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EXPERIMENTAL INVESTIGATION OF NONLINEAR INTEGRABLE OPTICS IN A PAUL TRAP*

J. A. D. Flowerdew†, S. L. Sheehy, University of Oxford, Oxford, UK
D. J. Kelliher, S. Machida, STFC Rutherford Appleton Laboratory, Oxford, UK

Abstract

Octupoles are often used to damp beam instabilities caused by space charge. However, in general the insertion of octupole magnets leads to a nonintegrable lattice which reduces the area of stable particle motion. One proposed solution to this problem is Quasi-Integrable Optics (QIO), where the octupoles are inserted between a specially designed lattice called a T-insert. An octupole with a strength that scales as $1/\beta^3(s)$ is applied in the drift region to create a time-independent octupole field, leading to a lattice with an invariant Hamiltonian. This means that large tune spreads can be achieved without reducing the dynamic aperture. IBEX is a Paul trap which confines low energy ions with an RF voltage, simulating the transverse dynamics of an alternating gradient accelerator. IBEX has recently undergone an upgrade to allow for octupole fields to be created in the trap in addition to quadrupole focusing. We present our first experimental results from testing QIO with the IBEX trap.

INTRODUCTION

In order to push the limits of the intensity frontier in accelerators, we must be prepared to address the problems associated with space charge and collective effects, which are the result of the interactions between the charged particles in a beam. It is important that we understand these collective effects because they can lead to resonant behavior and beam loss. These instabilities caused by space charge prevent accelerators from reaching their desired intensities. Achieving higher-intensity accelerators will require studying possible mitigation techniques to limit the effects of space charge. For example, the theory of Nonlinear Integrable Optics (NIO) [1] proposes a design for a nonlinear lattice that could damp the coherent resonances caused by space charge, without exciting the higher order resonances associated with the insertion of nonlinear elements. The challenge with testing these techniques experimentally is that studying beam loss in accelerators risks activating and damaging components. Thus, simulations are often relied upon instead to study collective effects. However, these simulations require large amounts of computational power to replicate the multiple Coulomb forces between particles and can never fully replace experimental validation. In order to circumvent these practical obstacles, linear Paul traps have been developed, to experimentally study beam dynamics without the large amounts of computational power needed for high-intensity simulations. A number of linear Paul traps have been created to study transverse dynamics such as the Simulator for Particle Orbit Dynamics (S-POD), Japan [2], the Paul Trap Simulator Experiment (PTSX), US [3], and the Intense Beams Experiment (IBEX), UK [4]. The transverse Hamiltonian in a Paul trap is equivalent to that of an alternating gradient accelerator [2]. Thus, these traps allow for the transverse effects in accelerators to be simulated without the risk of activating components and without the granularity of time steps, macro-particles, and grid sizes assumed in simulation. This paper presents the design and commissioning of a nonlinear upgrade to the IBEX Paul trap in order to enable experimental testing of a lattice designed according to the theory of Quasi Integrable Optics (QIO). We discuss the results from the successful trapping of ions in a quasi-integrable lattice and the promising potential contributions that the theories of NIO and QIO could make to the future of high-intensity beams.

NONLINEAR INTEGRABLE OPTICS

Current designs for accelerators such as synchrotrons and linacs are generally based on a system of alternating focusing and defocusing quadrupole magnets which confine the beam of charged particles. For an ideal system, a lattice constructed of these linear components would be fully integrable — in other words, the Hamiltonian of a single particle would be time-independent and hence conserved. However, in reality small misalignments between magnets and space charge forces create perturbations that require the use of nonlinear components like sextupoles and octupoles to make higher-order corrections. The addition of these components means that the system becomes nonintegrable, which has the effect of limiting the available phase space in which the motion of the particles is bounded. It would therefore be beneficial to be able to design a lattice that remained integrable even with the inclusion of nonlinear components, so that the system stays close to an integrable solution despite magnet misalignment and space charge.

Nonlinear Integrable Optics (NIO) proposes a method for using nonlinear components in a lattice while maintaining integrability. First proposed by Danilov and Nagaitsev in 2010 [1], NIO prescribes constructing a lattice that consists of a linear T-insert section and a drift region designed for a nonlinear magnet insert. A fully integrable solution in 2D requires a nonlinear insert with a complex elliptical potential which is being tested at the Integrable Optics Test Accelerator (IOTA) [5]. However, a quasi-integrable lattice, with one invariant of motion (the Hamiltonian), can be achieved with a scaling octupole field as the nonlinear insert. The conditions of the quasi-integrable lattice are as

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† jake.flowerdew@physics.ox.ac.uk

follows: (1) $n\pi$ (where n is an integer) phase advance over the linear section to provide quasi-periodic motion through the drift region. (2) Equal beta functions in the drift region, $\beta_x(s) = \beta_y(s) = \beta(s)$. (3) Octupole strength scaling with $1/\beta^3(s)$ in the drift region. IBEX will first test the quasi-integrable case, which uses an octupole potential of the form

$$V(x, y, s) = \frac{\kappa}{\beta^3(s)} \left(\frac{x^4}{4} + \frac{y^4}{4} - \frac{3x^2y^2}{2} \right), \tag{1}$$

where κ is a constant and $\kappa/\beta^3(s)$ defines the strength of the octupole field.

A two-cell T-insert was designed for the IBEX Paul trap with horizontal and vertical tunes of $Q_x = 1.276$ and $Q_y = 1.277$. The lattice was designed to have a tune slightly above the 1/4 integer resonance so that by increasing the space charge tune shift, the ion distribution can be brought onto resonance. In order to test QIO in the IBEX trap, a nonlinear upgrade was needed to allow for the creation of octupole fields within the trap.

NONLINEAR UPGRADE TO THE IBEX PAUL TRAP

The original IBEX trap was comprised of a quadrupolar rod configuration, consisting of four stainless steel cylindrical rods and two sets of end caps (ionisation region in Fig. 1). An RF voltage is applied to the central rods for transverse confinement of ions. A DC voltage applied to the end caps provides longitudinal confinement. The nonlinear upgrade recreated the structure of the linear trap but added four plate electrodes, which were positioned between the rods (Fig. 1), to allow for the creation of octupole fields. This design was adapted from that of the multipole trap constructed by the Hiroshima group [6].

Argon gas is leaked into the vacuum chamber and is first ionized with an electron gun and trapped in the linear ionization region before being transferred to the nonlinear experimental region. The number of ions stored in the trap, and hence the space charge tune shift, can be varied by increasing the pressure of gas which is leaked into the trap or by varying the time the electron gun is on. Ions are then stored in the experimental region for the desired period of time, at the end of which point the DC voltage on the end cap is dropped, accelerating ions onto on the Micro-Channel Plate (MCP) detector to be measured. Due to the ions stored being of low energies (<1 eV), it is possible to carry out investigations of resonant beam loss without risking damage or activation of the components. Following the full commissioning of this nonlinear upgrade in early 2023, we were then able to test the T-insert lattice in the presence of space charge which allowed us to observe the benefits of applying a quasi-integrable octupole.

EXPERIMENTAL RESULTS

In these experiments the ion density within the trap was varied in order to increase the space charge tune shift. Increasing the space charge tune shift causes the ion distribution to cross the 4th order incoherent resonance and the 2nd order coherent resonance. The resonant condition for the 4th order incoherent resonance, typically driven by octupole elements, is given by

$$Q_0 + \Delta Q_{rms} = \frac{5}{4}, \tag{2}$$

where Q_0 is the nominal tune and ΔQ_{rms} is the RMS space charge tune shift. The 2nd order coherent resonance, driven by space charge, is given by [7]

$$Q_0 + C_2 \Delta Q_{rms} = \frac{1}{2}(\frac{5}{2}), \tag{3}$$

where C_2 is the mode of the coherent oscillation and is equal to $C_2 = 3/4$ [8].

Previous simulation work showed that turning on a QI octupole was able to damp the 2nd order coherent resonance, without exciting the 4th order incoherent resonance [9]. In contrast, breaking the conditions of integrability was seen to excite the 4th order incoherent resonance.

For these experiments, an ionisation time of 0.4 s was chosen and the amount of argon leaked into the trap was varied to change the initial number of ions in the trap. Ions were stored first for 10 T-insert super-periods in order to estimate an initial number of ions in the trap, before storing for 1000 super-periods to study the effects of the incoherent and coherent resonances. All data points were averaged over 10 experiments and the error bars indicate the standard deviation in these measurements. In Fig. 2 (Top) the ion number after storing for 10 two-cell T-insert super-periods is plotted for varying argon gas pressures. This is measured for the linear T-insert lattice (Oct OFF), QI octupoles turned on (Oct ON QI), and two lattices that break the integrability conditions. For the lattice ‘Oct ON nonQI half’, the octupole is turned off every other cell in order to break the integrability condition, however this lattice only has half the integrated octupole strength as ‘Oct ON QI’. The lattice ‘Oct ON nonQI’ has the octupole turned off every other lattice period but the remaining octupole strength is doubled to create the same octupole tune spread as the ‘Oct ON QI’ lattice.

From Fig. 2 (Top) the ion number measured after 10 T-insert super-periods is seen to increase linearly with the argon gas pressure introduced into the vessel for all four lattices, as expected. The results were then repeated, storing the ions for 1000 T-insert super-periods, shown in Fig. 2 (Bottom). For an argon gas pressure of 0.82×10^{-7} mbar, corresponding to an initial ion number of around 1.5×10^6, the number of ions surviving 1000 T-insert super-periods can be seen to plateau in both of the non-QI lattices. This initial ion number corresponds to a space charge tune shift of around 0.02. This suggests that the octupoles in these lattices, which do not meet the conditions of QIO, are exciting the 4th order incoherent resonance. Therefore, in the case of both of the non-QI lattices, the benefit of a large tune spread to damp coherent resonances is negated by the octupoles driving the 4th order resonance. The maximum number of

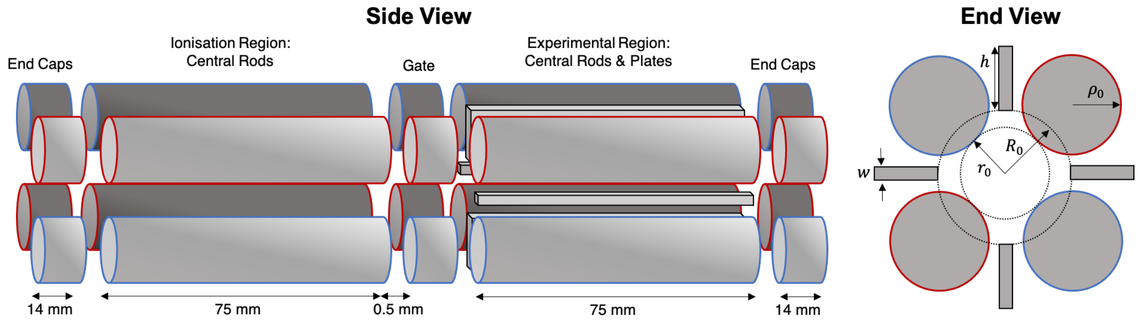

Figure 1: Schematic of the new IBEX trap. Opposing RF voltages are applied to the red and blue outlined rods for transverse confinement of ions. A DC voltage is applied to the end caps and gate electrodes to provide longitudinal trapping. In IBEX $r_0 = 5$ mm, $\rho_0 = 5.75$ mm, $w = 1$ mm and $h = 6$ mm. Four additional plates between the rods are present in the nonlinear trap at an inscribed radius of $R_0 = 8.5$ mm to enable the creation of octupole fields.

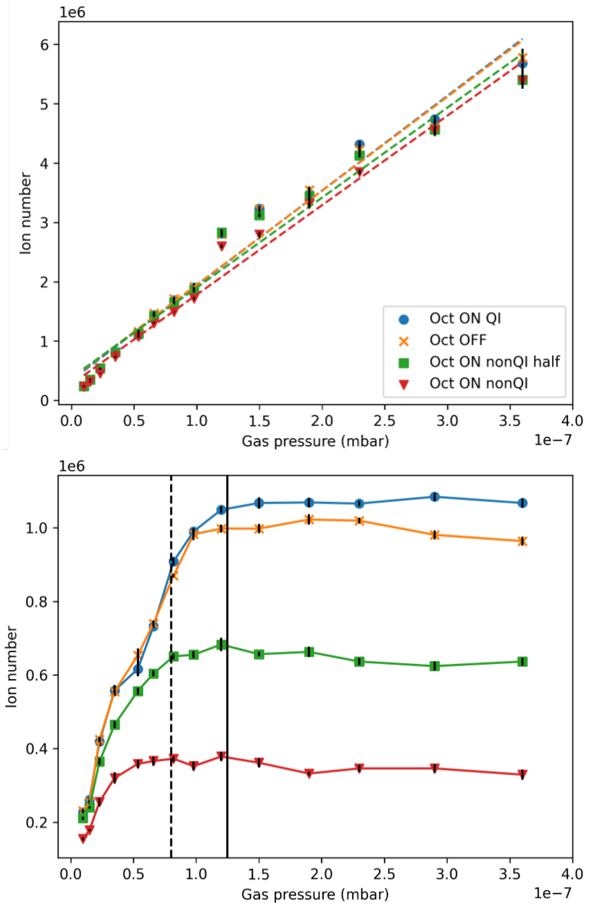

Figure 2: (Top) Ion number after 10 two-cell T-insert lattice super-periods at varying gas pressures. (Bottom) Ion number after 1000 two-cell T-insert lattice super-periods at varying gas pressures. Dashed black line indicates predicted location of 4th order incoherent resonance. Solid black line indicates predicted location of 2nd order coherent resonance.

ions surviving the ‘Oct ON nonQI’ lattice is approximately half of that of the ‘Oct ON nonQI half’ lattice. This is because the strength of the octupole for ‘Oct ON nonQI’ lattice is twice that of ‘Oct ON nonQI half’ and hence the driving term for the resonance is twice as large.

When octupoles are turned off (Oct OFF) the ion number is seen to plateau at a gas pressure of around 1.0×10^{-7} mbar, corresponding to an initial ion number of 2×10^6. This larger ion number, required to create a larger space charge tune shift in order to reach the coherent resonance, is consistent with the factor, $C_2 = 3/4$, in Eq. (3). When the QI octupole is turned on, it is seen to be able to store up to 10(2)% more particles compared to when the octupoles are off, over the 1000 T-insert super-periods studied. The increased ion survival for the Oct ON QI lattice is expected to be due to Landau damping of the coherent resonance from the octupole tune spread. The particle survival for the lattice with the QI octupole turned on was seen to be larger in simulation, compared to the coherent losses seen in the lattice when octupoles were off [9, 10]. The limited improvement of the QI octupole experimentally is suspected to be due to losses caused by residual gas collisions at higher gas pressures, along with errors in the implementation of the T-insert lattice, perturbing it away from integrability [10]. However, despite the limited ability to damp the coherent resonance in these experimental results, the addition of octupoles to the lattice using the method prescribed by QIO is shown to not drive the 4th order incoherent resonance in the vicinity.

CONCLUSION

The new nonlinear upgrade to the IBEX trap has been fully commissioned to allow for the study of lattices with quadrupole and octupole elements. This work presents the first successful trapping of ions in a quasi-integrable lattice in the new IBEX trap. As was shown in simulation, the quasi-integrable octupole was seen to help damp the 2nd order coherent resonance driven by space charge. This was aceived without exciting the 4th order incoherent resonance in the vicinity, excited by octupole elements that break integrability. The experimental results presented in this work show promising signs of the benefits of using QIO lattices in space charge dominated beams. We hope that future experimental runs of IBEX can further build on these results.

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