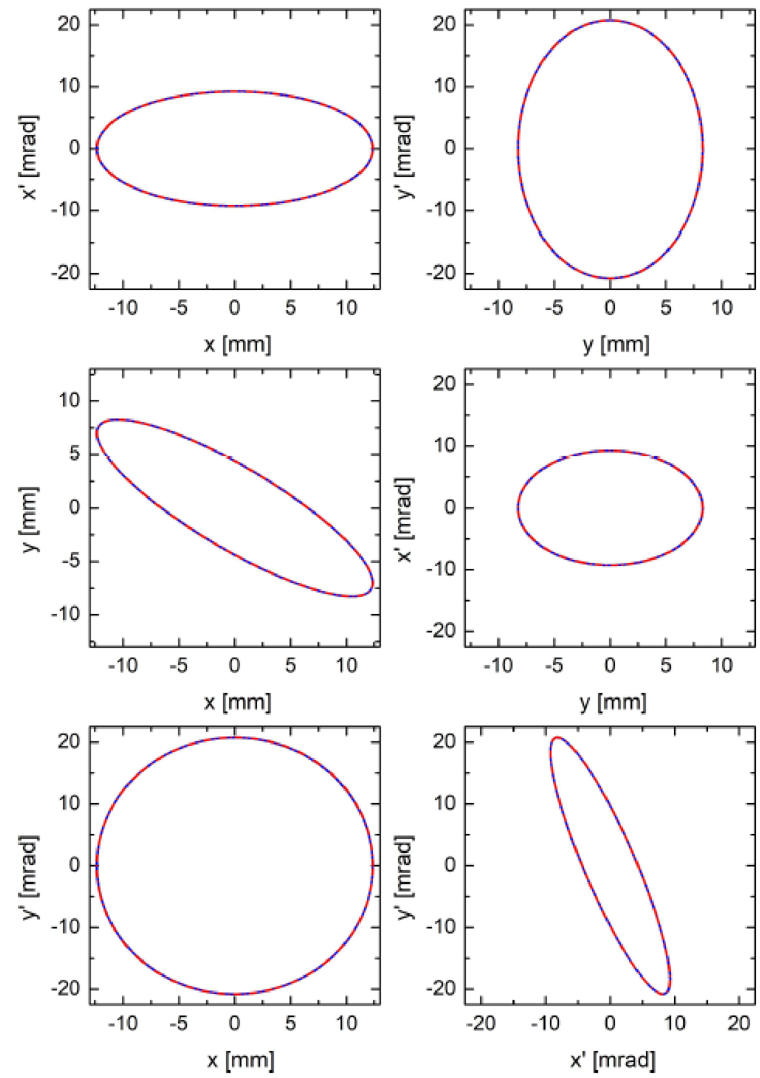
- match beam_0 for zero-current; channel modeled by R_0
- place beam_0 in front of the channel
- switch on current and rms-track beam_0 through channel
- this tracking is modeled by an effective FODO-transport matrix → store it as R_1
- re-match initial beam to the channel (modeled by R_1) such that beam matrix at entrance and exit are equal
- this will deliver new beam_1 in front of the channel
- rms-track beam_1 through channel → store effective FODO-matrix as R_2
- re-match initial beam to the channel (modeled by R_2) such that beam matrix at entrance and exit are equal
- keep doing the loop until $R_{n+1} \approx R_n$ → done!



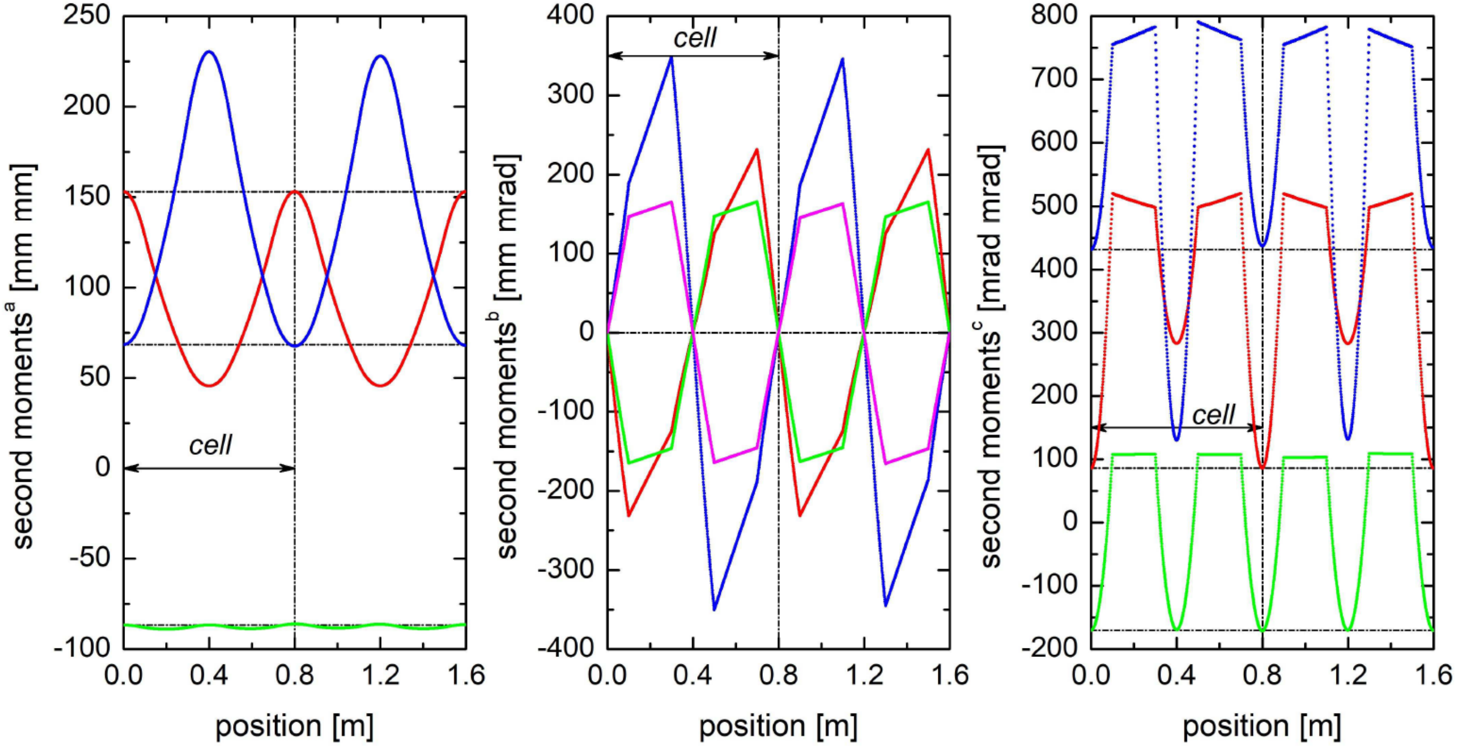
Result

- algorithm converges within less than 10 loops
- has been applied to a regular quad channel,
  however even tilted quads or solenoids may be used
- example:
  - protons dc
  - 150 keV
  - 10 emA
  - tune depression 12%
  - → 4 loops

FODO entrance
FODO exit
rms-tracked

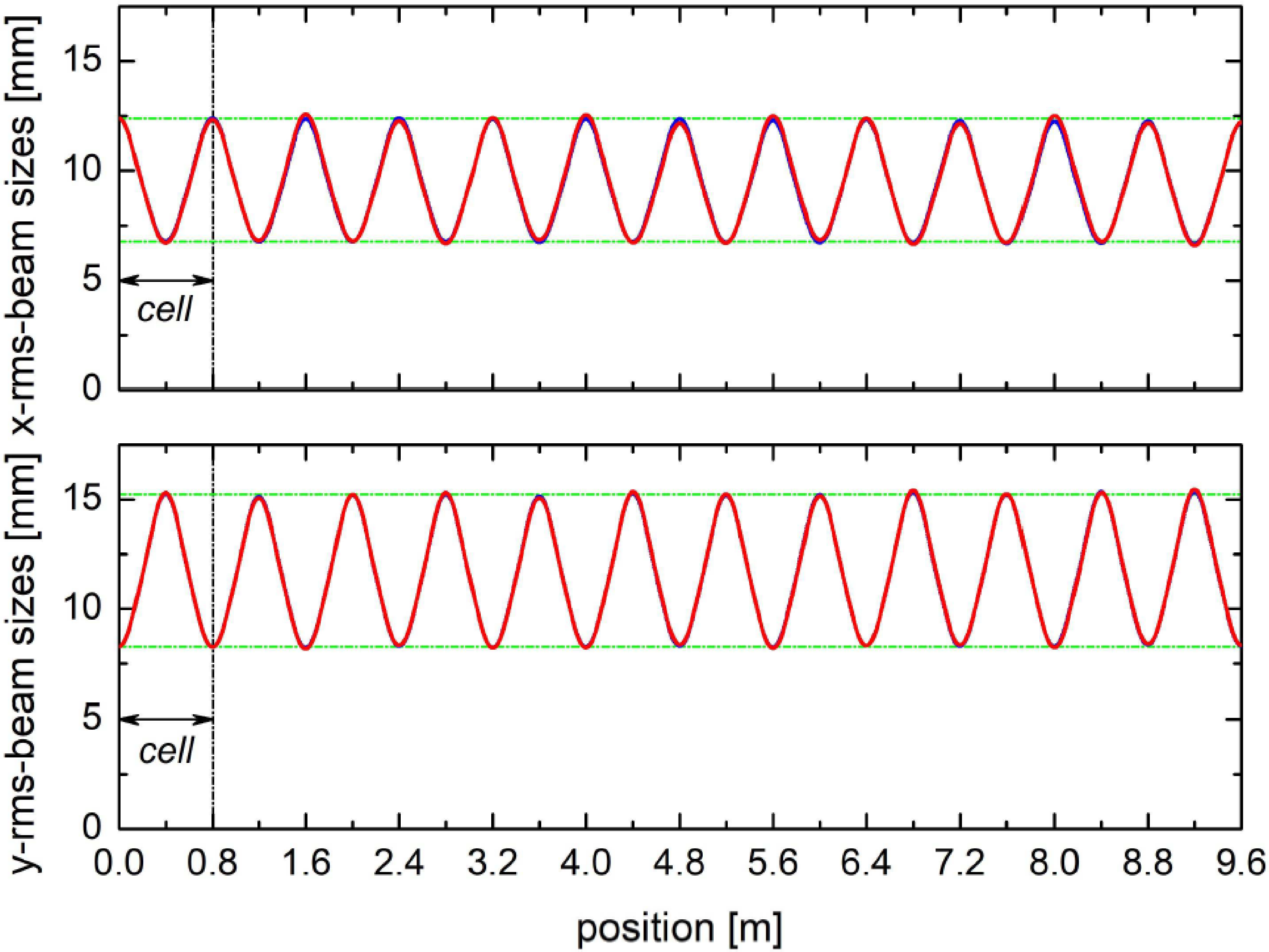


Result



- left: <xx>, <yy>, and <xy> along two FODO cells
- centre: <xx>, <yy’>, <x’y>, and <xy’>
- right: <x’x’>, <y’y’>, and <x’y’>

Benchmarking with Gaussian (BEAMPATH)



rms-tracking
Gaussian BEAMPATH

Conclusion

- Full cell-to-cell 4D-matching can be achieved for a coupled beams with considerable space charge forces
- Accomplished by rms-tracking of coupled beam with KV-distribution combined with dedicated iterative procedure of tracking and re-matching
- Benchmarking with an initial Gaussian distribution revealed that the method works very well
- Tool for systematic investigations of intense, coupled beam transport along periodic lattices
- One appealing application is imposing well defined spinning to beams
- Details: https://arxiv.org/abs/2309.11277

Thank you !