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SYNCHRONOUS PHASE AND TRANSIT TIME FACTOR

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Abstract

Synchronous phases (φ_s) and transit time factors (T) are the key parameters for linac designs and operations. While the couple (φ_s, T) is still our way of thinking the longitudinal beam dynamics, it is important to have in mind that the original "Panofsky definition" of these parameters is no longer valid in the case of high accelerating gradients leading to high particle velocity changes. In this case, a new (φ_s, T) definition allowing to keep both acceleration and longitudinal focusing properties is proposed. Examples are given in the SPIRAL2 linac case.

INTRODUCTION

Synchronous phases (φ_s) and transit time factors (T) are by far THE main parameters for linac designs and operations. The choice of the synchronous phase evolution is a compromise between linac efficiency (higher acceleration then lower linac length and cost at higher φ_s) and linac longitudinal acceptance (reduced acceptances then higher risk of beam losses at higher φ_s). The choice of the transit time factors linked to the accelerating cell or cavity geometries is also a compromise between linac efficiency (higher acceleration then lower linac length and cost at higher T), technical risk (higher peak fields at higher T) and construction / operation complexity (more β-families for a linac design with higher T).

The couple (φ_s, T) has been defined by Panofsky in the framework of Drift Tube Linac cells [1] : φ_s is "the rf phase by which the synchronous particle crosses the electrical center of the gap relative to the time at which the electric field reaches its crest value", and T is a coefficient such that the on-axis energy gain is

$$\Delta W = q\, E_0 L \;\, \mathrm{T}\, cos(\varphi_s) \tag{1}$$

with q the particle charge and $E_0 L$ the on-axis gap voltage

$$E_0 L = \int E_z(z, r = 0)\, dz \tag{2}$$

T is then the coefficient introduced to compute the energy gain taking into account the phase evolution during the gap traversal.

Using the smooth approximation, the Panofsky equation (1) also allows to write the equations of motion of the phase oscillations ($\delta\varphi$) around the synchronous particle.

$$\frac{d^2\delta\varphi}{dz^2} + \sigma_{0l}{}^2 \; \frac{\cos(\varphi s + \delta\varphi) - \cos(\varphi_s)}{-\sin(\varphi_s)} = 0 \tag{3a}$$

$$\frac{d^2\delta\varphi}{dz^2} + \sigma_{0l}{}^2\, \delta\varphi = 0 \tag{3b}$$

$$\sigma_{0l} = \sqrt{\frac{-2\pi q\, E_0\, T \sin(\varphi_s)}{m_0 c^2 \lambda\, \beta_s^3 \gamma_s^3}} \tag{3c}$$

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The nonlinear equation of motion (3a) allows to compute the large amplitude oscillations and the longitudinal acceptance defined by the so called separatrix. Its linear form (3b) shows that longitudinal focusing force for the small amplitude oscillations is function of the zero-current longitudinal phase advance σ_{0l} (3c).

To make it short, one can say that T $cos(\varphi_s)$ reflects the gap/cavity accelerating property through (1), and that T $sin(\varphi_s)$ reflects the gap/cavity longitudinal focusing property through (3c).

ENERGY GAIN AND CAVITY TUNING IN THE CASE OF HIGH ACCELERATING GRADIENT CAVITIES

As pointed out in Ref. [1], the Panofsky equation (1) is valid only if the fractional changes in velocity are small, a condition which is far to be fulfilled in linacs using superconducting cavities at low beta as in the SPIRAL2 case. Fig. 1 gives the example of a SPIRAL2 low beta QW cavity operated at 40% of its nominal field (2.6 MV/m) for deuterons at RFQ output energy (732 keV/u).

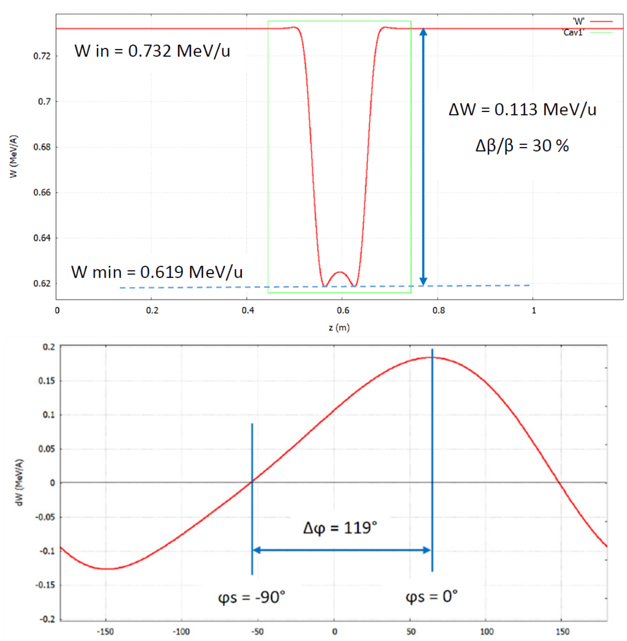

Figure 1: SPIRAL2 low beta cavity at 40% nominal field with deuterons at RFQ output energy. Top: energy evolution along a cavity tuned in buncher mode (φ_s = -90°). Bottom: energy gain for a 2π cavity phase scan.

Figure 1 shows that the fractional change in velocity is very large in this case (even in buncher mode) and that the energy gain has no longer a sinusoidal evolution function of the cavity phase. At the opposite of what is noted in [2], a phase angle defined with respect to the maximum energy gain in a given resonator can be far to be equal to the synchronous phase (-119° cavity phase shift to switch φ_s from

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0° to -90° in this example). A cavity phase tuning done making a synchronous phase shift with respect to the phase giving the maximum energy gain can then lead to important cavity phase errors with respect to the required ±1°.

An accurate cavity phase tuning must be done with respect to the buncher phase (φ_s = -90°, ΔW = 0) making a phase shift computed by a reference particle tracking in field map taking into account the cavity field and particle input energy. The choice "with respect to the buncher phase" instead of "with respect to the maximum energy gain" has two advantages: better accuracies on the cavity phase at the ΔW = 0 crossing and on the phase measurements when the beam is bunched [3].

(φ_s, T) IN THE CASE OF HIGH ACCELERATING GRADIENT CAVITIES

In the case of high accelerating gradients and multi-gap cavities there is no straightforward definition of the synchronous phase φ_s as done by Panofsky since the link between the cavity phase and φ_s must be computed making a reference particle tracking using the cavity field map.

In any case the first equation (4) is used in such a way that the (φ_s,T) couple verify the Panofsky equation (1)

$$\mathrm{T}\, cos(\varphi s) = \Delta W_{track} \,/\, q\, E_0 L \quad (4)$$

with

$$\Delta W_{track} = \int q\, E_z(z, r=0)\, cos\big(\phi(z)\big)\, dz \quad (5)$$

and

$$\phi(z) = \phi(z=0) + \int_0^z \frac{2\pi}{\beta(z)\lambda}\, dz \quad (6)$$

A second equation must be chosen to compute φ_s and T.

TraceWin "Historic" (φ_s, T) Computation

In the Panofsky synchronous phase definition as "the rf phase by which the synchronous particle crosses the electrical center of the gap", the gap "electrical center" z = 0 is defined such that

$$\int E_z(z, r=0)\, sin\left(\frac{2\pi z}{\bar{\beta}\lambda}\right) dz = 0 \quad (7)$$

with $\bar{\beta} = (\beta_{in} + \beta_{out})/2$.

The TraceWin "historic model" [4] uses the same φ_s definition (8) taking into account the reference particle phase evolution $\phi(z)$ computed during the tracking (6).

$$\int E_z(z, r=0)\, sin(\phi(z) - \varphi_s)\, dz = 0 \quad (8)$$

In this case the synchronous phase is computed from

$$tan(\varphi s) = \frac{\int q\, E_z(z, r=0)\, sin\big(\phi(z)\big)\, dz}{\Delta W_{track}} \quad (9)$$

TraceWin "new" (φ_s, T) Computation

As shown in [5], the choice of a synchronous phase definition based on the cavity "electrical center" don't allow to obtain the right cavity longitudinal focusing properties in the case of high particle velocity changes.

To do this, the first step is to use the longitudinal transfer matrixes computed making the reference particle tracking in the cavity field map

$$\begin{bmatrix} \delta\varphi_{out} \\ \delta W_{out} \end{bmatrix} = \begin{bmatrix} fm_{11} & fm_{12} \\ fm_{21} & fm_{22} \end{bmatrix} \begin{bmatrix} \delta\varphi_{in} \\ \delta W_{in} \end{bmatrix} \quad (10)$$

with $\delta\varphi$ and δW the small amplitude motions around the reference particle. The longitudinal focusing is

$$\frac{\delta W_{out}}{\delta\varphi_{in}} = fm_{21} \quad (11)$$

Using (4) the elementary energy shift is such that

$$\Delta W + \delta W_{out} = q\, E_0 L\, \mathrm{T}\, cos(\varphi_s + \delta\varphi_{in}) \quad (12)$$

leading to the longitudinal focusing

$$\frac{\delta W_{out}}{\delta\varphi_{in}} = -q\, E_0 L\, \mathrm{T}\, sin(\varphi_s) \quad (13)$$

This means that, to obtain the right cavity longitudinal focusing properties (11), the (φ_s,T) couple must verify

$$\mathrm{T}\, sin(\varphi_s) = -fm_{21} /\, q\, E_0 L \quad (14)$$

The combination of (4) and (14) finally allows to obtain a synchronous phase defined by

$$tan(\varphi_s) = \frac{-fm_{21}}{\Delta W_{track}} \quad (15)$$

where fm_{21} the focusing term and ΔW_{track} the energy gain are both computed during the reference particle tracking in the cavity field maps.

Comparison Between "Historical" and "New" (φ_s, T) for Increasing Accelerating Gradients

Figure 2 and Table 1- illustrate the errors done using the TraceWin "historic model" as the cavity field increases.

- The error on the φ_s computation at ΔW = 0 is low up to kE = 0.50 and diverges at kE = 0.55 (see Fig. 2 bottom),
- The error at ΔW maximum and intermediate φ_s is always high, even at low field (5.6°at 10% nominal field).

Table 1: SPIRAL2 low beta cavity, proton at RFQ energy, cavity field = kE * 6.5 MV/m. Reference particle input phase (φ_{in}) for 0 and maximum energy gains, TraceWin "historic model" φs at φ_{in} and input phase for $\varphi_s = 0$ ($\varphi_{in}0$), $\Delta\varphi_{in} = \varphi_{in}\Delta\mathrm{Wmax} - \varphi_{in}\,\Delta\mathrm{W0}$.

kE	ΔW = 0		ΔW maximum			$\Delta\varphi_{in}$
	φ_{in}	φ_s	φ_{in}	φ_s	$\varphi_{in}0$	
0.10	-48°	-90°	57°	5.6°	51°	105°
0.25	-59°	-90°	67°	9.0°	56°	115°
0.50	-81°	-91°	79°	8.8°	67°	148°
0.55	-85°	+90°	81°	8.5°	69°	154°

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Table 1 also gives the evolution of the cavity phase shift ($\Delta\varphi_{in}$) to switch φ_s from 0 to -90°. One can notice that even at 10% nominal field (kE = 0.1) this shift is 105°, i.e. 15° above the expected 90°.

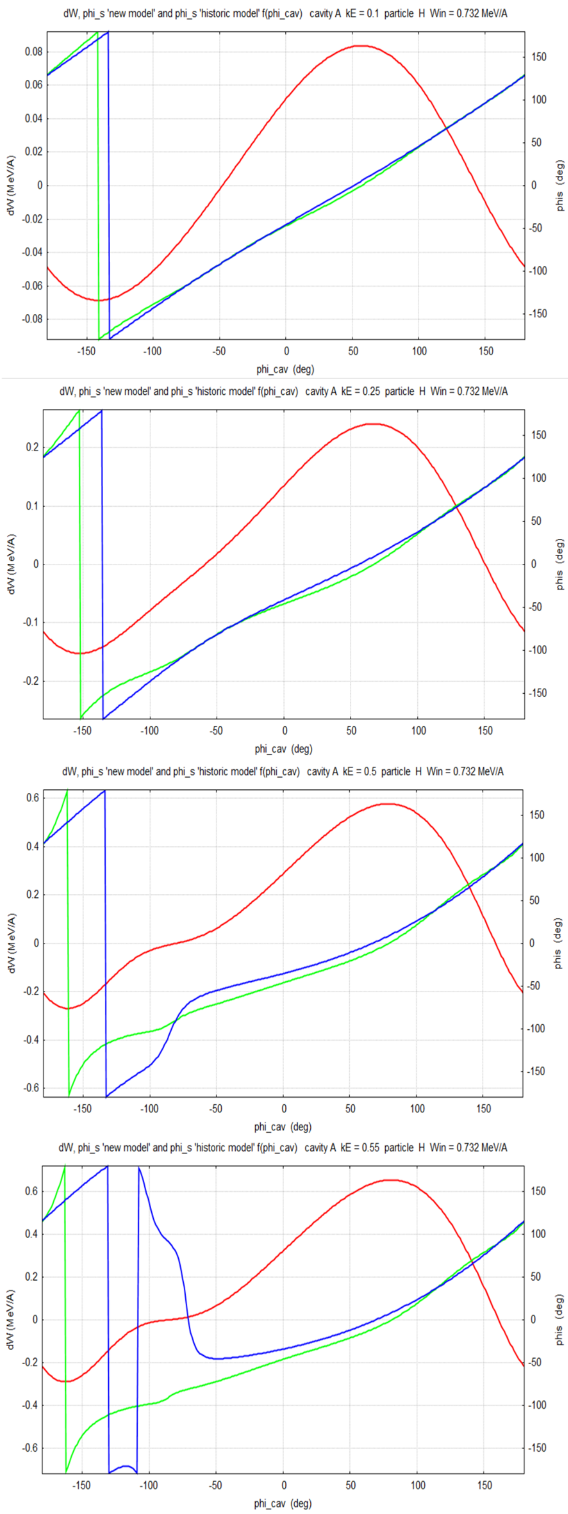

Figure 2: 360° cavity phase scan, Table 1 conditions. Energy gain (red), "new" (green) and "historic" (blue) TraceWin synchronous phase computations. From top to bottom: kE = 0.10, 0.25, 0.50 and 0.55.

COMPARISON IN THE CASE OF THE A/Q=7 SPIRAL2 LINAC TUNING

The SPIRAL2 linac tuning has been studied for the acceleration of heavy ions up to A/Q = 7 in the framework of the NEWGain (new SPIRAL2 injector) project undertaken at GANIL [6].

In the optimized tuning, all the cavities operate at their maximum field (6.5 MV/m, except one at kE = 0.6 for the longitudinal matching at the low-beta / high-beta cavity transition) and the "new model" synchronous phases are progressively ramped from -32° to -9° to reach the highest possible output energy and required longitudinal acceptance.

Figure 3 shows the large differences between the "new synchronous phases" (in blue) used for the design (reference particle cavity input phase computation) and the "historic synchronous phase" values computed using (8) (in magenta), especially for the first low-beta cavities, but also for the first high-beta cavities.

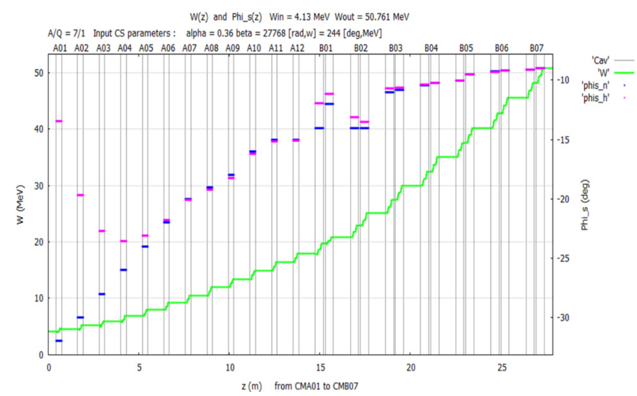

Figure 3: A/Q = 7 SPIRAL2 linac optimized tuning. Energy evolution (green), "new" (blue) and "historic" (magenta) synchronous phases.

The longitudinal acceptance (Fig. 4) confirms the validity of the "new model" chosen to obtain the right longitudinal focalization, even at high accelerating gradients (-25° to +45° phase acceptance for dW = 0).

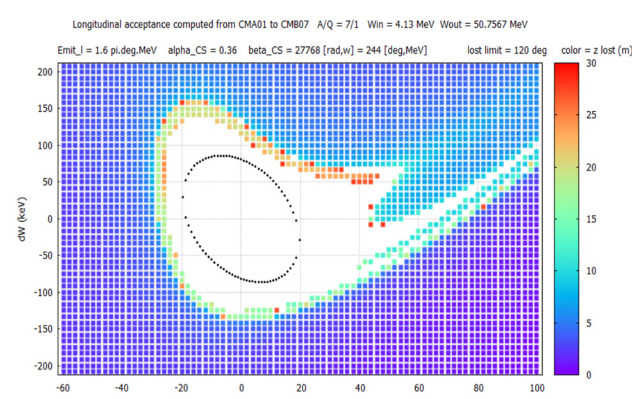

Figure 4: A/Q = 7 SPIRAL2 linac longitudinal acceptance. Optimized tuning using the TraceWin "new model" synchronous phases.

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Figures 2 and 3 show that in the -90° to 0° synchronous phase region the "historic model" always gives higher synchronous phase values than the "new model"; e.g. -15° instead of -32° for the first cavity in the case of the A/Q = 7 tuning (Fig. 3). This means that a linac design done using the "historic model" leads to lower cavity energy gains and larger acceptances in the linac front end, without benefice since the linac acceptance will be defined by the following cavities. A linac design done using the "historic model" is then not optimal from the linac energy point of view.

SUMMARY

An accurate tuning of high accelerating gradient cavities cannot be done using a method based on the Panofsky equation (1) no longer valid in the case of high particle velocity changes.

To stay below the ±1° phase error usually required to keep beam loss below an acceptable level, the best way is to tune the cavities with respect to their buncher phase (φ_s = -90°, ΔW = 0) making a phase shift computed by a reference particle tracking in field map. This method is successfully used to tune the SPIRAL2 linac cavities [3].

A new definition of the synchronous phase can be also used in order to obtain a better understanding and a better optimization of the longitudinal beam dynamics in linacs using high accelerating gradients. It has been implemented in TraceWin ("Use new synchronous phase definition" option) [4] and successfully used to optimize the SPIRAL2 linac tuning for heavy ions up to A/Q = 7.

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