ISBN: 978-3-95450-253-0 ISSN: 2673-5571 doi:10.18429/JACoW-HB2023-THBP47

STUDIES ON THE EFFECT OF BEAM-COUPLING IMPEDANCE ON SCHOTTKY SPECTRA OF BUNCHED BEAMS

C. Lannoy*,[1], D. Alves, K. Lasocha, N. Mounet, CERN, Geneva, Switzerland
T. Pieloni, EPFL, Lausanne, Switzerland
[1]also at EPFL, Lausanne, Switzerland

Abstract

Schottky monitors can be used for non-invasive beam diagnostics to estimate various bunch characteristics, such as tune, chromaticity, bunch profile or synchrotron frequency distribution. However, collective effects, in particular beam-coupling impedance, can significantly affect Schottky spectra when large bunch charges are involved. In such conditions, the available interpretation methods are difficult to apply directly to the measured spectra, thus preventing the extraction of beam and machine parameters, which is possible for lower bunch charges. To study the impact of impedance on such spectra, we perform here time-domain, macro-particle simulations and apply a semi-analytical method to compute the Schottky signal for various machine and beam conditions, including those corresponding to typical physics operation at the Large Hadron Collider. This study provides preliminary interpretations of the impact of beam-coupling impedance on Schottky spectra by incorporating longitudinal and transverse resonator-like impedance models into the simulations.

INTRODUCTION

Schottky spectra are a powerful diagnostic tool for investigating the longitudinal and transverse dynamics of charged particle beams. However, the accurate interpretation of these spectra becomes particularly challenging when collective effects such as beam-coupling impedance become significant as can be the case for proton bunches in the Large Hadron Collider (LHC). While theoretical frameworks for Schottky spectrum reconstruction exist [1–4], they do not include impedance effects which can strongly affect the spectra in certain conditions. In Ref. [5], we started investigating the effect of impedance on the longitudinal spectrum and addressed some of the gaps in existing theories.

The following section focuses on the effects of a longitudinal broad-band (BB) resonator on the Schottky spectrum and investigates, both theoretically and through simulations, the impact of such an impedance. Then, in the third section we explore through simulations how a transverse broad-band resonator affects the Schottky spectrum. Our concluding remarks then follow.

LONGITUDINAL BROAD-BAND RESONATOR

The equation of motion for the radio frequency (RF) phase ϕ of a given particle in the presence of longitudinal beam-coupling impedance was derived in Ref. [5] and is given by

$$\ddot{\phi} + \Omega_0^2 \sin\phi = \Omega_0^2 \frac{I}{\widehat{V}\cos\phi_s} \sum_{p=-\infty}^{\infty} Z_{\parallel}(p)\widehat{\lambda}(p) e^{j\frac{p}{h}\phi}, \quad (1)$$

with the nominal synchrotron frequency Ω_0, the average bunch current I, the amplitude of the RF voltage $\widehat{V}$, the synchronous phase ϕ_s, the revolution frequency ω_0, the longitudinal impedance $Z_{\parallel}(p) \equiv Z_{\parallel}(p\omega_0)$, the bunch spectrum $\widehat{\lambda}(p) \equiv \widehat{\lambda}(p\omega_0)$, and the RF harmonic number h. By expanding the sine and exponential functions into their Maclaurin series, the previous equation can be written in the compact form

$$\ddot{\phi} + \Omega_0^2 \sum_{n=0}^{\infty} S_n \phi^n = 0, \quad (2)$$

with the coefficients S_n defined by

$$S_n = \begin{cases} -Z_n & : n \text{ even}, \\ \frac{j^{n-1}}{n!} - Z_n & : n \text{ odd}, \end{cases}$$

and

$$Z_n = \frac{I j^n}{\widehat{V}\cos\phi_s \, n! h^n} \sum_{p=-\infty}^{\infty} \widehat{\lambda}(p) p^n \times \begin{cases} Re[Z_{\parallel}(p)] & : n \text{ even}, \\ j\, Im[Z_{\parallel}(p)] & : n \text{ odd}. \end{cases}$$

In order to derive an analytical relation between the amplitude of the synchrotron oscillation and its frequency, we assumed in Ref. [5] that the even terms in Eq. (2) can be neglected. In the following, we present how one can assess if this approximation is valid for a given impedance $Z_{\parallel}(p)$.

The complex exponential in Eq. (1) can also be expanded with the trigonometric functions

$$\ddot{\phi} + \Omega_0^2 \sin\phi = \frac{\Omega_0^2 I}{\widehat{V}\cos\phi_s} \sum_{p=-\infty}^{\infty} Z_{\parallel}(p)\widehat{\lambda}(p) \left(\cos(\frac{p}{h}\phi) + j\sin(\frac{p}{h}\phi)\right).$$

The cosine term is an even function and corresponds to the sum of all the even terms from Eq. (2). A first approximation of the RF phase shift for a particle of given synchrotron time-amplitude $\widehat{\tau}$ is given by the time average of the sum of the even terms:

$$\Delta\phi_s(\widehat{\tau}) = \frac{1}{S_1} \sum_{n=0}^{\infty} S_{2n} \left\langle \phi^{2n}(t) \right\rangle_t = \frac{I}{S_1 \widehat{V}\cos\phi_s} \sum_{p=-\infty}^{\infty} Re[Z_{\parallel}(p)]\, \widehat{\lambda}(p) \left\langle \cos\left(\frac{p}{h}\phi(t)\right) \right\rangle_t, \quad (3)$$

* christophe.lannoy@cern.ch

ISBN: 978-3-95450-253-0 ISSN: 2673-5571 doi:10.18429/JACoW-HB2023-THBP47

where $\langle\,\rangle_t$ denotes the average value over time. In the limit of small amplitude oscillations, Eq. (3) reduces to the linear theory, leading to Eq. (59) in Ref. [6]. By approximating the longitudinal motion of the particle with a harmonic motion [1, 3], we can write

$$\phi(t) = h\omega_0\hat{\tau}\sin(\Omega_s t + \varphi_s),$$

with φ_s, the initial phase of the synchrotron oscillation. The average value of the time-dependent part of Eq. (3) becomes

$$\left\langle \cos\left(\frac{p}{h}\phi(t)\right)\right\rangle_t = \frac{1}{2\pi}\int_0^{2\pi}\cos\left(p\omega_0\hat{\tau}\sin(t)\right)\mathrm{d}t = J_0(p\omega_0\hat{\tau}),$$

using Eq. (3.715.10) from [7] for the integral and with J_0, the zero order Bessel function of the first kind.

The average RF phase shift is given by the weighted average of the single particle phase shift

$$\Delta\phi_s = \int_0^\infty \Delta\phi_s(\hat{\tau})g(\hat{\tau})\,\mathrm{d}\hat{\tau} = \frac{I}{S_1\hat{V}\cos\phi_s}\sum_{p=-\infty}^{\infty} Re\left[Z_\parallel(p)\right]\hat{\lambda}(p) \times \int_0^\infty J_0(p\omega_0\hat{\tau})g(\hat{\tau})\,\mathrm{d}\hat{\tau},$$

where $g(\hat{\tau})$ is the distribution of synchrotron oscillation amplitudes. Using the Fourier-Hankel-Abel cycle, it can be shown that the last integral corresponds to the bunch spectrum $\hat{\lambda}(p)$, which yields

$$\Delta\phi_s = \frac{I}{S_1\hat{V}\cos\phi_s}\sum_{p=-\infty}^{\infty} Re\left[Z_\parallel(p)\right]\hat{\lambda}^2(p), \qquad (4)$$

and the total energy lost during one revolution is given by

$$\Delta E_{\text{bunch}} = -Ne\hat{V}\cos\phi_s S_1\Delta\phi_s = -\frac{Q^2}{2\pi}\omega_0\sum_{p=-\infty}^{\infty} Re\left[Z_\parallel(p)\right]\hat{\lambda}^2(p), \qquad (5)$$

where $Q = Ne$ is the total charge of the bunch. The S_1 factor in Eq. (5) accounts for the first order potential well distortion due to impedance. This expression is consistent with other formulas available in the literature, such as Eq. (4.35) in Ref. [8].

In the case of the LHC-like broad-band resonator defined in Table 1, we have $\Delta\phi_s = -0.94$ mrad with Eq. (4), in relative agreement with $\Delta\phi_s = -1.12$ mrad from the macro-particle simulation as shown in Fig. 1. Such a phase shift is small enough to be neglected, hence we can drop the even terms in Eq. (2).

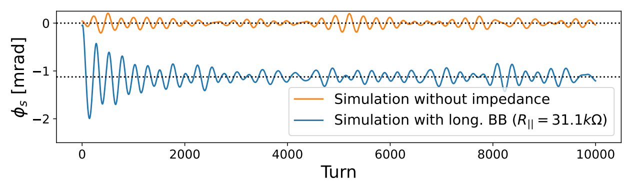

Figure 1: Synchronous phase shift from simulation.

A relation between the amplitude of the synchrotron oscillation and its frequency can thus be derived from Eq. (2) by keeping only the S_1 and S_3 terms [5], which allows to extend the theoretical matrix formalism [3, 4] to the case of a longitudinal broad-band resonator.

Figure 2 shows macro-particle simulations of an LHC proton bunch at injection energy, performed with PyHEAD-

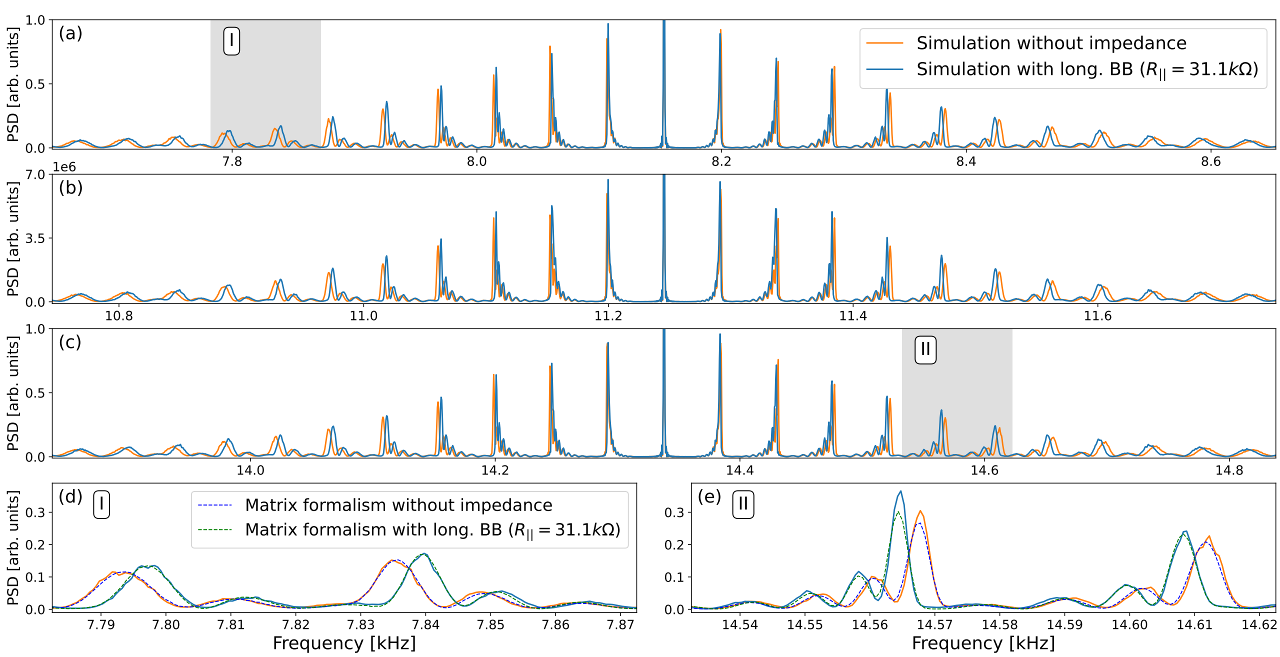

Figure 2: Simulated longitudinal (b), and horizontal (a and c), Schottky spectra with (blue) and without (orange) a longitudinal LHC-like BB resonator. Frequencies are shifted from the LHC Schottky harmonic 427725, to the 1st harmonic.

ISBN: 978-3-95450-253-0 ISSN: 2673-5571 doi:10.18429/JACoW-HB2023-THBP47

TAIL [9, 10] and following the method presented in Ref. [11]. Simulations have been conducted with and without a longitudinal broad-band resonator, and the relevant simulation parameters are given in Table 1. The shunt impedance and cut-off frequency of the resonator are based on Ref. [12, p. 71] and correspond to a first estimate of the broad-band part of the LHC longitudinal impedance. We can observe that the longitudinal and transverse bands are modified in the same manner - the impedance reduces the nominal synchrotron frequency above transition and the shape of the satellites is modified due to the amplitude dependent synchrotron frequency shift. The theoretical matrix formalism including impedance effects from Ref. [5] is also shown on the bottom plots and agrees well with the simulations.

TRANSVERSE BROAD-BAND RESONATOR

The effects of a transverse broad-band resonator on the Schottky spectrum can be observed in Fig. 3. The resonator parameters, which also correspond to a first estimate of the broadband part of the LHC transverse impedance [12, p. 71], are given in Table 1. The longitudinal band is not affected by the impedance, while a betatron tune shift is visible on the transverse bands (all satellites in a given transverse sideband are displaced by about 5 Hz in the same direction). The direction of the satellite's shift - toward the right (resp. left) for the lower (resp. upper) sideband - indicates that the broad-band resonator decreases the betatron tune. Similarly to the case of the longitudinal impedance, the satellites are not simply shifted but their internal structure is also affected by impedance.

Table 1: PyHEADTAIL Simulation Parameters

Intensity	1.5×10^{11} protons per bunch
Energy per proton	450 GeV
Emittances	$\epsilon_x = \epsilon_y = 2\,\mu\text{m}$
Tunes	$Q_x = 64.28,\ Q_y = 59.29$
Chromaticities	$Q'_x = Q'_y = 0$
Slippage factor	3.436×10^{-4}
RF harmonic	35640
RF voltage	4 MV
LHC circumference	26.659 km
Bunch length (RMS)	$\sigma = 0.31$ ns
Broad-band resonator	
Long. shunt impedance	$R_\parallel = 31.1\,\text{k}\Omega$
Trans. shunt impedance	$R_\perp = 1.34\,\text{M}\Omega\,\text{m}^{-1}$
Cut-off frequency	$\omega_r = 2\pi \times 5\,\text{GHz}$
Quality factor	$Q = 1$

CONCLUSION

This study delves into the influence of beam-coupling impedance on Schottky spectra of bunched beams. It highlights the challenges posed by impedance, particularly for large bunch charges. Macro-particle simulations and analytical methods were used to investigate this effect in both longitudinal and transverse spectra.

The findings reveal that longitudinal impedance can induce shifts in the synchrotron frequency while the presence of a transverse broad-band resonator leads to betatron tune shifts. Both longitudinal and transverse impedance also alter the internal structure of transverse satellites.

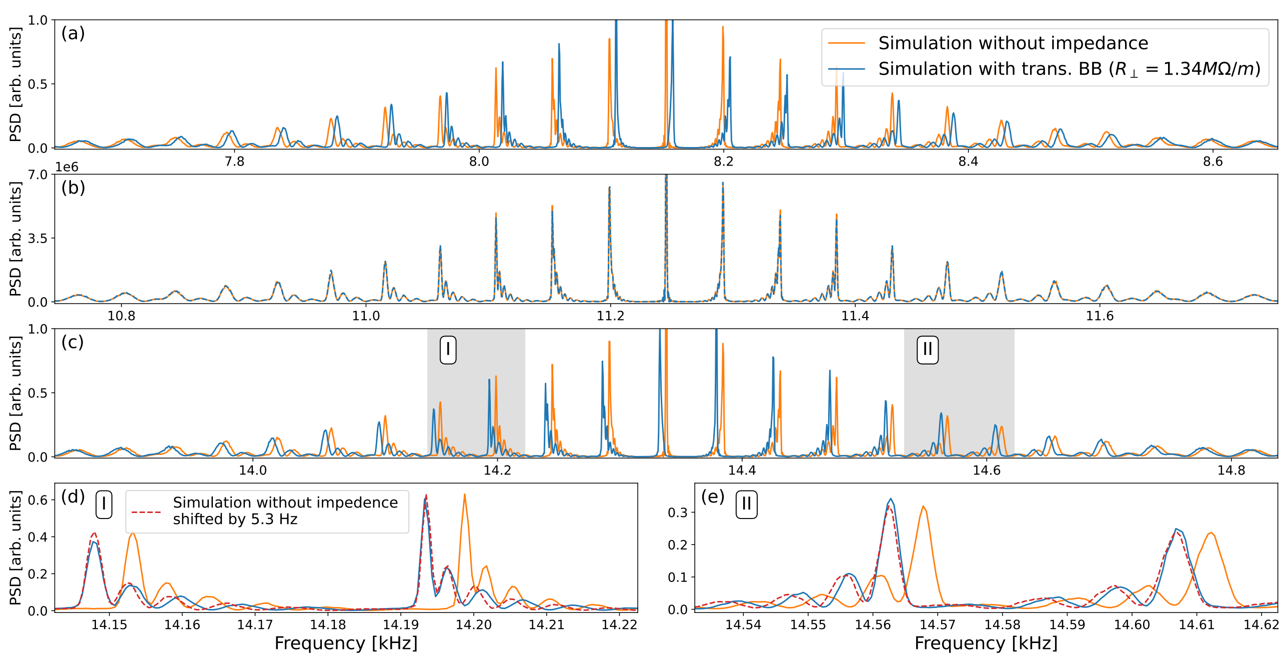

Figure 3: Simulated longitudinal (b), and horizontal (a and c) Schottky spectra with (blue) and without (orange) a transverse LHC-like BB resonator. Frequencies are shifted from the LHC Schottky harmonic 427725, to the 1st harmonic.

ISBN: 978-3-95450-253-0 ISSN: 2673-5571 doi:10.18429/JACoW-HB2023-THBP47

REFERENCES

[1] D. Boussard, "Schottky noise and beam transfer function diagnostics," CERN, Geneva, Switzerland, Rep. CERN-SPS-86-11-ARF, 1986. doi:10.5170/CERN-1987-003-V-2.416

[2] M. Betz, O. R. Jones, T. Lefevre, and M. Wendt, "Bunched-beam Schottky monitoring in the LHC," *Nucl. Instrum. Methods Phys. Res., A*, vol. 874, pp. 113–126, 2017. doi:10.1016/j.nima.2017.08.045

[3] K. Lasocha and D. Alves, "Estimation of longitudinal bunch characteristics in the LHC using Schottky-based diagnostics," *Phys. Rev. Accel. Beams*, vol. 23, p. 062 803, 2020. doi:10.1103/PhysRevAccelBeams.23.062803

[4] K. Lasocha and D. Alves, "Estimation of transverse bunch characteristics in the LHC using Schottky-based diagnostics," *Phys. Rev. Accel. Beams*, vol. 25, p. 062 801, 2022. doi:10.1103/PhysRevAccelBeams.25.062801

[5] C. Lannoy, D. Alves, K. Lasocha, N. Mounet, and T. Pieloni, "Effect of Longitudinal Beam-Coupling Impedance on the Schottky Spectrum of Bunched Beams," in *Proc. IBIC'23*, Saskatoon, Canada, Sep. 2023, pp. 433–437. doi:10.18429/JACoW-IBIC2023-WEP034

[6] J. L. Laclare, "Bunched beam coherent instabilities," CERN Accelerator School, Oxford, UK, Sep. 1985. doi:10.5170/CERN-1987-003-V-1.264

[7] I. Gradshteyn and I. Ryzhik, *Table of Integrals, Series, and Products*. 7th Ed., Academic Press, 2007.

[8] G. Stupakov, R. Lindberg, and B. Podobedov, "Wakefields and collective beam instabilities," USPAS, Houston, TX, USA, Jan. 2023. https://www.slac.stanford.edu/~stupakov/uspas23/2023-01-20/2022-12-07-LECTURES.pdf

[9] PyHEADTAIL code repository. https://github.com/PyCOMPLETE

[10] K. S. B. Li *et al.*, "Code Development for Collective Effects," in *Proc. HB'16*, Malmö, Sweden, Jul. 2016, pp. 362–367. doi:10.18429/JACoW-HB2016-WEAM3X01

[11] C. Lannoy, D. Alves, N. Mounet, T. Pieloni, and K. Lasocha, "LHC Schottky Spectrum from Macro-Particle Simulations," in *Proc. IBIC'22*, Kraków, Poland, Sep. 2022, pp. 308–312. doi:10.18429/JACoW-IBIC2022-TUP34

[12] N. Mounet, "The LHC Transverse Coupled-Bunch Instability," PhD thesis, EPFL, Lausanne, Switzerland, 2012. doi:10.5075/epfl-thesis-5305