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DESIGN OF A FIXED-FIELD ACCELERATING RING FOR HIGH POWER APPLICATIONS*

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Abstract

A fixed field accelerating ring has an advantage to achieve high beam power over conventional ring accelerators. It would be also a sustainable option as a future proton driver. First we will look back to the historical background of high power accelerator developments. We will then discuss how we achieve the high power beam and critical design issues.

INTRODUCTION

Making the CW beams is the most effective way to increase the average beam current and thus the beam power. That is a cyclotron's way of operation where all the hardware is running in a DC mode. Injection, acceleration and extraction can be done simultaneously. Once some of the hardware has to be pulsed, an accelerator cycle is dictated by that timing. Injection, acceleration and extraction no longer take place simultaneously. Usually acceleration is the major part in a cycle. The average beam current is reduced to a fraction of the whole cycle, namely the injection duration, which is relatively small. It is hard to recover the same average current by increasing the peak current during injection.

When people realised that RF frequency modulation is needed to go beyond the non-relativistic regime, from an isochronous cyclotron to a synchrocyclotron or a synchrotron, reduction of the average beam current was a big issue. Nevertheless, a history tells that a synchrotron becomes the major type of ring accelerators because it is easier to increase the beam energy than any other type of accelerators. A synchrotron can compensate the reduction of the average beam current to some extent by its higher energy. Note that the beam power is a product of the average beam current and the beam energy. It is interesting to compare the beam power of the PSI cyclotron which is a CW accelerator gives around 2 MW with 590 MeV while SNS and J-PARC RCS which are a pulsed ring accelerator give 1 to 1.4 MW with 1 GeV (SNS) and 3 GeV (J-PARC RCS).

There was another method to recover a reduction of the average beam current proposed in the early stage of synchrotron developments. That is the invention and development of a Fixed Field Alternating Gradient Accelerator (FFA) at Midwestern Universities Research Association (MURA) [1]. They accepted that an accelerator had to operate in a pulsed mode when the beam energy goes into the relativistic regime. They found, however, there is a way to reduce the size of a ring accelerator compared with an isochronous cyclotron or a synchrocyclotron without using the magnet ramping. The primary benefit of this scheme is that the whole operation cycle can be very high such as 1 kHz as long as enough RF power is provided and RF frequency is modulated fast, that is much easier than the magnet ramping. Fast acceleration increases the fraction of injection and thus the higher average beam current.

A proposal at MURA using the FFA concept did not go further mainly because it was so novel. More than 20 years later, however, an FFA drew an attention as a proton driver of a spallation neutron source. The optimum energy of spallation neutron production is around 1 to 3 GeV, that is above the energy range of a cyclotron, but not high compared with other synchrotrons especially for energy frontier particle physics research. On the other hand, the beam power must be higher than any accelerator previously built. As a spallation neutron source, the beam has to be pulsed. Considering all above, an FFA seemed to be the right choice. The proposal is called ASPUN that was studied at Argonne National Laboratory in the early 1980s' [2]. Unfortunately the project ended with a paper study only.

About 10 years later, another project to use an FFA as a proton driver for a spallation neutron source started. That was a part of European Spallation Source (ESS) project in the early 1990s' [3]. At that time, two different configurations were studied: one is a full energy linac plus an accumulator ring to make a short pulse. The other is a relatively low energy linac plus an FFA accelerator (Fig. 1). Eventually we know a long pulse spallation source with a full energy linac only was built as ESS in Lund.

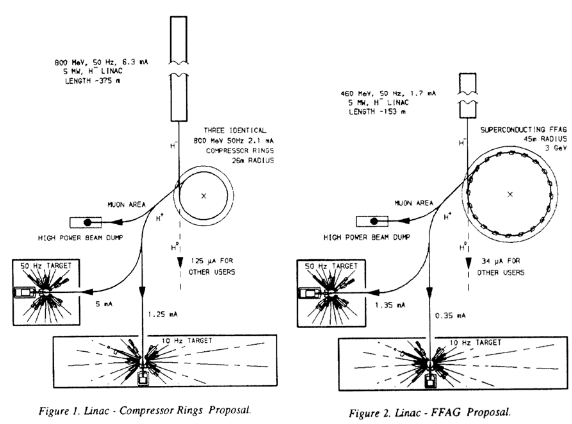

Figure 1: Two proposed schemes as ESS. (left) A full energy linac plus an accumulator ring, (right) medium energy linac plus an FFA [from Ref. [3]].

In the early 2000s', Mori and his teams built a proton FFA with a high repetition rate of 1 kHz [4]. The primary goal was to show that technology was mature enough to build a proton FFA. Computational power of the 3D magnet modelling and a magnetic alloy based RF cavity are some examples which was not available in the MURA days. On the other hand, the direction of the development since

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then was not necessarily toward the high power applications. Another aspect of an FFA, namely quick acceleration, was more emphasised. A couple of FFAs were built for demonstration of muon acceleration [5] and as an arc of ERL [6]. A high power FFA has not been realised.

TOWARD HIGH POWER FFA

Constant Tune

A requirement of frequency modulation in the RF cavity limits the maximum accelerating field gradient. Energy gain per turn is not as high as that of a cyclotron where the whole cycle finishes with a few 100 turns or up to 1000 turns. In a cyclotron, on the other hand, transverse tune moves as the beam is accelerated. At some points, the tune cross a resonance line. The resonance crossing, however, does not affects the beam because the resonance strength is not as strong as that of a strong focusing accelerator and the high energy gain per turn makes the beam cross resonances quickly.

In an accelerator with the RF frequency modulation like Rapid Cycling Synchrotron (RCS), the whole cycle takes the order of 10,000 turns. The transverse tune through the whole cycle has to be constant or at least the tune excursion has to be within major resonance lines. Although this requirement is important for any accelerators with 10,000 turns of revolution, it is more critical for a high beam current accelerator because it has more stringent criterion of the beam loss.

The constant tune was achieved in a synchrotron by ramping magnets synchronised with the beam momentum. In an FFA, the so-called scaling condition is imposed. When the transverse field increases with the power of k in the radial direction, the local field gradient becomes independent of the beam momentum due to the shift of the equilibrium orbit and the steeper increase of the field.

$$B_z(r, \theta) = B_{z0}\left(\frac{r}{r_0}\right)^k F(\theta) \tag{1}$$

In other words, instead of using a variation of time in a synchrotron, an FFA uses a radial displacement which makes the focusing force independent of the momentum. There is other ways to make the tune constant without using the field with the power of k in the radial direction. An FFA exists even without the constant tune. We call all of them a nonscaling FFA in contrast to an FFA which satisfies the scaling condition. There is another FFA where the orbit moves in the vertical direction instead of the radial direction. This is called a vertical excursion FFA. We will not talk about it further.

FD (DF) Spiral Sector

A recent operational experience of SNS and J-PARC suggests that the operating point close to the diagonal line in the transverse tune space gives the least beam loss. That is against our previous findings, that is, the fourth order difference resonance at the diagonal line (2Qx-2Qy=0) is driven by space charge (Montague resonance). The operating point should stay away for the diagonal line. On the other hand, the transverse emittance in SNS and J-PARC is equal in both planes. The difference resonance may be harmless. Although the reason is still not clear why the current choice of the operating point gives the best performance, the importance lesson we learned is that it is essential to have a fine control of the operation point for the beam commissioning.

There has been two kinds of an FFA lattice: one is called a radial sector and the other is a spiral sector. The radial sector FFA uses normal (Bf) and reverse (Bd) bending magnets. Since the geometrical field index k in Eq. (1) is the same for Bf and Bd, Bf gives focusing and Bd does defocusing. By keeping the average circumference same, the strength of Bf and Bd can be adjusted. That changes the edge angle of the equilibrium orbit at Bf and Bd, resulting in the change of (mainly) vertical tune. If the geometrical field index k is also adjustable by trim coils on the magnets, which (mainly) changes horizontal tune, the transverse tune space can be explored. The price we have to pay is the size of the accelerator. Reverse bending magnets increase the circumference with the given maximum momentum.

The spiral sector FFA uses only a normal (Bf) bending, but alternating gradient focusing is achieved by enhanced edge angle due to a spiral shape. A spiral angle of the magnet typically of 30 to 60 degrees introduces focusing in one end and defocusing in the other. The spiral sector design does not increase the circumference. However, it loses flexibility of the operating point since there is only one parameter, that is the geometrical field index k.

FD (DF) spiral sector was invented as a compromise between two conventional FFA lattices [7]. There are normal and reverse bending magnets like a radial sector FFA, but both magnets have the finite spiral angle. The issue of the large circumference of a radial FFA is mitigated by introducing the spiral angle which reduces the strength of Bd to achieve the same tune. If we consider a spiral sector design first, an FD (DF) spiral sector has additional parameter, namely the strength of Bd, to explore the tune space. For example, this novel design successfully covers the tune change of a unit in both direction in Fig. 2.

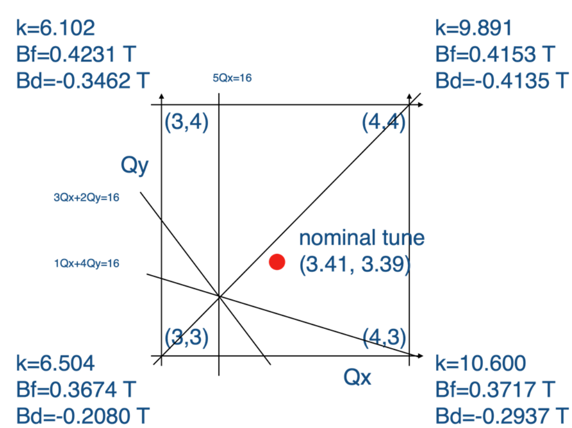

Figure 2: FD (DF) spiral sector design covers the tune from 3 to 4 in both Qx and Qy in the lattice of FETS-FFA [8]. The geometrical field index k and the strength of Bf and Bd are shown to achieve the tune at the corners.

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Superperiodicity

The space charge effect is the most crucial issue in a high current accelerator, which is most significant at the injection energy. Phase space painting with charge exchange injection is a common way to reduce space charge tune shift. The injection system with orbit bump magnets and collection of unstripped H^- beams as well as stripped electrons needs enough space for equipment in the lattice. A scaling FFA lattice is so far comprised of many simple focusing units, thus highly symmetric. It is difficult to provide a long straight section.

As a synchrotron lattice evolved from an initial high symmetric lattice like CERN-PS to the most recent one like SNS accumulator ring with the 4-fold symmetry and J-PARC synchrotrons with the 3-fold symmetry, introduction of superperiodicity in an FFA lattice is the necessary step for an FFA to incorporate the state of the art injection system. A long straight section is also helpful for extraction and other systems. It can be designed by adding lower periodic component in the azimuthal function of $F(\theta)$ in Eq. (1). In practice, it means that the distance between magnets and focusing units are no longer equal. The magnetic strength at Bf and Bd is different depending on the azimuthal location, namely more families of Bf and Bd.

Starting from a highly symmetric lattice with an identical 16 doublet focusing units, a 4-fold symmetry lattice is designed with 8 families of Bf and Bd as an example. Each magnet strength is optimised to flatten the beta functions. Figure 3 shows the footprint of a ring and beta functions.

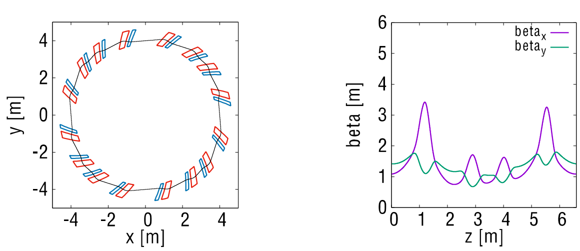

Figure 3: (left) Footprint of the 4-fold symmetry lattice. Red box shows Bf and blue box shows Bd. (right) Horizontal (red) and vertical (blue) beta functions of one superperiod.

Physical and Dynamic Aperture

It sounds rather obvious, but designing large enough physical aperture is the most effective way to reduce space charge effects and minimise the beam loss in a high power accelerator [9]. Both SNS and J-PARC have physical aperture of around 500 pi mm mrad and create beam emittance of 100 to 200 pi mm mrad by phase space painting. With nominal beta functions, physical size of the beam stay clear region is roughly 100 to 200 mm, which is large compared with accelerators for other applications.

In an FFA, the orbit moves as the beam is accelerated. Although the alternating gradient configuration squeezes the excursion, it is still several 0.1 m or up to 1 m. Physical aperture is not an issue horizontally. In the vertical direction, physical aperture depends on design of the magnet. There are two ways to make the field gradient necessary for an FFA. One is to shape the gap inversely proportional to the design field. A gap height becomes narrower at the high energy end. A ratio of the gap at injection and extraction depends on the geometrical field index and the range of beam momentum. The other way is to shape the field by trim coils on the surface of a pole. It is practical to use a flat pole and add trim coils.

The physical beam size shrinks as the beam is accelerated due to adiabatic damping of emittance although the beta functions increase, which are proportional to the radius. The physically shaped gap could still give the enough aperture. On the other hand, it depends on the parameter such as the field index. For the high power FFA, flat pole magnets with trim coils are a better choice in terms of the acceptance control. Flat pole magnets have another benefits. It is easier to control the fringe field extent. More details are discussed in Ref [10].

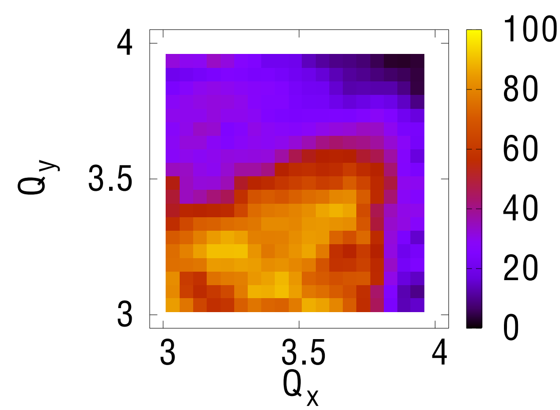

Figure 4: Dynamic aperture of the 4-fold symmetry lattice. It is normalised by beta*gamma. There is a good area between Qx,y=3.0 and 3.5.

Dynamic aperture is a critical issue. Because of the magnetic field profile: Eq. (1), an FFA has all order of multipoles intrinsically. Systematic nonlinear resonances are a concern. This becomes more serious in a superperiod structure. There is a strong amplitude dependent tune shift primarily due to octupoles. By a tracking study, we found that dynamic aperture depends on the spiral angle, lattice geometry as well as the operating point. For example, dynamic aperture depending on the tune space is depicted in Fig. 4. Because of the 4-fold symmetry, the integer line of 4 in both planes is a strong systematic resonance of fourth order and above. We found that there is a region below Qx,y=3.5 where the dynamic aperture normalised by beta*gamma is almost 100 pi mm mrad at 3 MeV, which corresponds to physical aperture of more than 1000 pi mm mrad, twice of SNS and J-PARC design value.

Collimation System

Since it is impossible to completely eliminate beam loss, localising the beam loss by the collimation system is a common practice. In the horizontal direction, the beam moves as it is accelerated. One collimator is located on the

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inner side of the aperture which works at the injection energy. Another collimator can be also located on the outer side of the aperture which works at the extraction energy. In the vertical direction, a continuous collimation during acceleration is possible. A horizontal position of the equilibrium orbit is a function of the beam momentum. The vertical beam height including the adiabatic damping effects can be calculated as a function of the horizontal coordinate. Continuous or stepwise jaws will be located. Compared with the collimation system for a synchrotron where the beam size shrinks and there is no way to collimate the beam except at the injection energy, the collimator system in an FFA can shape the beam well before extraction.

Whether it will be one or two stage collimator depends on the beam energy. For a few MeV FFA, protons stop at the first collimator. For a few GeV proton driver FFA, two stage collimator is necessary to collect large amplitude particles efficiently.

Beam Stacking

In a fixed field accelerator, the beams with any momentum from injection to extraction can circulate simultaneously. This makes it possible to create high peak current at extraction energy by beam stacking. Beam stacking involves the following process.

1) Inject and accelerate the beam to the extraction energy.

2) Adiabatically debunch the beam and keep it as a coasting beam circulating at the orbit corresponding the extraction energy. Longitudinal emittance should be preserved as much as possible.

3) Inject and accelerate the second beam to the energy slightly below the extraction energy.

4) Adiabatically debunch the beam and create another coasting beam below the first coasting beam in the longitudinal phase space. The empty area between first and second coasting beams in the longitudinal phase space should be minimised.

5) Repeat the same process of 3) and 4) until the design current is obtained at the extraction energy.

6) Adiabatically create a stationary RF bucket or a barrier bucket and capture all the beams at the extraction energy.

7) When an empty gap in the time axis is created, excite extraction kicker magnets and extract all the beams in a single turn.

Beam stacking gives a nice interface between accelerators and users who want a high peak current with a low repetition rate. For example, users will see the beam in 30 Hz from the accelerator while the accelerator itself operates in 120 Hz with beam stacking of 4 bunches. From an accelerator point of view, it is easier to increase the repetition and reduce the bunch current. It may not be possible to accelerate 4 times larger bunch current in 30 Hz because a space charge tune shift is too high at injection. By beam stacking, the beam current is not limited at injection. In theory, the number of the beam to be stacked is only limited by the space charge effect at the extraction energy.

Beam stacking was demonstrated at MURA using an electron FFA [1]. However, it was not the demonstration of the full energy acceleration in an FFA and controlled stacking at the top energy. Using the FFA at Kyoto University, Institute for Integrated Radiation and Nuclear Science (KURNS), we did the beam stacking experiment. The beam was injected at 11 MeV and accelerated up to about 35 MeV. After about 33 ms later, the second beam was injected and accelerated up to slightly lower energy of the first beam. Although the beam current was not high enough to see space charge effects, our goal was to demonstrate beam stacking under strict control. The first measurement was the momentum spread after stacking. Ideally, the total momentum spread should be twice of the momentum spread of a single beam. The second measurement was the beam current after stacking. Ideally, the total current should be twice of the current of a single beam.

To measure the momentum spread and the beam current of the coasting beam after stacking, Schottky signal was analysed. Full Aperture Bunch (FAB) monitor signal is Fourier analysed. A frequency domain signal is averaged with Welch method and converted to Power Spectrum Density (PSD) as shown in Fig. 5. Since Schottky signal of a coasting beam reflects incoherent beam oscillations, the width of the PSD signal in frequency domain shows the momentum spread through

$$\frac{dp}{p} = \frac{1}{h\eta}\frac{df}{f} \tag{2}$$

and the area P is proportional to the beam current N.

$$\int \left(\frac{dP}{df}\right) df = 2Z_t e^2 f_0^2 \int \left(\frac{dN}{df}\right) df \tag{3}$$

Preliminary results shows that the total momentum spread is about twice as much after stacking of two beams. The beam current of the first beam is, however, significantly lowered than the second beam. Since the beam life time measurement shows the e-fold life time is about 10 times longer than the time interval of the first and second beam injection, it cannot explain the large difference of the first and second beam current after beam stacking.

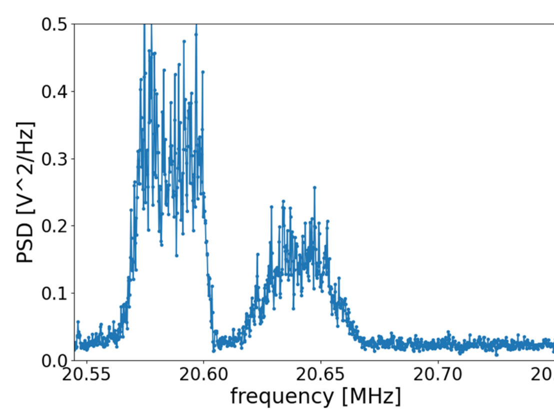

Figure 5: Schottky signal example. The peak signal on the left shows the second beam and the right shows the first beam. High frequency means high momentum.

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One possible explanation of the beam loss is the RF knock out, which was investigated at MURA. The RF voltage to accelerate the second beam does not make a cumulative effect to the coasting beam at the extraction energy because the revolution frequency of the coasting beam and the RF frequency is slightly different. There is no net interference in the longitudinal phase space. In the horizontal phase space, the equilibrium orbit of the coasting beam changes suddenly at the location of an RF cavity due to the finite dispersion function. In a scaling FFA, the dispersion function is $r/(k+1)$ everywhere and is not be zero. This change of the equilibrium orbit occurs with the frequency of the RF or the frequency difference between the RF and the revolution frequency (a beat frequency). Since the RF frequency increases below the revolution frequency to almost the revolution frequency (assuming the RF harmonic number is 1) during the acceleration of the second beam, at some point it could match to the horizontal betatron frequency.

$$\frac{\omega_{rev} - \omega_{rf}}{\omega_{rev}} = \frac{\omega_{\beta,h}}{\omega_{rev}} \quad \text{or} \quad 1 - \frac{\omega_{\beta,h}}{\omega_{rev}} \tag{4}$$

The resonance occurs and the beam is kicked horizontally. Figure 6 shows the simulation result using the parameter of the KURNS FFA.

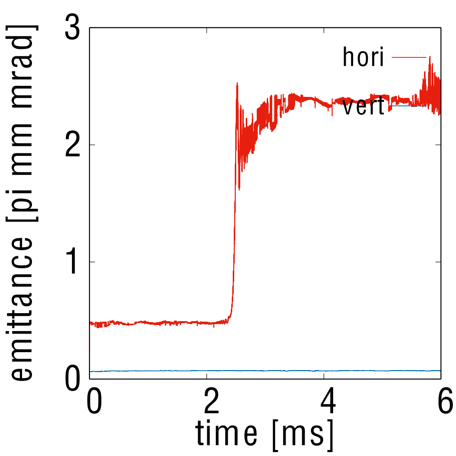

Figure 6: Simulation of the RF knock out. At about 2.3 ms, the resonance condition of Eq. (4) is satisfied and horizontal emittance blows up rapidly. There is no effect in vertical.

Although we did not measure the horizontal beam size, momentum spread measurement shows that of the first and second beams are almost equal after the beam loss of the first beam occurs. This agrees with the RF knock out prediction, which is purely a horizontal resonance. Study of mitigation is the next step. A local cancellation of the RF knock out between two RF cavities with a horizontal phase advance of pi is one method. With the more number of RF cavities, a local cancellation can be done with combination of horizontal phase advance among the cavities, not necessarily pi. A further study is needed.

MODELLING OF HIGH POWER BEAMS

Since the beta function and beam emittance of an FFA have a similar value to that of a synchrotron and the RF frequency is also in the similar range: a few MHz, the transverse beam size and the longitudinal bunch length are similar to that of a synchrotron. Modelling techniques of the high power beams, for example, 2D and 2.5D space charge models developed for the high power beams in a synchrotron is applicable. Note that it is different from the beams in a cyclotron where an aspect ratio of the transverse and longitudinal size is closer to 1 because of the high RF frequency, ~ 50 MHz. The 3D ellipsoidal approximation as a beam shape is relevant.

A big difference from a synchrotron in a modelling point of view is, however, non-existence of a closed orbit. In theory, the equilibrium orbit radius can be calculated locally for each turn when the synchronous momentum is given by the RF voltage and frequency programme. It is, however, not practical to do so, especially for the scalloped orbit with reverse bending magnets.

In order to model the long bunch beam in an FFA, we take the following steps.

1) Track many macro particles with time as the independent variable. At each time step, a snapshot of the macro particle distribution in the 3D configuration space is obtained.
2) Translate the coordinate system from Cartesian to cylindrical system. The ring centre of an accelerator is the origin of the coordinate.
3) Do binning along the azimuthal angle coordinate. Typically one bin has a width of 1 degree or less.
4) Calculate the average azimuthal and radial position of all the particles within each bin. This gives the azimuthal and radial position of an approximate closed orbit with respect to the local coordinate system.
$$(\theta_{av}, r_{av}) = \left(\frac{\sum \theta_i}{n}, \frac{\sum r_i}{n}\right)$$
5) Calculate the average azimuthal and radial momenta of all the particles within each bin. This gives the angle ψ of an approximate closed orbit with respect to the local coordinate system.
$$\tan(\psi) = \frac{\sum p_{r,i}}{\sum p_{\theta,i}}$$
6) Within each bin, the average position and the angle of an approximate closed orbit is subtracted from each macro particle coordinate.
7) Now all the macro particles are aligned with a long straight line.
8) Transverse and longitudinal space charge forces are calculated for this long straight bunch and the 3D momenta of all the macro particles are updated accordingly.
9) Translate the coordinate of each macro particle in the reverse order.
10) Update the 3D positions within a time step.

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Figure 7 shows macro particles and the beam size defined by the beta function and emittance. Using the coasting beam, the rms emittance evolution is calculated with the different initial number of particles.

Simulation parameters are shown in Table 1. Since the distance to the nearby systematice resonance (4Qx,y=12) is 3.26-3.00=0.26, the emittance increase beyond the number of particles of (1~2) x 10^{12} is reasonable because coherent tune shift with 1 x 10^{12} is -0.23 (Fig. 8).

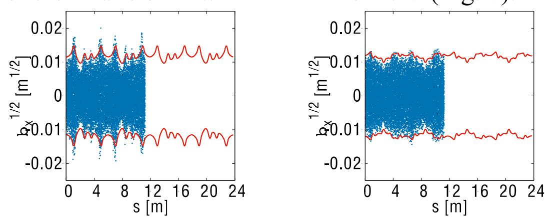

Figure 7: Macro particles after coordinate translations (horizontal only) and the beam size defined by the beta function and emittance. (left) Horizontal, (right) vertical.

Table 1: Simulation Parameters

Parameters	Value	Unit
Lattice	FETS-FFA	
Circumference	~ 23	m
Energy	3	MeV
Longitudinal	Coasting	
Transverse	KV	
Emittance (100%)	10	pi mm mrad
Injection	Single turn	
Operating point	(3.26, 3.26)	

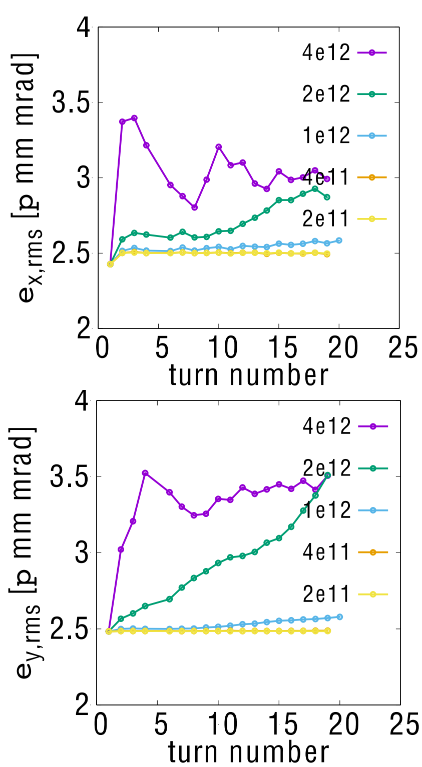

Figure 8: RMS emittance evolution vs the initial number of particles. (top) Horizontal, (bottom) vertical.

SUSTAINABILITY

Sustainability becomes the most important keyword nowadays when we consider a future accelerator based facility. Although sustainability has many aspects, an FFA has a certain advantage in terms of energy efficiency and thus a cost of operation. The main part of an electricity bill comes from excitation of the main magnets. In an RCS, for example, it is more than a half because of the AC magnets. A DC magnet based FFA could use superconducting technology. Even with normal conducting magnets, the experience at PSI shows its advantage of a cyclotron over an RCS [11]. This area of study should be pursued further.

SUMMARY

MURA originally promoted the idea of an FFA as an accelerator for high beam current with higher beam energy than a cyclotron. Since the end of MURA, an FFA as a proton driver for a spallation neutron source was proposed a few times at different places. Nothing, however, went further than a paper study. In the early 2000s', Mori and his teams built a proton FFA with a high repetition rate of 1 kHz. This was a milestone as a revival of an FFA. On the other hand, the developments of FFA have not been toward high power applications. No high power FFA was built. We focused on the realisation of a high power FFA for the last several years. Based on the original FFA optics, several advanced concepts and its design have been made. We discussed FD (DF) spiral sector, superperiod structure, large dynamic aperture, collimation system and beam stacking at the extraction energy. We also developed modelling tools of the high power beams in an FFA.

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