THE TRACKING CODE RF-Track AND ITS APPLICATION

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Abstract

RF-Track is a CERN-developed particle tracking code that can simulate the generation, acceleration, and tracking of beams of any species through an entire accelerator, both in realistic field maps and conventional elements. RF-Track includes a large set of single-particle and collective effects: space-charge, beam-beam, beam loading in standing and travelling wave structures, short- and long-range wakefield effects, synchrotron radiation emission, multiple Coulomb scattering in materials, and particle lifetime. These effects make it the ideal tool for the simulation of high-intensity machines. RF-Track has been used for the simulation of electron linacs for medical applications, inverse-Compton-scattering sources, positron sources, protons in Linac4, and the cooling channel of a future muon collider. An overview of the code is presented, along with some significant results.

INTRODUCTION

RF-Track [1] is a tracking code developed at CERN currently used for the simulation, design, and optimisation of several diverse accelerators: electron-driven medical facilities, the positron sources of CLIC and FCC-ee, low-energy proton and ion linacs, electron coolers, and other exotic setups like the muon cooling channel of a future muon collider. The code was initially created as a tool to perform tracking simulations of a medical linac for hadron therapy featuring backwards travelling-wave structures [2]; then, it evolved into a multi-purpose accelerator toolbox capable of handling a large number of challenging simulation scenarios [3]. RF-Track can simulate beams of particles with arbitrary energy, mass, and charge, even mixed, and transport them through conventional matrix-based elements as well as through special elements and field maps. It implements two different particle tracking methods: tracking *in time* and tracking *in space*. The first is preferred in space-charge-dominated regimes, where the relative positions of the particles in space matter. The second is preferred when collective, intra-bunch effects are unimportant. Consistent and intuitive methods are available for seamlessly transitioning from one model to another.

Equipped with a friendly user interface based on the Octave or Python scientific languages, RF-Track is the ideal tool for performing complex numerical experiments in accelerator physics.

Integration Methods for the Equations of Motion

RF-Track offers several methods for integrating the equations of motion, depending on the tracking environment and beamline element. When tracking in space, it provides conventional thick-matrix-based elements such as quadrupoles, sector bends and drifts. In field maps, RF-Track numerically integrates the particle trajectories elements using numerical methods such as leap-frog, Runge-Kutta, Bulirsch-Stoer, and other higher-level routines for adaptive step-size control [4], up to the 12th order. When tracking in time, the equations of motion are always solved through numerical integration using the above-mentioned methods, with time being the independent variable.

BEAM LINE ELEMENTS

Two distinct environments have been created for space and time tracking: Lattice and Volume. Lattice represents a conventional sequence of beamline elements and is suitable for space tracking. Volume is a more general environment where elements can overlap, have arbitrary orientation in space, and particles can move in any direction. Elements' misalignment can be simulated in both environments.

In the following sections, elements unique to RF-Track: field maps and several special elements, are described.

Field Maps

RF-Track has the capability to import 1D, 2D, and 3D field maps of static and oscillating RF fields, standing or travelling in both the forward and backward directions. Field maps can be linearly or cubically interpolated.

1D Field Maps In the case of 1D maps, RF-Track accepts the longitudinal on-axis electric field and performs an off-axis field expansion, assuming cylindrical symmetry. Both the electric and the magnetic components of the field are computed. This expansion fulfils Maxwell's equations and is based on the method presented in [5], similar to what is done in ASTRA [6].

2D Field Maps In the case of 2D maps, the user provides the longitudinal and radial components of the electric and magnetic fields, and cylindrical symmetry is automatically applied.

3D Field Maps In the case of 3D maps, RF-Track guarantees a divergence-free interpolation of the field at any point, automatically correcting inaccuracies resulting from conventional interpolation or field measurement errors that would violate Maxwell's equations [7]. Additionally, in the case of symmetric fields, RF-Track accepts partial views of the field and automatically applies appropriate electric-wall and magnetic-wall symmetries for efficient memory usage. Figure 1 shows an example of tracking through the field map of an alpha magnet.

Special Elements

Electron Cooler Electron coolers are used in storage rings to reduce the phase space volume of heavy particles

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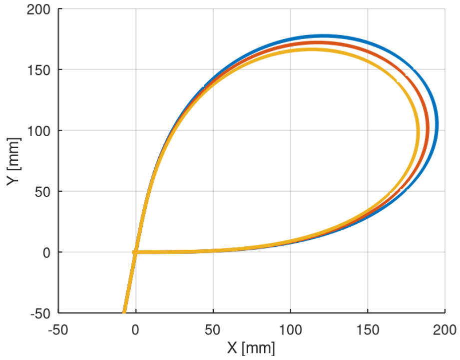

Figure 1: Three electron bunches with average kinetic energy 650 keV ±5%, travelling through an alpha magnet, simulated in RF-Track. As expected, the alpha magnet is perfectly achromatic.

such as protons, antiprotons and ions. Their effect depends on the Coulomb interactions between the circulating beam and the cold electrons at small relative velocities. RF-Track implements electron cooling using a hybrid-kinetic model, where the ions are represented as single particles and the electron plasma as a 3D mesh of thermal electrons [8].

Adiabatic Matching Device (AMD) An AMD, or Flux Concentrator, is a special pulsed magnet for capturing positrons in a positron source. This device features a very strong field in the proximity of the tungsten target that generates the positrons (around 10 T) while canalising them towards the capturing acceleration structures. An analytic 3D model of the AMD field was implemented in RF-Track to enable an integrated and fast start-to-end optimisation of an entire positron source in one go, without requiring the creation of *ad hoc* field maps [9, 10].

Coils, Solenoid The purpose of these two special elements is to implement the analytic expressions for the 3D field of a coil and a thick solenoid in the whole 3D space. When these special elements are used in a Lattice, the conventional thick matrix is used. When these elements are used in a Volume, the analytic 3D field is used to enable the simulation of realistic fringe fields and field overlap. This is crucial in the simulation of systems where strong solenoids are needed, like photoinjectors, electron guns, as well as the muon cooling channel of a future muon collider.

Toroidal Harmonics In the context of electron or hadron therapy, a static toroidal field configuration has been considered using superconducting magnets to replace the need for rotating gantries (GaToroid [11]). These fields can be naturally expanded in series using Toroidal Harmonics [12]. A fast expansion in toroidal harmonics has been implemented in RF-Track, with the double purpose of (1) computing the harmonics starting from a field map generated by electromagnetic field solvers and (2) optimising the field directly looking at the impact of the toroidal field components on the charged particle dynamics.

Space-Charge Fields Space-charge fields are elements capturing the electromagnetic field generated by a particle distribution. Within the distribution, this element provides the space-charge (or beam-beam) field; outside, it provides the external electromagnetic fields generated by the charge distribution. This element can be used for the simulation of the so-called *weak-strong* or *strong-strong* beam-beam interactions.

Inverse-Compton Scattering A simulation module implementing Inverse-Compton scattering (ICS) was added to the code. The module consists of a special beamline element that simulates the interaction between the tracked beam and a laser, making RF-Track capable of simulating a complete ICS source in one go, from the particle source to the photons [13]. This module was intended for the start-to-end optimisation of electron-based ICS sources, but it works with any charged particle. An example of usage was presented in [14].

SINGLE-PARTICLE AND COLLECTIVE EFFECTS

Space Charge and Beam-beam

In RF-Track, the space-charge kick is computed by solving the 3D Maxwell's equations for the electric-scalar and magnetic-vector potentials via a cloud-in-cell method based on integrated Green's functions in free space. The kick on each particle is then computed as the Lorentz force due to the fields obtained from the electromagnetic potentials. The simulation of mirror charges in rectangular or circular beam pipes is also possible using predefined Green's functions. As this module computes the electric and magnetic field components simultaneously, it also computes beam-beam effects.

Beam-loading

Beam loading effects result in energy losses for long trains of bunches in standing- and travelling-wave structures. To address these effects, a beam-loading module was developed using a power-diffusive model. Particular attention has been devoted to guns for high-intensity bunches in photoinjectors and X-band linacs. A detailed description of the model and benchmarks performed at the CERN CLEAR [15] test facility are presented in [16, 17]. Figure 2 shows a comparison with results in the literature.

Wakefields

Short- and long-range wakefield effects can be important in high-energy, high-intensity electron accelerators such as those planning to use X-band technology. RF-Track implements three different models of wakefields.

Short-range Wakefields Based on an analytic description of the wakefield [19], RF-Track computes the short-range effects based on the user-provided key geometric parameters of the accelerating cell. Transverse and longitudinal

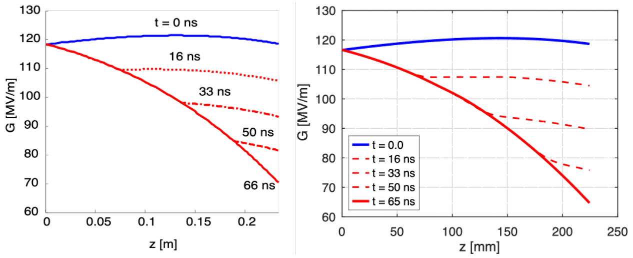

Figure 2: Time evolution of the effective gradient G_{eff} in the CLIC accelerating structure. The left-hand-side plot corresponds to the analytic results presented in [18]. The right-hand-side plot shows the beam-loaded field calculated with RF-Track.

wakefields are applied via a fast convolution of the longitudinal charge distribution with the single-particle wake potential using an FFT.

Long-range Wakefields RF-Track computes the long-range bunch-to-bunch effects by requiring the user to specify the transverse high-order modes present in the structure via the frequency, the amplitude, and the Q factor of each mode.

Arbitrary Wake Function RF-Track allows the user to specify an arbitrary wake potential in the form of a 1D vector. This model is extremely useful for assessing threshold tolerances and allows the simulation of long-range effects where a simple formula provides the wake function, for instance *resistive-wall wakes*. A more detailed description is given in Ref. [20].

Incoherent Synchrotron Radiation

RF-Track considers synchrotron radiation emission due to *any* electromagnetic field, or force, acting on the single particle. The emission of incoherent synchrotron radiation can be taken into account in any element, including accelerating structures.

Multiple Coulomb Scattering

Particle-matter interaction was recently introduced in RF-Track. The implementation consists of adding three new single-particle effects: multiple Coulomb scattering (MCS), stopping power (SP), and energy straggling (ES). Since effects can be added to any element and activated independently or simultaneously, RF-Track can now perform particle tracking and beam optics calculations in any material, for instance, in air or water [3]. Figure 3 shows an example of tracking electrons in air.

ALGORITHMS

Beam-based Alignment

The implementation of element misalignment has been completely redesigned to improve flexibility. Now, elements can be grouped, while each element can have arbitrary offsets and orientation in space, and nesting is possible. This means that individual elements can be misaligned w.r.t. a common reference, this reference misaligned w.r.t. a supporting girder, while the girder is misaligned w.r.t. the ground. Supporting routines to implement beam-based alignment quickly have been provided.

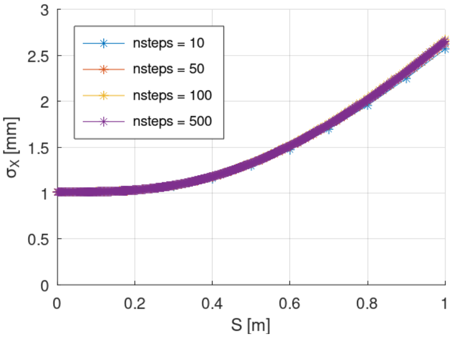

Figure 3: RMS transverse beam size evolution of a 100 MeV electron bunch with zero initial divergence travelling through 1 m of air. The plot shows the good convergence of the model already with 10 integration steps.

EXAMPLE

The Linac4 RFQ

The CERN Linac4 352.2 MHz RFQ was taken as the first test case for benchmarking RF-Track in the world of proton and H$^-$ tracking, currently simulated with PATH [21]. The RFQ was described by a field map built out of FEM electrostatic simulations performed with COMSOL Multiphysics© [22], with the physical vane geometry taken into account to define the apertures. The stepsize of the field map was 0.2 mm. The file was directly imported in PATH and RF-Track, and an initial beam distribution of 500 k macroparticles was used. Figures 4 and 5 show the excellent agreement reached when tracking without space charge. The difference in overall transmission through the RFQ between the two codes is less than 0.5%, and the output beam distributions overlap nicely in all three transverse and longitudinal phase spaces. This benchmark is described in greater detail in [23].

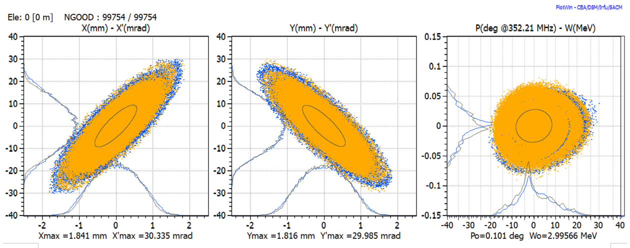

Figure 4: Zero space charge RFQ beam output distribution in x-x', y-y', E-ϕ planes (left to right) for RF-Track (in blue) vs PATH (in yellow).

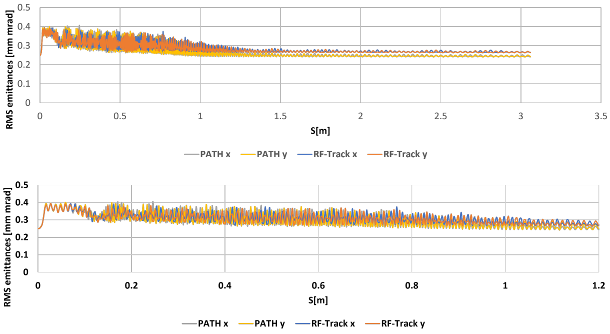

Figure 5: Zero space charge normalized RMS transverse emittances. The lower plot shows a detail of the first 1 m of RFQ.

CONCLUSION

In this paper, we have presented the RF-Track tracking code. Initially developed for tracking light ions and protons in medical linacs, it evolved to include electrons, positrons, and muons. A set of dedicated single-particle and collective effects allowed the simulation of high-intensity machines, establishing RF-Track as the reference tool for the simulation of medical linac based on electrons, the FCC-ee pre-injector linacs [24], the CLIC and FCC-ee positron sources, and of the muon collider's cooling channel. An interest in high-intensity proton linac simulations motivated detailed tests of RF-Track back in the area of hadron acceleration. A benchmarking of PATH and RF-Track beam dynamics tracking results in Linac4 has been kicked off. The results obtained from the comparison are very encouraging.

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