A linearized Vlasov method for the study of transverse e-cloud instabilities

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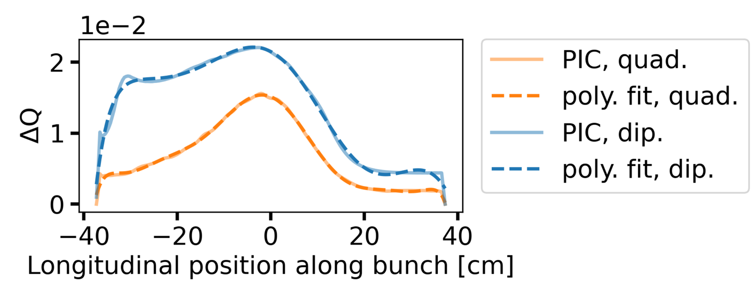
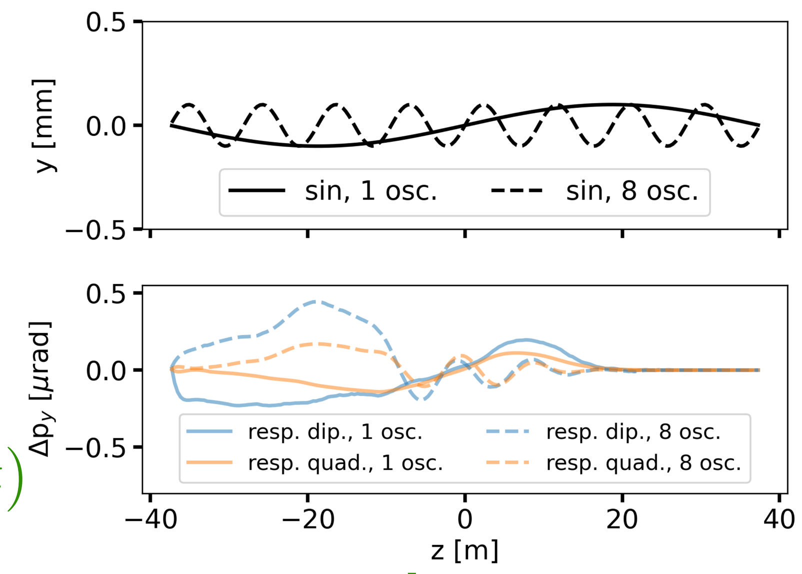
Abstract

Using a Vlasov approach, electron cloud driven instabilities can be modeled to study beam stability on time scales that conventional Particle In Cell simulation methods cannot access. The Vlasov approach uses a linear description of electron cloud forces that accounts for both the betatron tune modulation along the bunch and the dipolar kicks from the electron cloud. Forces from electron clouds formed in quadrupole magnets as well as dipole magnets have been expressed in this formalism. In addition, the Vlasov approach can take into account the effect of chromaticity. To benchmark the Vlasov approach, it was compared with macroparticle simulations using the same linear description of electron cloud forces. The results showed good agreement between the Vlasov approach and macroparticle simulations for strong electron clouds, with both approaches showing a stabilizing effect from positive chromaticity. This stabilizing effect is consistent with observations from the LHC.

Simulation Model

- The e-cloud forces are divided into **quadrupolar forces** and **dipolar forces**.
- The quadrupolar forces are described as a detuning along the bunch.
- The dipolar forces are described using a set of responses calculated from sinusoid beam distortions passing through an e-cloud.
- $\psi(x, x', z, \delta; t)$ represents the particle density of the bunch.
- **The linearized Vlasov equation** (1st order) ([6]) :





$$\frac{\partial \Delta\Psi}{\partial t} - \omega_0(Q_{x0} + \Delta Q(r,\phi))\frac{\partial \Delta\Psi}{\partial \theta_x} + \omega_s \frac{\partial \Delta\Psi}{\partial \phi} = -\frac{\eta g_0(r)}{\omega_s m_0 \gamma}\frac{df_0}{dJ_x}\sqrt{\frac{2J_x R}{Q_{x0}}}\sin\theta_x F_x^{coh}(z,t)$$

- Solve for $\Delta\psi$ by using ansatz: $\Delta\psi(J_x, \theta_x, r, \phi; t) = e^{j\Omega t}\Delta\psi(J_x, \theta_x, r, \phi)$
- The Vlasov equation is now reduced to an **eigenvalue problem** [10] [11]

$$\Delta x(z) = \sum_{n=0}^{\infty} a_n k_n(z)$$

$$\bar{x} = \sum_{n=0}^{\infty} a_n h_n(z)$$

- The e-cloud forces are introduced **in the same manner** in MP simulations utilizing **PyHEADTAIL** as a tracker for benchmarking.

transverse phase space coordinates:

$$\begin{cases} x = & \sqrt{\frac{2J_x R}{Q_{x0}}}\cos\theta_x \\ x' = & \sqrt{\frac{2J_x Q_{x0}}{R}}\sin\theta_x \end{cases}$$

(J_x = horizontal action)

Longitudinal phase space coordinates:

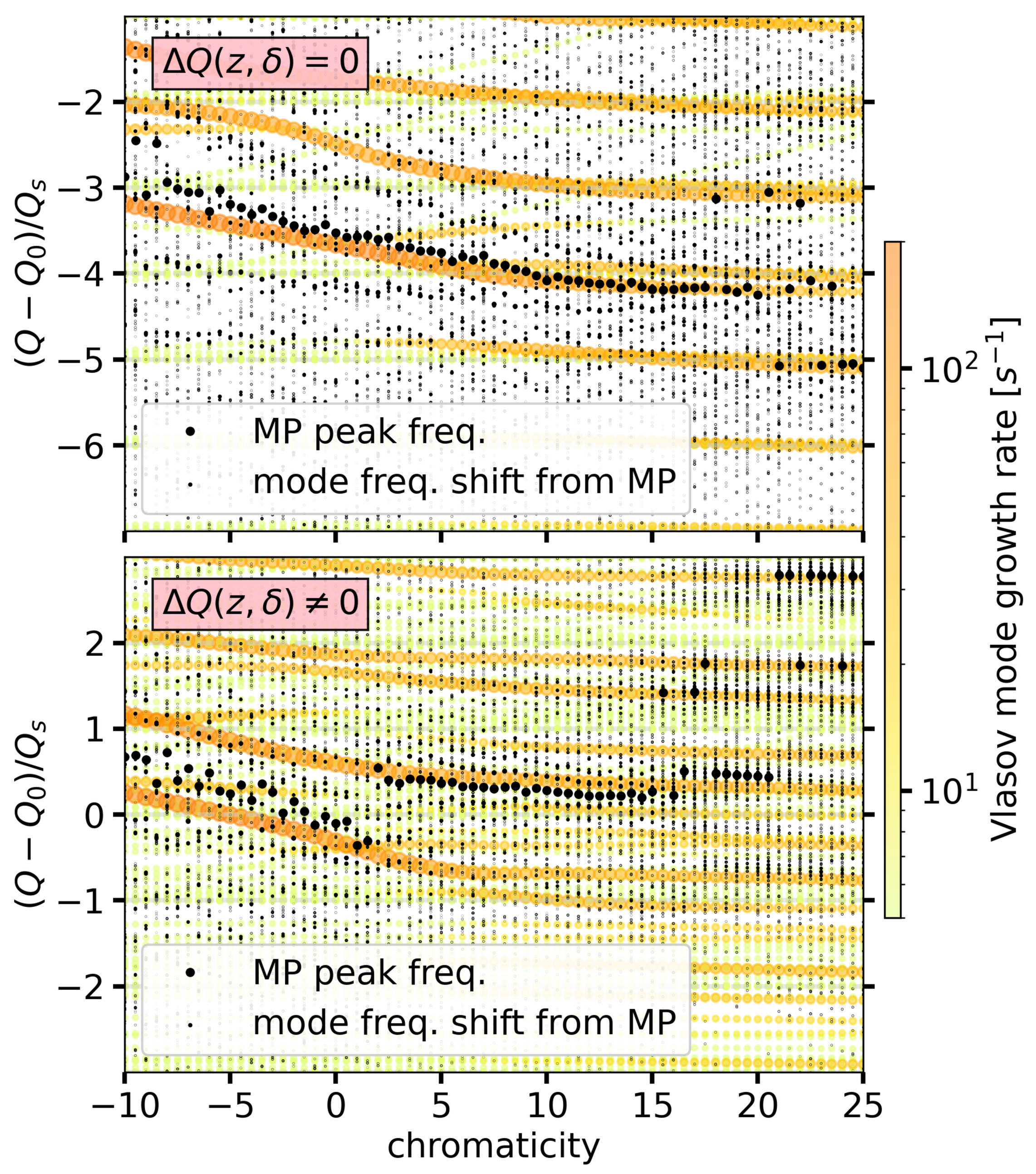
$$\begin{cases} z = & r\cos\phi \\ \delta = & \frac{\omega_s}{v\eta} r\sin\phi \end{cases}$$

Q_{x0} = unperturbed betatron tune

R = the accelerator radius
v = the velocity.
η = slippage factor
m_0 = particle rest mass
γ = the relativistic factor

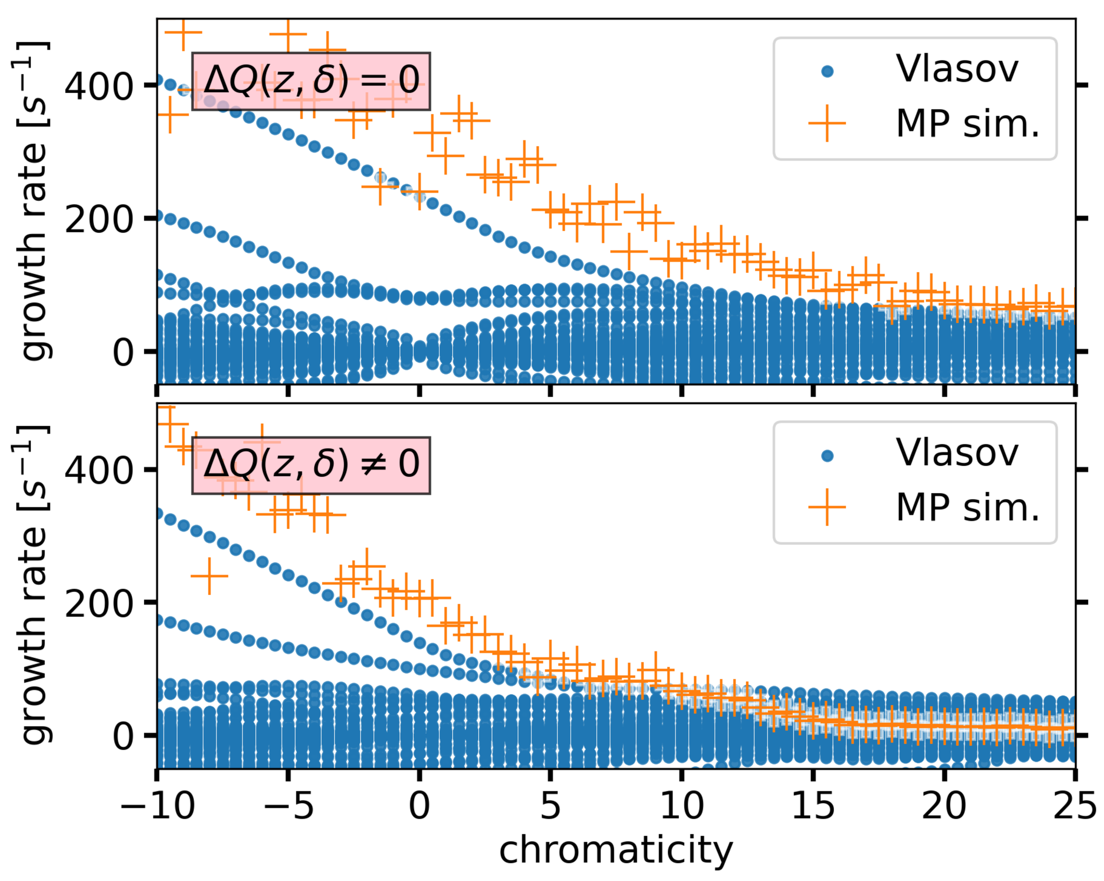
ω_0 = revolution angular frequency
ω_s = synchrotron angular frequency
The unperturbed bunch distribution ψ_0 has been factorized as:
$\psi_0 = \frac{\eta v}{\omega_0} f_0(J_x) g_0(r)$.

Results

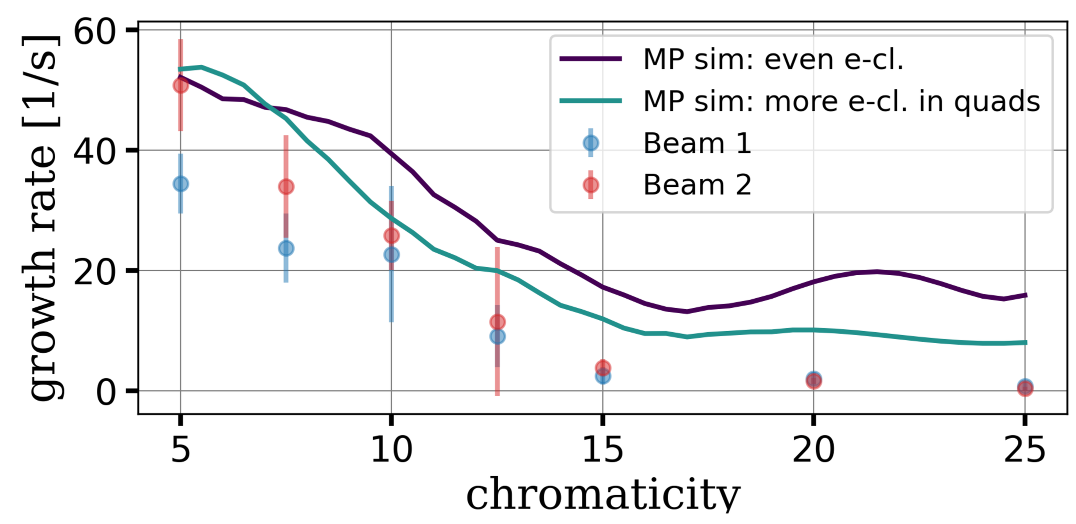


- The linearized Vlasov equation yields a **set of possible modes**, each with a complex angular frequency Ω.
- The Vlasov equation is then **solved for various chromaticities.**
- In the **Vlasov** simulations, the frequency shift of the **strongest modes matches** the frequency shift obtained from the **MP simulations**.
- When a **detuning from e-cloud is present**, several **weak Vlasov modes** between the Q_s lines are **visible for high chromaticity**
- Some of these weak modes are **visible also** in the **MP simulations**.

- The **imaginary** part of each complex frequency Ω corresponds to **the growth rate** of the corresponding mode.
- The growth rate from MP simulations **agrees with the most unstable Vlasov mode** for chromaticities smaller than Q' = 15.
- The Vlasov modes predict weak instabilities not visible in the MP simulations when Q' > 15 **and** $\Delta Q(z, \delta) \neq 0$.
- This indicates the presence of **damping mechanisms that are not captured by the linearized Vlasov equation.**



Measurements



- **Instability measurements** were conducted at the **LHC** under conditions with **strong e-cloud**.
- The measured growth rates **decrease with chromaticity**.
- **MP** simulations using the **Vlasov e-cloud formalism of forces** exhibit a **similar dependence on chromaticity**.

Conclusions

- **E-cloud** forces have been expressed in a **dedicated Vlasov formalism** for e-cloud forming in both **dipoles** as well as **quadrupole magnets**.
- Results from the Vlasov approach are compared against MP simulations using the **same description of e-cloud forces.**
- The Vlasov approach showed **good agreement** with MP simulations for chromaticities < 15.
- The **Vlasov** approach predicts the existence of **weak instabilities that are not observed in MP simulations**, indicating the presence of **damping mechanisms** that are **not captured** by the linearized Vlasov equation.
- Simulations predict a **stabilizing effect of chromaticity**, which is **also observed** in **measurements**.