LONGITUDINAL LOSS OF LANDAU DAMPING IN DOUBLE HARMONIC RF SYSTEMS BELOW TRANSITION ENERGY

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♦ Abstract

Landau damping plays a crucial role in ensuring single-bunch stability in hadron synchrotrons. In the longitudinal plane, loss of Landau damping (LLD) occurs when a coherent mode of oscillation moves out of the incoherent synchrotron frequency band. The LLD threshold is studied for a purely inductive impedance below transition energy, specifically considering the common case of double harmonic RF systems operating in counter-phase at the bunch position. The additional focusing force due to beam-induced voltage distorts the potential well, ultimately collapsing the bucket. The limiting conditions for a binomial particle distribution are calculated. Furthermore, the contribution focuses on the configuration of the higher-harmonic RF system at four times the fundamental RF frequency operating in phase. In this case, the LLD threshold shows a non-monotonic behavior with a zero threshold where the derivative of the synchrotron frequency distribution is positive. The findings are obtained employing semi-analytical calculations using the MELODY code.

1. Main definitions

- **Loss of Landau damping (LLD) [1]** occurs when coherent mode frequency Ω moves outside the incoherent frequency band → $\Omega = \min[f_s(\phi)]$

$$\lambda(\phi) = \lambda_0\left(1 - \frac{U_{tot}(\phi)}{U_{tot}(\phi_{max})}\right)^{\mu+1/2}$$

Line density — Total potential [2] — Maximum phase deviation in the bunch

$$U_{tot}(\phi) = U_{rf}(\phi) + \zeta\, \mathrm{Im}Z/k\,[\lambda(\phi) - \lambda_0]$$

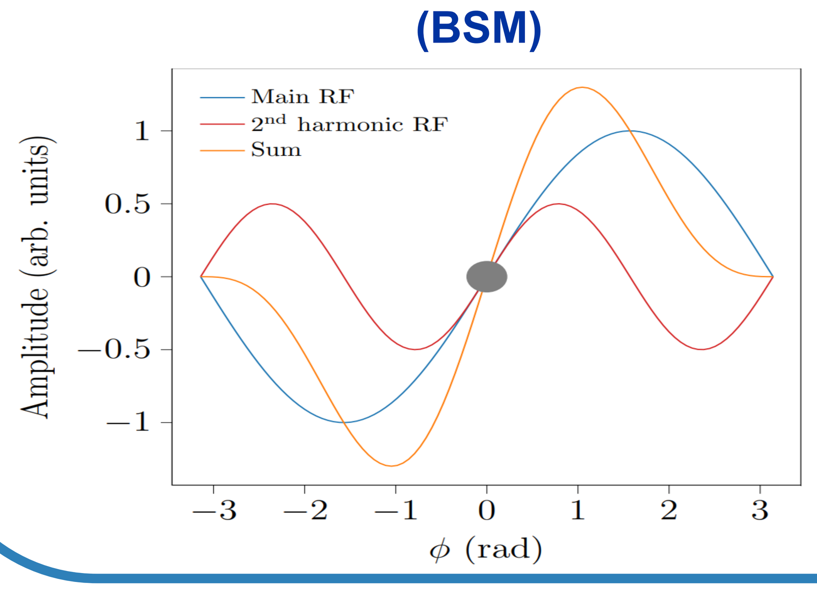
$$\zeta = 2\frac{\pi N_p h\, f_{rf}}{V_0}$$

Number of particles

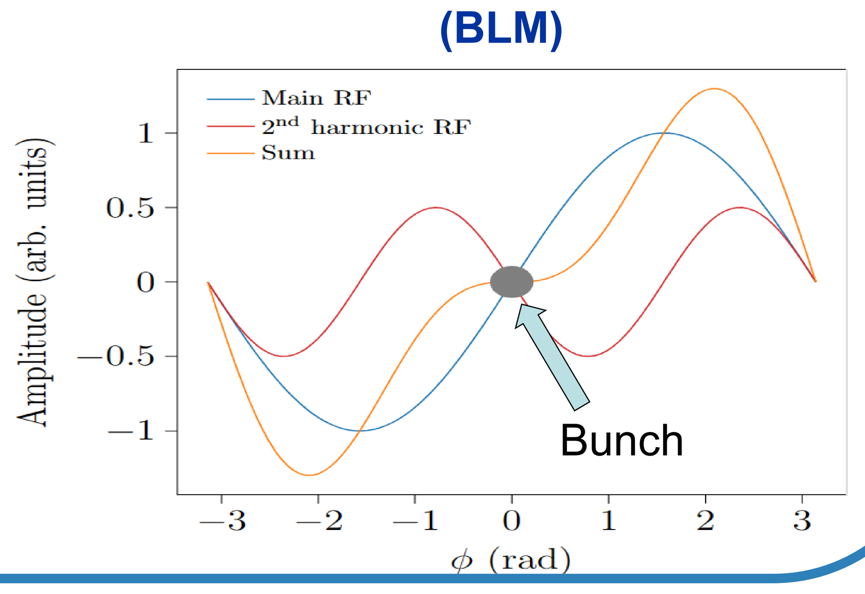
Main accelerator parameters

Parameter	Unit	Value
Circumference, $2\pi R$	m	26658.86
Beam energy, E	TeV	0.45
Main harmonic number, h		35640
Main RF frequency, f_{RF}	MHz	400.79
RF voltage at fundamental harmonic, V_0	MV	6
Effective impedance, $\mathrm{Im}Z/k$	Ω	-0.07

RF voltage in bunch shortening mode (BSM)



RF voltage in Bunch lengthening mode (BLM)



2. Upper limit intensity due to potential well distortion

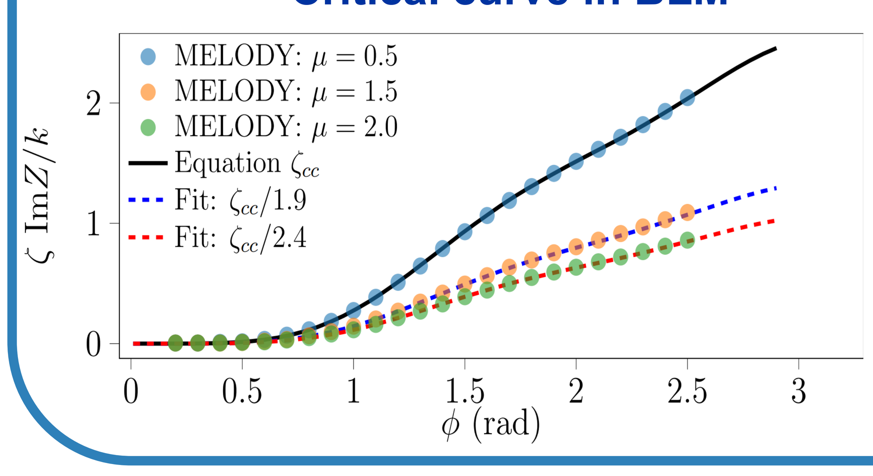
- The beam induced voltage acts as a **focusing force** → The total potential well shrinks with the intensity ultimately collapsing the buckets.
- For **binomial particle distribution** with $\mu = 1/2$, the exact solution for these **critical** intensities is derived:

$$\zeta_{cc} = \left(U_{rf}(\phi_{max}) - U_{tot}(\phi_{max})\right) - \int_{-\phi_{max}}^{\phi_{max}} \left(1 - \frac{U_{rf}(\phi)}{U_{rf}(\phi_{max})}\right) d\phi$$

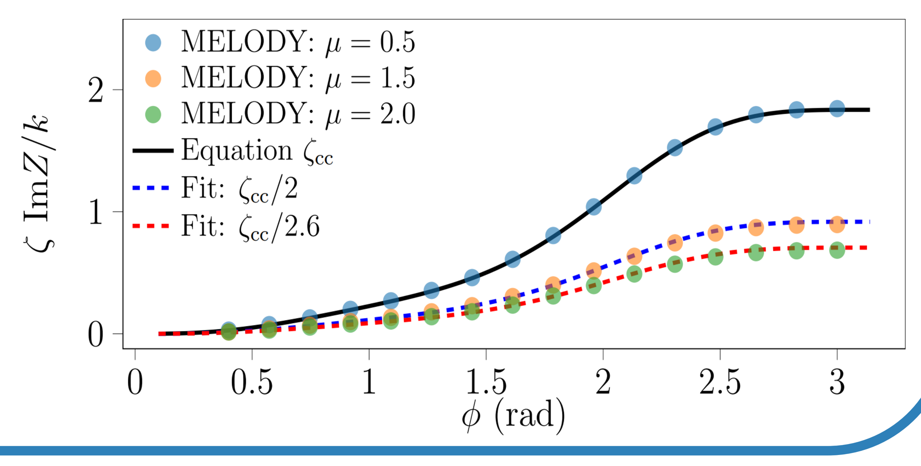
→ Holds for any RF potential

- In the case of $\mu \neq 1/2$, only empirically fitted functions are proposed for the moment
- Excellent agreement with semi-analytical calculation (**MELODY** [3])

Critical curve in BLM



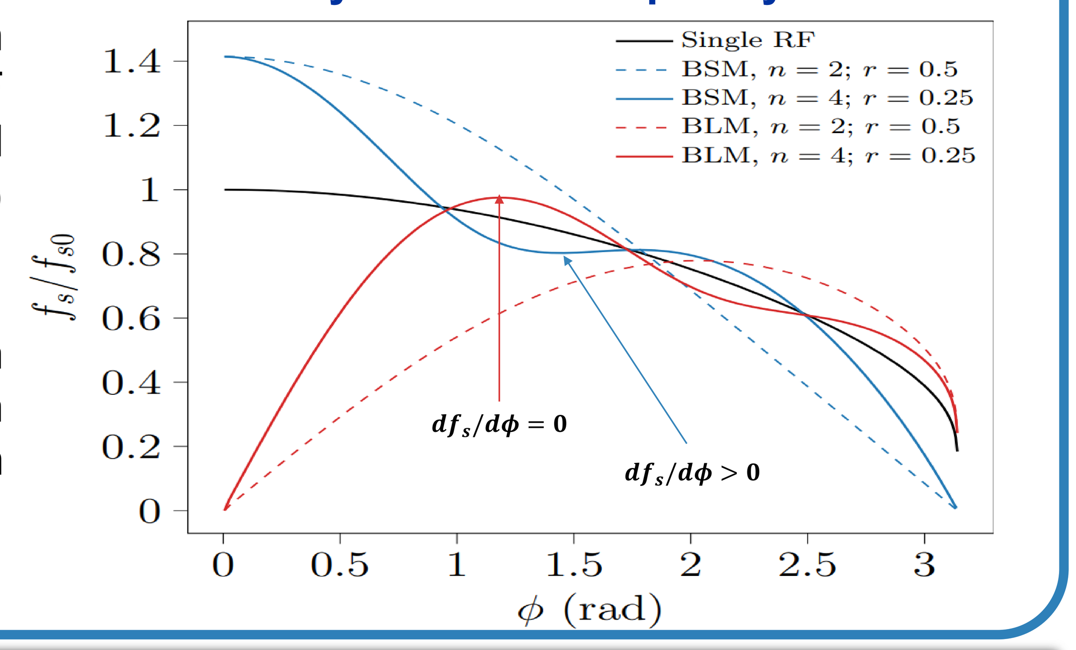
Critical curve in BSM



3. Synchrotron frequency distribution

- Synchrotron frequency distribution changes, $f_s(\phi)$ changes with RF configuration, e.g., voltage (r) and harmonic number ratio (n) between the RF systems.
- Enlarging the synchrotron frequency spread is a common technique to enhance beam stability

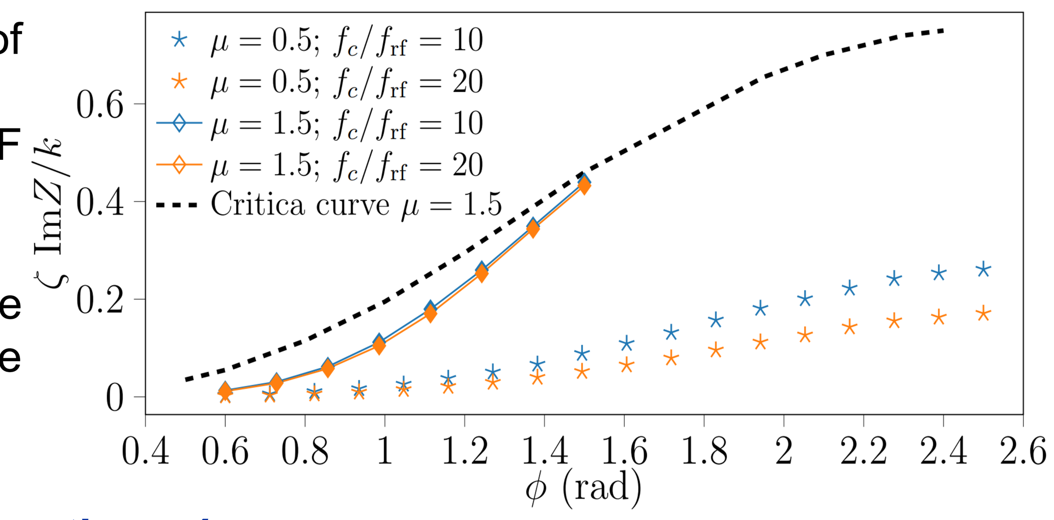
Normilzed synchrotron frequency distribution



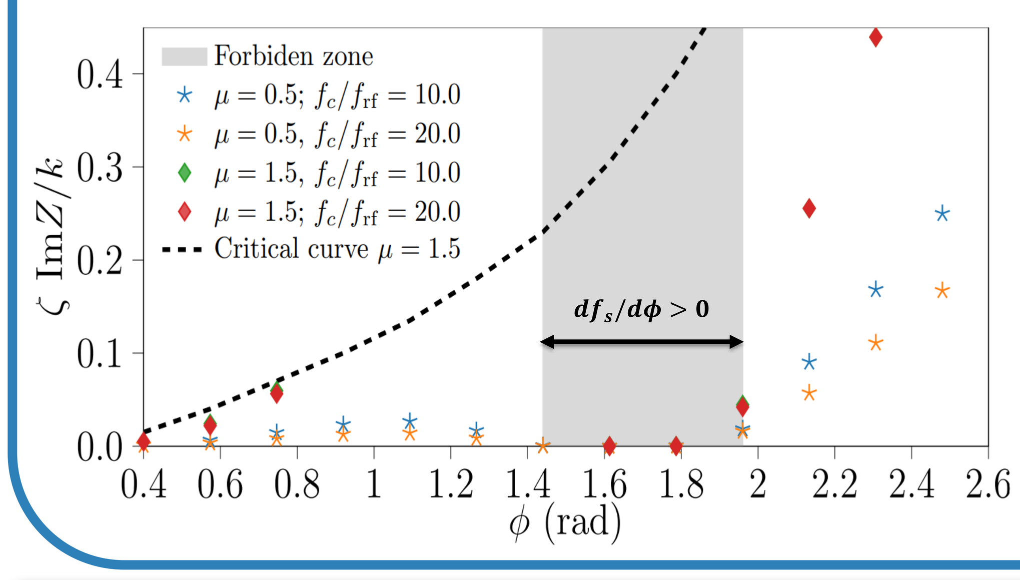
4. The LLD in bunch shortening mode

- LLD threshold is independent of the cutoff frequency f_c for $\mu > 1$ → in agreement with single RF case [4]
- For larger bunch lengths, the threshold disappears beyond the critical curve

LLD in BSM for harmonic number ratio *n*=2



LLD in BSM for harmonic number ratio *n*=4

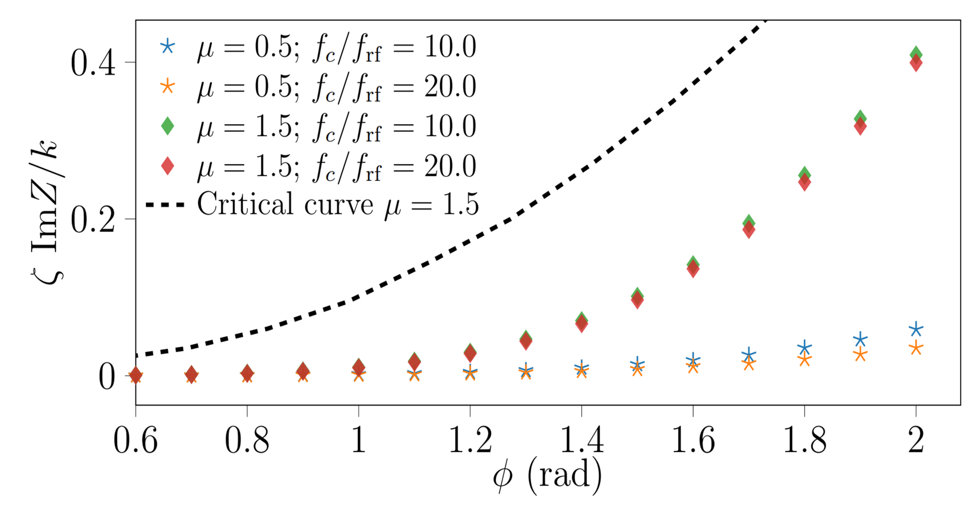


- The LLD threshold is not a monotonic function of the max phase deviation configuration for $n = 4$
- Threshold is significantly affected by the presence of $df_s/d\phi > 0$

5. LLD in bunch lengthening mode

- Monotonic function of the maximum phase deviation
- The LLD threshold is independent of the cutoff frequency for $\mu > 1$

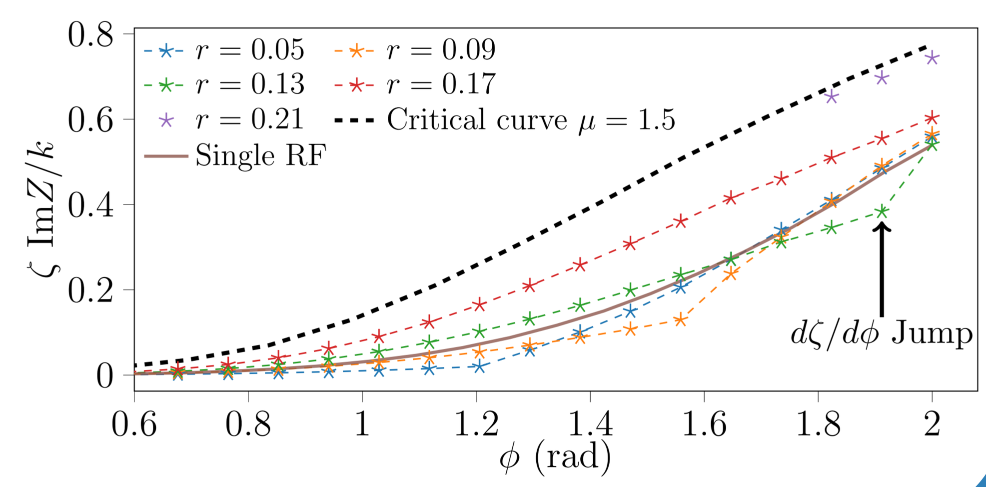
LLD in BLM for harmonic number ratio *n*=2



- Jump of the derivative $d\zeta/d\phi$ → The $\min[f_s(\phi)]$ moves from the center to the tail of the bunch

→No longer dependent on the impedance cutoff frequency

LLD in BLM for harmonic number ratio *n*=4



♦ Conclusions

The loss of Landau damping in synchrotrons is a critical condition that can lead to beam instabilities and particle loss. The present study focuses on the LLD threshold within the common configuration of BSM and BLM when inductive impedances below transition energy (or capacitive above) are involved. The limiting intensity for a binomial particle distribution was calculated analytically and compared with results from the semi-analytical code MELODY. In BSM, loss of dependency on the cutoff frequency in the LLD threshold agrees with the prediction showing a non-monotonic behavior. As expected, regions where $df_s/d\phi > 0$ led to a vanishing LLD threshold at any intensity. On the contrary, in BLM, the LLD threshold results in a monotonic function.

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References
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[3] I. Karpov, Matrix Equations for LOngitudinal beam DYnamics (MELODY) code, https://gitlab.cern.ch/ikarpov/melody.
[4] I. Karpov, T. Argyropoulos, and E. Shaposhnikova, "Thresholds for loss of landau damping in longitudinal plane," Phys. Rev. Accel. Beams, vol. 24, p. 011002,1, 2021. doi:10.1103/PhysRevAccelBeams.24.011002.